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ELEMENTS.

I   N U S.

ε GREEK.

he PLATONIC
A S, &c.

Y   L   O   R.

συνίςηκι.
PYTHAGORAS.

THE

Philofophical and Mathematical

COMMENTARIES

OF

PROCLUS;

SURNAMED,

PLATO'S SUCCESSOR,

ON THE

FIRST BOOK OF EUCLID'S ELEMENTS.

AND HIS

LIFE BY MARINUS.

TRANSLATED FROM THE GREEK.

WITH

A PRELIMINARY DISSERTATION on the PLATONIC
DOCTRINE of IDEAS, &c.

By THOMAS TAYLOR.

'Επὶ τὸ γνῶναί τι κ̓ Διωρῆσαι πᾶς ἄνθρωπۻ ὑπὸ τῦ Θεῦ συνίστηκι.

PYTHAGORAS.

A little learning is a dangerous thing,
Drink deep, or tafte not the PLATONIC fpring:
There fhallow draughts intoxicate the brain,
And drinking largely, fobers us again.

VOLUME I.

TO

THE SACRED

MAJESTY

OF

TRUTH.
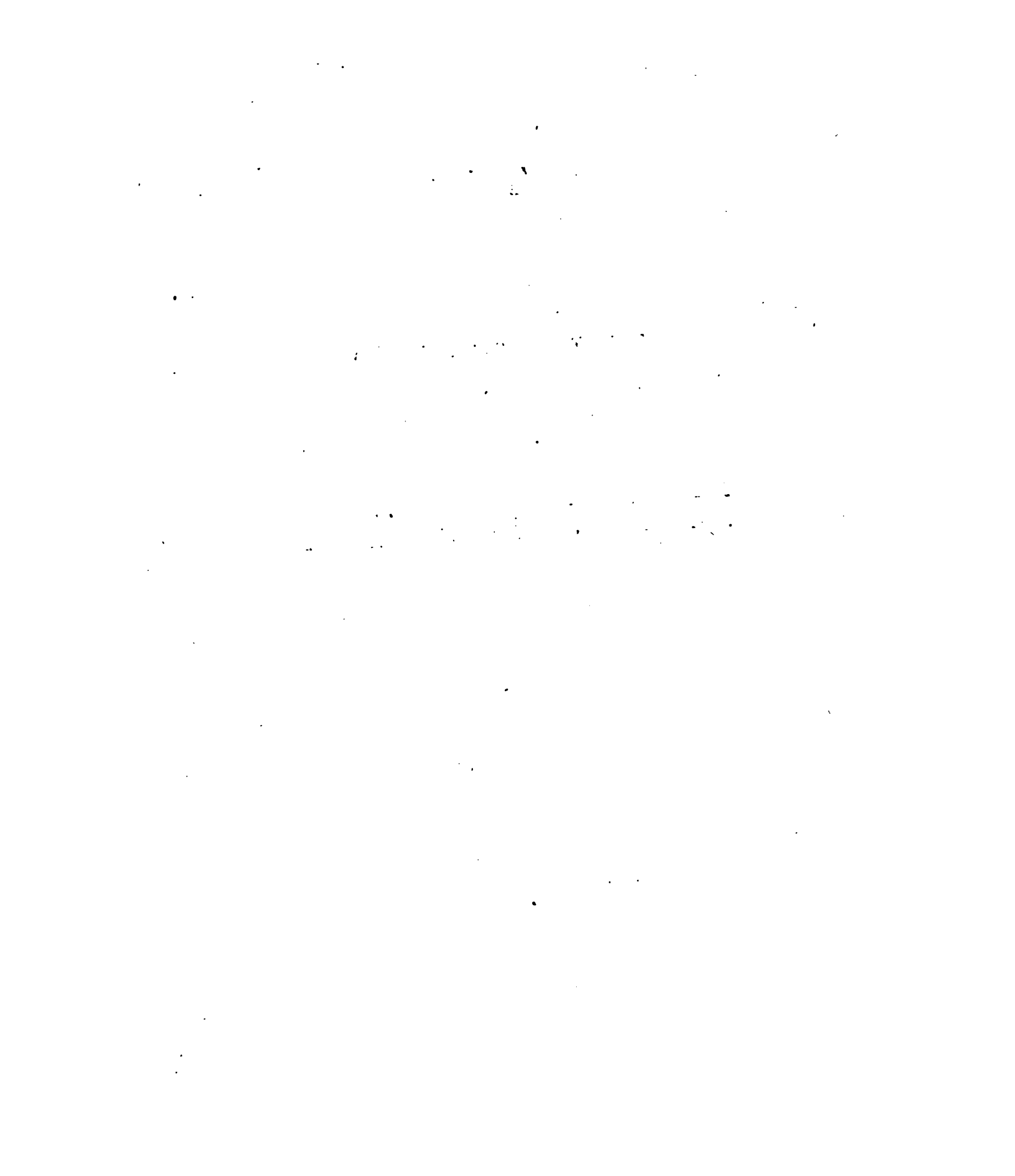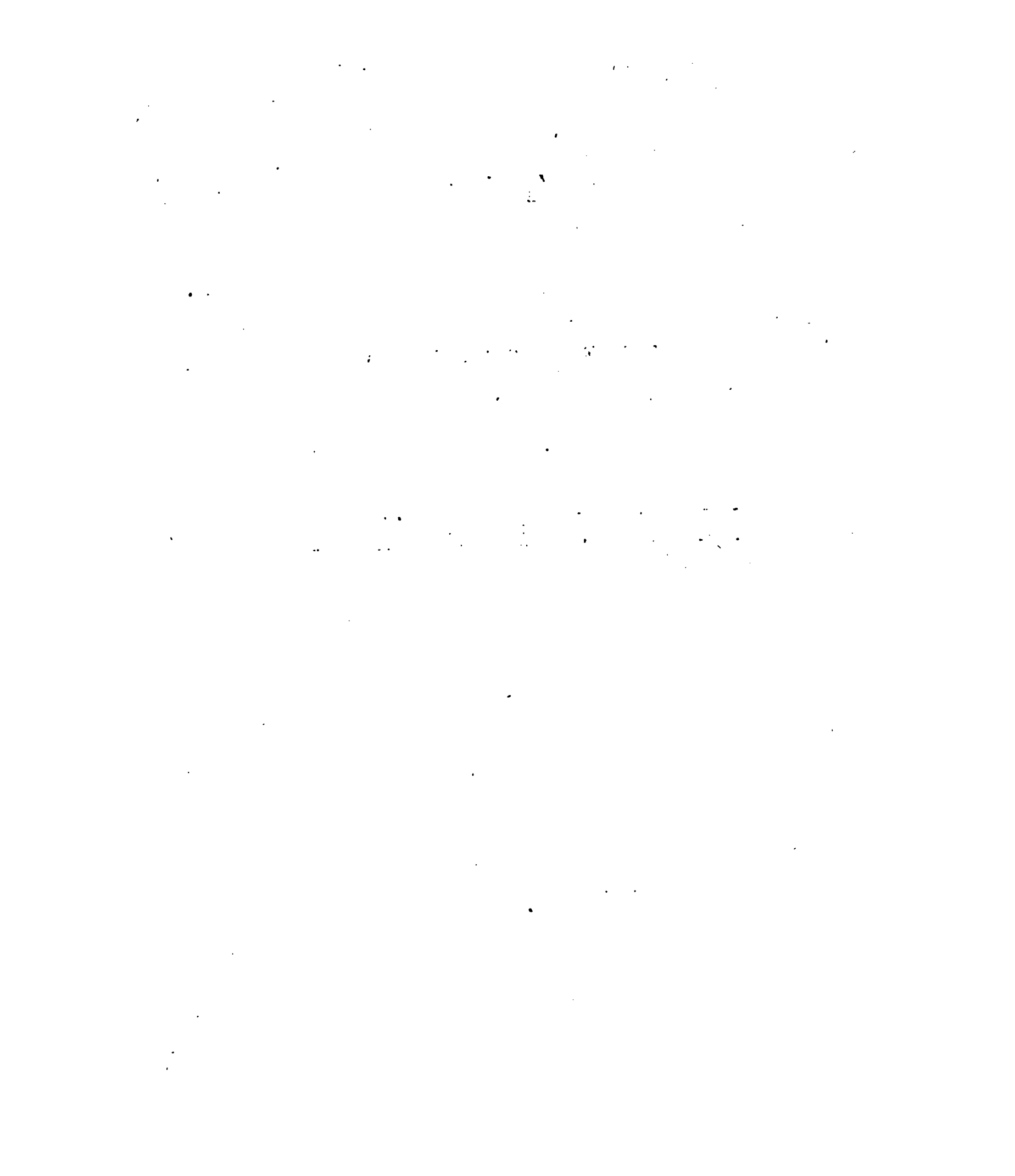
PREFACE.

THE defign of the prefent work is to bring us acquainted with the nature and end of Mathematics in general, and of Geometry in particular : and in the execution of this defign our Author has difplayed an uncommon elegance of compofition, and a moft valuable ftore of recondite learning. He is not content with every where unfolding the full, and moft accurate méaning of Euclid; but he continually rifes in his difcourfe, and leads us into the depths of the Pythagoric and Platonic philofophy. We are furprifed to find an ufe in Geometry, which at prefent it is by no means fufpeƈed to afford. For who would conceive that it is the genuine paffage to true theology, and the veftibule of divinity? This, indeed, is by no means the cafe when it is ftudied for lucre, and applied to mechanical purpofes; for then the foul is neither elevated nor enlightened, but degraded and filled with material darknefs. Hence thefe Commentaries are alone valuable to the liberal part of mankind, who look beyond *fenfe* for certainty; and who prefer things defirable for their own fakes, before fuch as minifter to the neceffities of life.

P R E F A C E.

THE tranflation of this work is attended with great difficulty and labour; not only from the fenfe of the philofopher, which is always profound, and frequently obfcure, but from the great incorrectnefs of the Greek edition, in which, exclufive of numberlefs typographical errors, entire fentences, effential to the connection, are frequently omitted; and in one place two pages of the Latin tranflation are wanting in the original, as will be fhewn in our following notes. Indeed, the Latin tranflation of Francis Barocius the Venetian, (Patav. 1560.) which was made from a variety of manufcripts, is inconceivably valuable; for the diagrams, fo neceffary to a work of this kind, but which are omitted in the Greek, are here inferted; and the verfion is every where faithful, and fufficiently perfpicuous to thofe who are converfant in the ancient philofophy. Barocius juftly cautions the reader not to compare his verfion with the printed Greek, which he obferves is rather lacerated than printed; as indeed, without his tranflation, it is impoffible for any one to read the half of this invaluable work, even though he fhould be as perfect in Greek as in his native tongue. If I had not, therefore, fortunately acquired this tranflation, which is at prefent very rare, I would have by no means engaged in this arduous undertaking. Barocius, indeed, gives evident proofs of his poffeffing the philofophical genius, by the excellence of his tranflation, and his preface to the reader; and it is

greatly

greatly to be lamented that he did not adorn his verſion with explanatory notes, which this profound work frequently demands, and which he was doubtleſs well qualified to accompliſh. This defect I have endeavoured, as far as I was able, to ſupply; and at the ſame time have been cautious neither to weary the reader by prolixity, nor by too much brevity to leave him deſtitute of proper information. In the diſtribution of the firſt book of this work into chapters, I have followed the order of Barocius, becauſe it is natural and obvious; and muſt beg leave to ſolicit the reader's indulgence for uſing the words *partible* and *impartible*, differently from their common ſignification. Theſe words I have generally employed to expreſs the meaning of μεριϛος *, and αμεριϛὸς † in the Greek, as I do not conceive that the words *diviſible* and *indiviſible* always convey their full ſignification. I have likewiſe uſed *quadrangle* inſtead of ſquare, and *quinquangle* for the word pentagon. For if τρίγωνος be rendered *triangle*, why ſhould not τέτραγώνος be rendered *quadrangle?* And, as Barocius obſerves, why, for a ſimilar reaſon, ſhould not πεν]άγωνος and ἑξάγωνος be rendered *quinquangle* and *ſexangle*; and ſo of the reſt? Uniformity is always deſirable when it can be obtained; and is no where ſo neceſſary as in ſcientifical diſquiſitions.

* i. e. Capable of parts.   † i. e. Not capable of parts.

PREFACE.

IT is likewife neceffary to inform the reader, that though I have always endeavoured to give the faithful meaning of my Author, yet I have occafionally paraphrafed his fenfe, when moft obfcure, and added fuch elucidations of my own, as I either thought neceffary to the full comprehenfion of his matter; or which were naturally excited by the fire and fpirit of the Original. If it fhall appear that I have fucceeded in the execution of this work, and rendered it intelligible to the lovers of truth, I fhall rejoice in my fuccefs, and confider my labours fufficiently rewarded. The applaufe of the multitude I am neither likely nor defirous to gain; but I am anxious to procure the approbation of the difcerning few, who know that the age of philofophy is paft; and who efteem the works of her ancient heroes as the moft precious treafures which have efcaped the ravages of time.

TIME, indeed, is like a deep and rapid river; whatever is trifling and light, is precipitately borne on its furface, and what is valuable and weighty, finks to its bottom. Hence, the fuperficial obferver collects nothing more than the rubbifh, which it is forever devolving into the abyfs of oblivion; while the profound and contemplative genius explores the depths of the ftream, and accounts himfelf happy if he can gather any of the pearl which its bottom contains. Thus the difcoveries of experimental philofophy,

float

float like ftraws on the furface, while the wifdom of Pythagoras and Plato lies concealed in the depths of the river. I am well aware it will be faid, that the reverfe of this fimilitude is true; that the modern philofophy is the pearl, and the ancient the ftubble; and that the former will be celebrated by pofterity, and increafe in reputation when the latter fhall fcarcely be known. But let us attentively examine the truth of this affertion, and fhut our ears to the unfubftantial echoes of popular applaufe. Is it reafonable to fuppofe that men of fuch exalted abilities, as the Pythagoric and Platonic philofophers pöffeffed, even in the eftimation of their opponents, accompanied with the greateft advantages of birth and fortune, and the moft unwearied attention, have difcovered nothing valuable, and have left nothing behind them, but jargon and reveries? Is it to be fuppofed, that in an age when philofophy was almoft adored; when it was efteemed by kings, cultivated by noblemen, and even reverenced by the vulgar; when empire was relinquifhed for its purfuit, and every danger encountered for its poffeffion: is it to be fuppofed, that nothing but delufion was the offspring of fo glorious a period, and nothing but folly the reward of fuch generous endurance? Or fhall we fay, that the difcovery of truth was referved for the age of experiment; and that fhe is alone to be apprehended in the infinite labyrinth of particulars? That fhe is to be inveftigated with the corporeal

fenfes

fenfes, and not with the powers of intellect; and that the crucible, the alembic, and the air-pump, are the only mediums of detection? If this be the cafe, truth is material, and may be calcined, diftilled, and rarefied, like any other corporeal fubftance. It is no longer eternal and immutable, but perifhable and fluctuating; the phantaftic fubject of fenfible infpection, and not the fteady and real object of the permanent energies of fcience. Shall we call this the age of philofophy, in which talents are proftituted for fuftenance, and learning fubmits to the impudence of wealth? Shall we fay that we have ftrengthened the caufe of philofophy, by demolifhing her fchools; and increafed her independence, by enlarging the empire of commerce? Where fhall we find the man, who is at prefent reverenced for the profeffion of teaching fpeculative truth, or indeed who teaches it at all? Or fhould we chance to meet with fuch an obfolete character, fhall we find him fupported by the profeffion? It is a well known fact, that men formerly lived in the higheft efteem by its propagation: it is equally as notorious, that a man at prefent would ftarve by fuch an attempt. Dare we affert, that the reafon of this difference muft be afcribed to the greater liberality, and more philofophical fpirit of the prefent age? Shall we not rather fay, that the period, in which thefe ancient heroes lived, was the golden age of philofophy;—a period fo different from the prefent, as to appear fabulous on the comparifon? For mark the

dif-

diftinguifhing characteriftics of our inferiority. The great object of ancient philofophy, was an accurate fpeculation of principles and caufes: but that of the modern, is a confufed inveftigation of effects. And if purfuits participate of the nature of their fubjects, and caufes are more noble than effects, the ancient philofophy muft undoubtedly be more elevated than the modern. Again, the object of the Pythagorean and Platonic philofophy was to make its poffeffors wife and virtuous; and to elevate them above the common frailties and imperfections of degraded humanity; and this end was happily accomplifhed in its votaries, as their lives abundantly evince: but the object of modern philofophy, is a promotion of the conveniencies and refinements of life, by enlarging the boundaries of traffic; and the Mathematical Sciences are ftudied folely with a view to this enlargement. The defign of the ancient philofophy was to remove the caufes of wonder, by contemplating effects in their caufes: the grand object of the modern, is to increafe admiration, by attempting to inveftigate.caufes through the infinity of particular effects. So that philofophy, as Mr. Harris juftly obferves, now ends where it formerly began. For either there is no fuch thing as fcience, or if its exiftence be admitted, it can never be obtained by experimental enquiries; as thefe muft be liable to all the inaccuracy and imperfection of their material fubjects.

In

PREFACE.

In short, the philosophy of Pythagoras and Plato will be found, when impartially considered, to contain every thing which can enlighten the mind, improve the morals, and exalt the character of man. It is built on the steady basis of truth, and will survive the wreck of ages. Its foundation is deep, and its summit reaches the heavens. It is a mighty rock, which modern systems *may* assail, like a raging sea; but, like stormy waves, they will only be broken about its impenetrable sides. To war against wisdom is folly; for opposition in this case is the destruction of its author. The moderns may, indeed, expect, because their merit is raised by the present age, above that of the ancients, to appear as giants in the eyes of posterity; but they will only verify the elegant observation of the poet *, that

Pygmies are Pygmies still, though perch'd on Alps,
And Pyramids are Pyramids in vales.

* Dr. Young, in his Night Thoughts.

A DIS-

A

DISSERTATION

ON THE

PLATONIC DOCTRINE of IDEAS, &c.

SECTION I.

THE Platonic doctrine of Ideas has been, in all ages, the derision of the vulgar, and the admiration of the wife. Indeed, if we confider that ideas are the moft fublime objects of fpeculation, and that their nature is no lefs bright in itfelf, than difficult to inveftigate, this oppofition in the conduct of mankind will be natural and neceffary; for, from our connection with a material nature, our intellectual eye, previous to the irradiations of fcience, is as ill adapted to objects the moft fplendid of all, "as the eyes of bats to the light of day *." And yet (as I prefume, it will appear from the following difcourfe), unlefs the exiftence of thefe lucid beings is admitted, there can be no fuch thing as fcience; nor, indeed, any genuine knowledge at all. Hence, an enquiry concerning their nature and reality, is highly proper, as an introduction to the enfuing Commentaries, in which they are confidered as the ftable pillars of all truth, and the prolific principles of the univerfe.

* See book the fecond, of Ariftotle's Metaphyfics.

But previous to this enquiry, it is proper to obferve, that Plato was not the inventor, though he was a ftrenuous af-ferter, of ideas; for, in the Sophifta he affirms, that ideas were the difcovery of men who excelled in wifdom and piety, and who contended for an invilible effence. Diogenes La-ertius, indeed, afferts, that Plato received the doctrine of ideas from Epicharmus. But Epicharmus was not their inventor, becaufe Pythagoras, and others of ftill higher an-tiquity, were well acquainted with ideas; fo that it may be affirmed, with much greater truth, that Plato was inftruct-ed in their nature by Philolaus his preceptor, and the dif-ciple of Pythagoras. For Pythagoras, after his myfterious manner, fignified ideas by numbers. But, prior to Pytha-goras, Orpheus was an afferter of ideas, and called Jupiter, or the demiurgus of the world, " the idea of all things." And, according to Syrianus, the mundane fphere, celebrated by Empedocles, is no other than the ideal world; fo that the doctrine of ideas is as ancient as that of wifdom itfelf.

But to begin with our enquiry: in the firft place, with-out univerfals there can be no fcience; for the flowing and perifhing nature of particulars is perfectly foreign from that ftability and duration which is requifite to objects of invariable truth. Neither is it poffible, that infinite indi-viduals can exift without the fubfiftence of one caufe endued with infinite power; for all multitude muft neceffarily ori-ginate from one, and muft refemble its caufe in as great a degree of perfection as its nature can admit; by a diffufed infinity, fhadowing forth that infinite power which fubfifts in indivifible union. Hence, if this be the cafe, and if infinite men, horfes, and a multitude of other univocals, are produced in an infinite time, an unity of infinite power muft be the fource of each, according to which they are

generated:

generated in a terminated manner to infinity in the universe. Again, all animals are tranfmuted from that which is in capacity (i. e. feed), into energy. But if this be true, it is requifite there fhould be fome animal in the univerfe, fubfifting in ever-vital energy, which may call forth that which is concealed in dormant capacity, into perfect actuality. Thirdly, the celeftial orbs would not perpetually revolve in the fame fpaces, and after the fame manner, unlefs one and the fame univerfal number, or idea, ruled in each. So, likewife, there is a natural number in every animal; or thofe of the fame fpecies, would not always (when perfect) be diftinguifhed with the fame invariable organs; nor would they be fubject to puberty and old age, at the fame time, unlefs they were detained by the fame meafure of nature. Befides, the participation of univerfals, is evident in every fenfible object. Thus, the rational nature is united with every individual man. Thus, animal fubfifts in a lion and a horfe, in a man and a dog. And thus the pentad, or number five, is participated in the five fingers, and the duad in the noftrils, eyes, hands, and feet. But fince thefe do not fubfift without a caufe, but are perfected by certain determinate natures, it is neceffary there fhould be an univerfal animal, in the whole of nature, feparate from fenfibles, by means of which this fenfible animal is generated. And that there fhould fubfift in nature a pentad, through which the hands are always adorned with that number of extremities; and a duad, from which the two eyes and noftrils are derived. But if nature does not poffefs thefe numbers from herfelf, as fhe is not the firft caufe of all, but derives them from another caufe, in the fame manner as matter from nature, it is neceffary there fhould be univerfals and numbers prior to nature, fubfifting in far greater purity and perfection.

2. Again,

2. Again, we may demonftrate the exiftence of ideas as follows: if the Deity, in fabricating the univerfe, operated effentially (and there is no other way in which we can conceive him to operate), he muft fabricate the univerfe, an image of himfelf. But, if this be the cafe, he contains in himfelf, in the manner of an exemplar, the caufes of the univerfe; and thefe caufes are no other than ideas. Befides, this confideration is not to be admitted, that the perfect muft neceffarily antecede and prefide over the imperfect; unity over multitude; the impartible over the partible; and that which is perpetually the fame, over that which admits of variation and change. From whence it may be inferred, that things do not originate from bafer natures, but that their gradual proceffions end in thefe; and that they begin from the moft perfect, beft, and moft beautiful natures. But let us purfue this reafoning more minutely, as it affords the ftrongeft arguments for the exiftence of ideas.

When the Deity fabricated the various fpecies of animals, and beftowed on them the different fenfes, it was doubtlefs with a view to the benefit of their poffeffors, as he forefaw, that without thefe, the animal could neither provide for its own fupport, nor defend itfelf from furrounding dangers. But may we not enquire from whence this previous perception originated? For it is not to be fuppofed, that he firft made animals deftitute of fenfes, and fo, being admonifhed by their fudden deftruction, afterwards affigned them to their nature. Shall we fay, this fore-knowledge was the refult of a reafoning procefs? But then, we again afk, What were the principles of this ratiocination? For if they originated from other reafonings, it is neceffary, at length, to arrive at fomething prior to thefe difcurfive operations, on which they ultimately depend; fince all reafoning muft

be

be founded on indemonſtrable principles. Was ſenſe, then, or intellect the principle of this previous perception ? But ſenſe, in the preſent inſtance, had not then a being, for it could not exiſt prior to the animal nature : it was, therefore, intellect. But if intellect be the repoſitory of certain propoſitions, and the concluſion be ſcience, it muſt follow, that there could not then be a conſultation of any thing ſenſible. For the principle and the concluſion muſt both depend on ſomething intelligible. Beſides, may we not aſk, how ſuch a habit of thought aroſe before the exiſtence of a ſenſible nature ! It is abſurd in the extreme, to ſay from chance; and to reſolve it into a ſudden volition of the Deity, is an aſſertion that may, indeed, ſatisfy vulgar minds, but can by no means quiet the reſtleſs ſpirit of phi-loſophical inveſtigation. Since, to ſuppoſe the cauſe of the univerſe, actuated by ſudden volitions, is to place him on a level with the vileſt natures, and ſubject him to the irrational impulſes of the brute. Hence we infer that the formation of animals, and by the ſame arguments of the world, was not the reſult of any reaſoning proceſs. For, indeed, argument and foreknowledge cannot with propriety be attributed to the Deity ; but when they are aſcribed to him, we muſt conſider it as nothing more than an indication of his conſtituting particulars, in a manner ſomewhat ſimilar to the providence of a wiſe man, in inferior concerns. For, in ſubordinate natures, whoſe operations cannot take effect prior to enquiry, reaſon is neceſſary, on account of the inferiority of that power which precedes the reaſoning energy. In like manner, foreknowledge is ne-ceſſary, becauſe a power is wanting to its poſſeſſor, which might render him ſuperior to its uſe. For foreknowledge is directed to this end, that one particular circumſtance may take place in preference to another. But if it be requiſite

that

that every energy in the Deity fhould be void of defect, and if it is not lawful that any thing fhould be prefent with him, which is not total and univerfal, it is neceffary that all things fhould be contained in every thing effential to the nature of the Deity.   Hence, fince even futurity is with him prefent, there is nothing in him pofterior; but what is prefent in him becomes pofterior, by its participation in another.   If then futurity be prefent with the Deity, it is neceffary it fhould be fo prefent, as if foreknown in a pofterior nature ; that is, in fuch a manner that nothing may be wanting to any being; and that is, laftly, fo that every thing may be complete.

Befides, reafoning cannot, by any means, belong to an eternal effence like the deity; for if this be admitted, he muft be forgetful of his former operations.   And if, in confequence of reafoning, he produces more perfect natures afterwards, his works could not be perfectly beautiful before : but if they were beautiful before, they muft be co-exiftent with their caufe, i. e. they muft be eternally beautiful, antecedent to the reafoning energy.   Again, if we fuppofe the fupreme intellect, the demiurgus of the world, to operate by enquiry, his energy could not be fpontaneous, and truly his own; but his effence would be fimilar to that of the artificer, who does not derive his productions from himfelf, but procures them as fomething adventitious by learning and enquiry.   But if the univerfe was not formed by deliberation, it muft be co-exiftent with its caufe, and refide in his effence; for if it be not co-exiftent there muft have been fome particular time, in which its artificer determined on its production; and this determination muft have been the refult of a reafoning procefs, concluding that it would not be good to produce it before that particular time, (from whence, by the way, we infer the

eternity

eternity of the world.)   And if the univerfe be cô-exiftent with its author, it muft perpetually emanate from his nature, and be dependent on it, like the fhadow on its forming fubftance.   But in this cafe, its archetype muft be contained in the effence of its author; for every caufe is that primarily, which its effect is fecondarily.   And hence we infer, that if the fenfible univerfe be replete with forms of every kind, the exemplars of thofe forms, muft fubfift in immaterial perfection, in the artificer of the world.

Ifthis fenfible world, then, be formed according to the exemplar of that which is intelligible; may we not fay, with the great Plotinus, that it is requifite univerfal animal fhould there primarily fubfift in perfect vital energy, containing all things in its omniform effence.   " Hence (fays he *) the heavens are there a divine animal, replete with ideal ftars.   Earth too does not there fubfift folitary, but is much more vital than this corporeal earth, for it is full of intellectual life.   The fea too is there, and all water fubfifting in life, and an ever-abiding ftream.   For how is it poffible that any thing not vital, can be the progeny of life itfelf? He, therefore, who enquires from whence animals originate in the intelligible world, might as well enquire from whence all life, and foul, and univerfal intellect, arofe.   For here there is nothing indigent nor defective, but every thing is perfect and exuberant.   Here they all flow from one fountain, not as from a certain fpirit, or heat, but as if from an univerfal quality, poffeffing and preferving in itfelf, all qualities; fuch as fweetnefs, accompanied with fragrance of fmell, the vigour of wine, and the ftrength of all juices, bright colours, and whatever is perceived by the tafte."

* Ennead vi. lib. vii.

3. Such

3. Such then are the arguments which the Platonic phi-
lofophy affords in defence of ideas; the exiftence of which
was fo evident to Plato, that, in the Sophifta, he compares
thofe who oppofe the friends of ideas to the giants of old,
warring, as it were, on celeftial fouls, and fuch as are engag-
ed in fublime inveftigations. Let us now confider to what
univerfals thefe lucid beings are confined; fince, according
to the Pythagoreans and Platonifts, there are not ideas of
all univerfal conceptions. " For, in the firft place (fays Sy-
rianus *), there are no ideas of things evil and bafe, becaufe
thefe fubfift in nature rather by a privation and abfence of
ideas. And, on this account, they are faid to exift contrary
to nature. Nor, fecondly, of negations, for thefe are de-
ftructive of the bound and limitation which is attributed
to every thing from the unifying and comprehending na-
ture of ideas; and hence, feparation is rather the refult of
material infinity than of that which is formal or ideal.
Nor again, are there any ideas of things which at different
times receive a variety of conditions. For thefe participate
of tranfmutation from a moveable caufe, but not from the
immoveable and ftable illuftration of ideas. Nor again of
parts, fuch as the hand, head, fingers, and the like. For
the caufes of things exifting entire, produce whole fpecies
and forms, not divided about the parts of thefe, like the
reafons of nature. But neither did thefe wife men place
in intellect the determinate caufes of accidents in bodies,
fuch as fweetnefs and whitenefs. For they confidered that
natural reafons were fufficient for the production of acci-
dents. Nor again, of compofites, as of a wife man. For

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* In his commentary on the 2d, 12th, and 13th books of Ariftotle's Metaphyfics, page
60. A Latin tranflation only of this invaluable work is extant; but I have fortunately a
copy in my poffeffion, with the verfion every where corrected by the learned Thomas Gale,
and with large extracts from the Greek.

since ideas are simple, they preside over the simple essence of every thing. But the composition and division of things is the business of our intellect; ideas, at the same time, and that intellection which is co-ordinate to ideas, being exempt from all these, on account of superlative simplicity. Neither, therefore, must we establish ideas of things generated from dissimilars, such as mules; nor of fruit produced by engrafting from different trees. For all these have a posterior and adventitious generation, and are not the work of nature alone, nor of nature proceeding according to her own reasons, but, as it were, compelled to labour contrary to her own determinations. Hence it is manifest, that all art, which imitates nature, and alone ministers to the use of mortal life, is separated from the cause of ideas. But neither are the works which, depending on the purpose of the soul, are perfected by a concourse of many causes, and which we are accustomed to call the operations of fortune, to be conjoined to the cause of ideas. For things which are there perfected, are eternal, and subsist perpetually the same, free from the nature of contingent events. It remains, therefore, that ideas must be confined to universal and perfect essences, and to whatever confers to their natural disposition; as for instance, to man, and every thing perfective of man, such as wisdom and virtue. For ideas existing as the generative and energetic causes of the perfection of every thing, distribute being to essences, and convert them to the inexhaustible plenitude of their own omniform natures."

4. But let us now consider the nature of numbers; for as every form is a number, according to the Pythagoreans *, a speculation of this kind must afford no small light to the

* See Proclus on Plato's Theology, p. 226.

arduous inveſtigation of ideas. Will it not, therefore, be proper, in the firſt place, to enquire, with the great Plotinus [*], whether multitude is not a departure and diſtance from *one*, ſo that infinity itſelf is a ſeparation from unity in the extreme, becauſe it is no other than innumerable multitude; that on this account it becomes evil; and that we contract a ſimilar nature when departing from intellectual unity, we are divided by ſenſible multitude? For a being then properly becomes many, when no longer able to remain collected in itſelf, the ſame, it is diffuſed abroad, and thus, being diſperſed, is variouſly extended; ſo that when, by diffuſion, it is abſolutely deprived of unity, it becomes perfect multitude, deſtitute of that univerſal cement, which unites one part with another.' But whenever the conciliating one is preſent, then that which was ſcattered and diffuſed, becoming permanent by its bounding power, paſſes into magnitude. But if any one ſhould deny the ſubſiſtence of unity, aſſerting that one is no where to be found, which is not ſome particular one; and ſhould hence affirm, that what is called one abſtractedly, is only a certain affection of the ſoul towards any being; we aſk, what prohibits the appellation of eſſence, from being nothing more than an affection of the ſoul, and conſequently the exiſtence of *being*, a deluſion? For we predicate unity of particulars with as great propriety as being. I am well aware, that philoſophers of the preſent day will anſwer, that we have an evident proof of the reality of being, from its agitating the ſoul, and becoming apparent in the phantaſy: to which we reply, that in like manner, the ſoul is agitated, and the imagination influenced about the *one*. For every individual as much excites the perception of one, as of being.

[*] Ennead. vi. lib. 6.

Beſides,

Befides, it is neceffary to enquire whether we behold this paffion and conception of the foul, as one or multitude. And again, when we fay not one, we do not then poffefs one from the thing itfelf; for we fay that one is not contained in that individual. And hence we muft poffefs one in our own nature, and this muft refide in the foul, feparate from that which is denominated fome particular one. But here it may be objeted, that the one we poffefs is received from externals, and is nothing more than a conception of the mind, produced by the thing itfelf. For it will be faid, that as multitude is nothing befides a number of individuals, which are called many, fo one is nothing befides one thing; and is formed by thought feparating that one particular from others. To this we reply as follows :

How can it be confonant to reafon to fuppofe that the conception of *one* arifes from the fenfation of fome one particular fubjet? For one particular man, who is difcerned by fenfe, is by no means the fame with one itfelf, fince, if this were the cafe, thought could never predicate one of that which is not a man. Befides, as cogitation, on beholding the different pofitions of things, affirms that this is here or there, fo when it perceives an individual, it pronounces one; for that paffion is not vain, nor does it affert one of a non-entity. Nor muft we think it predicates one, becaufe this individual is different from another; for when cogitation affirms fuch a thing is this, and not another, it declares, in the mean time, that the other is *one*. Likewife when it affirms that any thing is this *alone*, it then declares, that what is alone is one: on which account, it predicates one, prior to alone. Befides, if there be multitude, it is neceffary that one fhould antecede; fince when it predicates *many*, it pronounces more than one. And

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when it affirms that an army contains a multitude of men, it conceives the foldiers reduced to one order.

For thought, indeed, does not permit multitude to remain perfect multitude, deftitute of the conciliating power of unity; in which very circumftance, the fubfiftence of one is evinced; for acutely and fwiftly perceiving the one which refults from order, it reduces the nature of the many into one. Befides, we affirm that a houfe and an army are each one, but that a houfe is more one than an army, on account of the continuity of its parts. If therefore, one is contained more in that which is continued than in that which is difcrete, and ftill more in what is perfectly indivifible, it is evident that *the one* is a certain nature, and has a real being. For it is impoffible that the more and the lefs fhould take place among things which have no fubfiftence. If then it be not poffible to underftand any thing without one or two, or fome other number, it is by no means proper to deny exiftence to that, without which we cannot comprehend the exiftence or properties of any being: but it is requifite *that* nature fhould antecede all difcourfe, and intelligence, which is every where neceffary to their exiftence.

Again, if unity has no real fubfiftence, and is nothing more than a name or conception of the mind, it may be deftroyed without the deftruction of its fubject. The unity, therefore, of a houfe may be taken away, without the ruin of a houfe. But if a houfe is nothing more than certain materials, reduced into one form, this is impoffible. And, on the contrary, the alteration of that fubject, of which unity is predicated, can make no real alteration in unity (on this hypothefis) any more than the death of a man can affect his name. When, therefore, a body, of which *one* was predicated, is divided into a multitude of

parts,

parts, there is no real alteration made in the unity of the body, becaufe unity is nothing more than a name.

It was in confequence of this reafoning, and perceiving that unity was participated by every being, that the Pythagoreans placed a fuper-effential one at the top of the univerfe, intelligibly abftracted from all beings in fimplicity and excellence of nature. For they confidered, that unlefs there was a felf-fubfifting one in all things, there could neither be univerfals nor particulars. Not the firft, becaufe they are by nature *one* and *many*. But it is requifite that the one itfelf, fhould prefide over that which is not one alone. Nor again, the fecond, becaufe they are many and one, (that is, they participate more of multitude than unity, and their nature is determined more by the *many* than the *one*.) And becaufe of things in participation, unlefs an unparticipated one is added, there can be no caufe of union to beings; in the fame manner as the caufe of effence to beings, is taken away by thofe who deny that being itfelf, is the principle of all effence. For as the good itfelf, is the one principle of good to the univerfe, and is nothing befides good; and as a felf-motive nature, which is nothing befides felf-motion, is the caufe of motion to all things; fo all things proceed from being itfelf, and all united natures receive their union from *the one*, abftracted from all things.

Hence (fuch is the abfolute dominion of unity), continued quantities would have no exiftence without its participation; for when they are divided, fo far as they lofe unity, they change their being into fome other form. Hence, the bodies of plants or animals, which are each of them one, when they fly from unity, and are diffipated into multitude, immediately lofe the effence they formerly poffeffed, and become fomething elfe; which new ftate of
being.

being they likewife poffefs fo far as they are one. Add too, that health then flourifhes in the corporeal frame, when the body is conciliated into one; then beauty flourifhes, when the power of one conne{s the members into proportion and confent; and then virtue reigns in the foul, when the foul is reduced into one fimilitude with that which is divine.

5. But let us now inveftigate the nature of numbers. All number, according to the Pythagoreans, originates from unity and the indefinite duad; the firft having the relation of form, and the fecond, that of matter to all the orders of numbers. But they likewife divided number into two kinds, effential and monadic. The effential number they confidered as firft fubfifting in the intelligible world, together with being, and from thence diftributed into all the various gradations of forms. But the monadic, or that which is compofed from certain units, they juftly confidered as nothing more than the image of effential number. And with refpeét to the numbers which the human foul participates, thefe from its imperfeét condition have a middle fubfiftence; i. e. they exift in a vital, gnoftic, and fpeculative, but not in an operative manner. Hence, when receiving one thing with another, we affirm, that they are two, as a dog and a man, or two men; or when we compute more than two, as ten, and fay that there is a decad of men, this number is not effential to the two or ten individuals, nor is it to be conceived as fubfifting in fenfible natures; but it is purely quantity. But when we diftribute this ten, into units, we produce the principle of quantity, and generate a fubjeét in opinion *, capable of

parti-

* In giving monadic number a fubfiftence in opinion, I have followed the diftribution of Proclus, in the conclufion of his comment on a point; and, I think, not without fufficient reafon. For fince monadic numbers are more immaterial than geometrical lines and figures,

they

participating the effential decad of our foul. But when, confidering man in himfelf, we affirm that he is a certain number, as the duad, compofed of animal and rational, we do not obferve one mode in this predication; but fo far as by a difcurfive operation of the foul, we numerate, we effect a particular quantum; but fo far as the fubjects are two, and at the fame time both one (fince one fills the effence of both, and in both unity is contained), we pronounce another, and an effential number: and this duad is not of a pofterior origin, nor alone fignifies a certain quantity, external to the fubject, but a duad fubfifting in the effence of man, and containing his nature. For here we do not produce a number by a difcurfive operation, while we purfue effential natures. But when we number any ten things, which are not connected by any conciliating unity, like a choir, or an army, then this decad, which we predicate of the ten particulars, fubfifts alone in our numerating foul, which renders the ten individuals in opinion, a definite quantum. But in a choir, or an army, effential number is participated exclufive of that which fubfifts in our foul. And if it be enquired how number fubfifts in the human foul, we muft fay, that the foul, by her felf-moving energies, procreates number, while fhe numerates, and by this energy, caufes the exiftence of quantity; in the fame manner as in walking, we give rife

they muft have a more immaterial fubfiftence. But as they are correfpondent to matter, they cannot refide in the effential reafons of the foul; nor can they fubfift in the phantafy, becaufe they are fuperior to geometrical figures. It remains, therefore, that we muft place them between διανοια, or cogitation, and the phantafy; and this middle fituation is that of opinion. For cogitation, which Plato defines, in his Sophifta, to be an inward difcourfe, without voice, is an energy of the rational foul, extending itfelf from propofitions to conclufions. And, according to Plato, in the fame place, opinion is the filent affirmation, or negation of διανοια, or thought. Hence, fays he, " opinion is the conclufion of cogitation; but imagination, the mutual mixture of fenfe and opinion." So that opinion may, with great propriety, be faid to contain monadic number, to which it bears the proportion of matter. And hence the reafon is obvious, why the Pythagoreans called the duad opinion.

to a certain motion. Thus, monadic number, or a collection of units of various kinds, fubfifts in opinion, in a manner correfpondent to that of geometrical figures; and by this means participates the effential number of the foul. For as a triangular figure in the phantafy, is the recipient of a triangular nature, or of triangle itfelf; fo every three units in opinion, receive the effential triad of the foul, and, by this means, from a definite quantum.

In fhort, as in every being we may difcern the refemblances of matter and form, fo in the pentad, or any other number, the five units, which are the fubject of participation, and the quantity of the number, originate from the duad; but the form, that is the pentad itfelf, from unity. For every form is an unity, which unites its fubject quantity, and connects it with its ideal fpecies. It is, therefore, requifite to underftand, that the two principles of mathematical numbers are refident in our fouls, with which every mathematical number is co-exiftent; I mean unity, comprehending in itfelf all the forms of numbers, and which correfponds to unity in intellectual natures; and the duad, endued with a generative power, of a formlefs nature, and of infinite virtue; and which is called boundlefs, on account of its being the image of never-failing and intelligible duality. Hence, the unity of the foul, with a never-ceafing energy, continually diftinguifhes and forms all the orderly proceffions of her numbers, fuffers no vacuum to intervene, and leaves no quantity formlefs and innumerable. Hence too, no effential number of the foul, as for inftance, the pentad, is compofed from fubftance and accident, as a white man; nor from genus and difference, as man from animal and biped; nor again, from five unities mutually touching each other, like a bundle of wood; nor from things mixt, like water and

wine,

wine, nor from things fubfifting by pofition, in the man-
ner that ftones compofe a houfe ; nor laftly, does it fubfift
like things numerable; for it is not becaufe they are
compofed from indivifible units, that they poffefs any thing
befides units. For many points are indivifible, yet quan-
tity is not produced on this account ; but becaufe they
participate of two natures, the one correfponding to matter,
and the other to form. Laftly, it is not proper to fay,
that the number feven (and fo of any other number), is
compofed from the triad and the tetrad; for units, indeed,
compofed with units, form a fubject adapted to the recep-
tion of the heptad, or the ideal and effential number feven;
but the definite numerical quantity feven, is formed from
fo many units, and the ideal heptad. Hence, as the foul
of the fhipwright gives form to the timber, from her
inherent art; fo the numerative foul, from the unity
endued with the relation of a principle which fhe poffeffes,
gives form and fubfiftence to all her inherent numbers.
But there is this difference between the two, that the fhip-
wright's art is not effential to our nature, and requires
manual operation, becaufe it is converfant with fenfible
matter; but the numerative art is effentially inherent in
the foul, and is therefore prefent with all men, and poffeffes
an intellectual matter, which it eafily forms without the
affiftance of time. And this, perhaps, is what deceives
many, who think that the heptad is nothing more than
feven units. For the imagination of the vulgar, unlefs it
firft perceives a thing deftitute of ornament, and after-
wards the operations of the adorning artificer fupervening
its nature; and laftly, beholds the thing perfect, and in-
vefted with form, cannot be perfuaded that it poffeffes
two natures, the one formlefs, but the other endued with
an energetic and forming power.

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And here it is neceffary to obferve, that though unity is the form of all arithmetical forms, yet it is chiefly the form of the decad. For what unity is fimply to all the feries of numbers, that the decad is to the following hundreds, thoufands, and millions; from whence, according to a fecondary progreffion, it is denominated unity. As intellect, therefore, is the form of all things, but efpecially of the foul, fo unity, though it is the idea of all numbers, yet efpecially of the decad. But the reafon why the Pythagoreans extended ideal numbers no farther than ten, is becaufe this number is the ultimate perfection of beings, containing all things in its omniform nature. For all proportion fubfifts within the number ten; the arithmetical in a natural progreffion of numbers from unity; the geometrical in the numbers 1, 2, 4, and 1, 3, 9, and the harmonical in the numbers 2, 3, 6, and 3, 4, 6. And fince the caufes of all things are contained in numbers, as far as to the decad *, it is fuperfluous to fuppofe exemplars of the following numbers.

If it fhould be afked in what manner we muft conceive number as fubfifting in the intelligible world, we anfwer, with the great Plotinus, that we muft conceive it as fubfifting in being itfelf, with a power of impelling it to the production of multitude. " Hence (fays he, Ennead vi. lib. vi.) number is either the effence or the energy of being, and animal itfelf, and intellect is number. But, perhaps, we muft call being, number united (ἀριθμὸς ἡνωμένος), but beings, number evolved, or unfolded; (ἐξεληλεῖμένος ἀριθμὸς) intellect, number moving in itfelf; (ἀριθμὸς ἐν ἑαυτῷ κινύμενος)

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* Ἄφροπον, ἀκαμάτον Δικάδα κλιύσιν μιν ἀγηὴν,
 Ἀθάνατοί τι Θιοὶ καὶ γηγινίεις ἀνθρωπόι.

Syrian. in Meta. Ariftot. p. 113. Gr.

i. e. (According to the Pythagoreans) " the immortal gods and earth-born men, call the venerable decad, immutable and unwearied."

and laftly, animal, number comprehending (ἀριθμὸς περιέχων.") It was in confequence of this reafoning, that the Pythagoreans called ideas numbers; becaufe the gradual evolution of thefe from ineffable unity, produced all the beautiful variety of forms. Their exalted conceptions of numbers, likewife, originated from the fame fublime theory. Hence, * Pythagoras, in the facred difcourfe, calls number " *the ruler of forms and ideas.*" But † Philolaus, " *the commanding and felf-begotten container of the eternal duration of mundane concerns.*" And ‡ Hippafus, and all thofe who were called ἀκυσματικοὶ (or fuch as were yet under the probation of the quinquennial filence), " *the firft exemplar of the mundane fabric, and the judiciary inftrument of its artificer.*"

6. And here I cannot but take notice, with regret, of the very unphilofophical miftake committed by that great mathematician Dr. Barrow § : I fay, with regret, on account of the extraordinary obligations I am under to his writings, for my proficiency (whatever it may be) in mathematical learning. But refpect muft yield to the truth. " Unity, fays he, is not indivifible. (For how ex. gr. can ⅙ added to ⅙ be equal to unity, if unity be indivifible and incompofed, and reprefent a point) but rather only unity is properly divifible, and numbers arife from the divifion of unity." Here the Doctor evidently confounds fenfible units, which are the fubjects of vulgar practical arithmetic, with thofe units which are the objects of fcience. Every individual fenfible object, is indeed an unit, fo far as it

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* Αυτὸς μὶν Πυθαγόρας ἐν τῷ ἱερῷ λόγῳ διαρρήδην μορφῶν καὶ ἰδιῶν κράτορα τὸν ἀριθμὸν ἐιʼγ ⸱ ⸱⸱
                    Vid. Syrian. in Arift. Meta.

† Φιλόλαος δὲ, τῆς τῶν κοσμικῶν αἰωνίας διαμονῆς τὴν κρατιστεύσαν καὶ αὐτογενῆ συνοχὴν ⸱ ⸱⸱ τον ἀριθμόν,
                    Syrian. in eodem loco.

‡ Οἱ δὲ περὶ Ἱππασον ἀκυσματικοὶ, ἀριθμὸν εἶπεν παράδειγμα πρῶτον κοσμοποιίας⸱ ⸱ ⸱ ⸱⸱ κριτικὸν κοσμουργοῦ θεοῦ ὄργανον.   Jamb. in Nicomach. Arith. p ⸱

§ In his Mathematical Lectures, page 48.

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participates the connecting and conciliating power of an immaterial *one*: but the unity which ftands at the top of fpeculative arithmetic, is perfectly indivifible, or arithmetic would ceafe to be a fcience. The truth of this is evident from Euclid's definition: " Unity (fays he) is that according to which each of the things which are, is called one." But if unity be a compofite, the definition is falfe; fince a compofite, or a certain multitude, can never be the caufe of unity, but the contrary. And that this immaterial *one* fubfifts in fenfible natures, has, I hope, been fufficiently proved in the preceding part of this difcourfe. But the Platonic Theo * of Smyrna, fully eftablifhes the indivifibility of unity, as follows : " Unity is terminating quantity, the principle and element of numbers, which remains undiminifhed by the moft immenfe multitude of fubtractions, and being deprived of all number, continues firm and fixt, becaufe it is impoffible for divifion to proceed beyond the bound of unity. Thus, if we feparate any one corporeal fubftance into parts, the *one* again becomes *many*; and by fubtracting the feveral parts, we end in one part; and from this remaining part, again divided, arifes multitude; and by taking away every part, we again arrive at *one*. So that *one*, confidered as *one*, is incapable of diminution, and perfectly indivifible. On the contrary, every number is diminifhed by divifion, and is feparated into parts lefs than itfelf; as the number 6 into 3 and 3, or into 4 and 2, or into 5 and 1. But unity in fenfible particulars, if divided, is diminifhed after the manner of body, and by fection is diftributed into parts lefs than itfelf: but it receives increafe after the manner of number; for inftead of the one, multitude is produced. In this fenfe, therefore, is unity indivifible; for nothing is divided into parts greater than

* In Arithmet. p. 23.

itfelf.

itſelf. But that which is cut into parts greater than the whole, and into parts equal to the whole, is divided as number. Thus, for inſtance, if any one ſenſible body is divided into ſix parts, 1, 1, 1, 1, 1, 1, theſe ſhall be equal to the whole; but by a ſection into 4 and 2, it is divided into parts greater than the whole, conſidered as *one*; for 4 and 2 conſidered as numbers, exceed unity, and the body was ſuppoſed to be one. Unity, therefore, as number is perfectly indiviſible. But unity is called by the Greek word μονας, only, or alone, either becauſe it remains immoveable, and does not deſert itſelf, nor ſurpaſs the bounds of its nature (for it remains the ſame, however multiplied into itſelf, through an infinite progreſſion) or becauſe it is placed ſeparate and apart from the multitude of other numbers, it is denominated the *monad*, or *one*."

In conſequence of this very miſtaken hypotheſis, which oppoſes not only all the wiſdom of antiquity, but the ſublimeſt truths, the Doctor aſſerts, that an arithmetical cypher is the principle of numbers; and that it is analogous to a point in geometry. Juſt as if a cypher, which is nothing more than a mark expreſſive by its poſition with numbers, of a certain quantity, had a real exiſtence, and was productive of number: when, at the ſame time, any other arbitrary character would ſerve the ſame purpoſes, if applied in a ſimilar manner. It muſt ſurely afflict every thinking mind, to ſee how dreadfully the mechanical ſyſtem of philoſophy, which has been ſo long in faſhion, enſlaves and perverts the minds of its votaries; for there cannot, I think, be a more egregious inſtance of its fatal tendency, than the preſent, in which *nothing* is conſidered as the foundation of that noble ſcience, arithmetic; which was deſervedly placed by the ancients, in the firſt rank of the mathematical diſciplines. Such a foundation, indeed, *may*

be

be proper to the *mechanical philofophy*, but is very ill adapted to fupport the folid fabric of the arithmetical fcience. But let us attend to the arguments of this moft learned man, in defence of fo ftrange an affertion, " A cypher, or arithmetical nothing (fays he) is really the bound of every number coming between it and the numbers next following, but not as a part. A cypher being added to, or taken from a number, does neither increafe nor diminifh it; from it is taken the beginning of computation, while itfelf is not computed; and it bears a manifeft relation to the principal properties of a geometrical point." But in what manner are we to conceive the *nothing* which intervenes between any two numbers, to be their term or boundary? For Euclid defines a term to be the extremity of any thing; implying by the extremity, fomething belonging to that of which it is the bound. But how can a cypher, or *nothing*, in any refpect belong to number, or *fomething?* For if *nothing* be a boundary, merely from its intervention, a point exifting between any two disjoined lines, though at the greateft diftance from each, muft be their common boundary, which is evidently abfurd. Befides, what relation does it bear to a point, which is endued with a generative power, by its flux forming the fimple extenfion of a line, and, at the fame time, every where limiting its progreffion, and fubfifting in infinite capacity in its every part? Where are the real and divine properties to be found in an arithmetical nothing, which Proclus, in the following Commentaries, exhibits in a point? And how can computation originate from a mere non-entity?

But a little confideration will convince us, that this Saracen, or Indian cypher, is nothing more than an arbitrary character, invented for the purpofe of facilitating computation. For, fuppofe the letter (*a*) to be placed in

its

its ftead, and to fignify, when connected with the mark for unity, ten, or ten multiplied by one; when connected with the mark for two, ten multiplied by two, and fo on. And again, when placed twice after unity, let it exprefs the fecond power of ten, or one hundred, in this manner, *a a*; when thrice connected, one thoufand, or the third power of ten, and fo on: fhall we fay, in confequence of this, that (*a*) is the bound of numbers, and the principle of arithmetic? Or, fhall we not rather fay, that it is an arbitrary fymbol, like any other algebraic character, having no real connection with numbers, and depending, for its exiftence and application, entirely on the will of its inventor. But this opinion is too abfurd to need any farther refutation.

7. It may here, perhaps, be expected, that I fhould explain how, in the language of Syrianus [*], " divine number proceeds from the immortal retreats of unity, until it arrives at the divine tetrad [†];" and that I fhould unfold the properties of the tetractys, according to the Pythagoreans: but an undertaking of this kind, would not only far exceed the limits of this differtation, but, perhaps, in the prefent age, might be juftly deemed, by the lovers of wifdom, a proftitution and profanation of the moft exalted truths. Enough, I hope, has been faid to excite the curiofity, and roufe the attention of the thinking and liberal part of mankind; and thofe who underftand what is here briefly delivered, may apply themfelves, with advantage, to Proclus on Plato's Theology, where they will find all the myfteries of numbers unravelled: and to the works of the great Plotinus, who will lead them into the pene-

* In Arifot. Meta. p. 112. Gr. vel 59. b. Lat.

† For the tetrad contains all numbers within its nature, in the manner of an exemplar; and fince it is, that in monadic numbers, 1, 2, 3, 4, are equal to ten.

Italic.

tralia of the moft recondite wifdom. But, in perufing the works of thefe great men, the reader muft not expect to find the fublimeft truths explained in a familiar manner, and adapted, like many modern publications, *to the meaneft capacities.* For this, indeed, is impoffible to be effected. " Mankind (fays Petvin *), are not to be made any more truly knowing than happy by another's underftanding.— There is no man can at once convey light in the higher fubjects, to another man's underftanding. It muft come into the mind from its own motions, within itfelf: and the grand art of philofophy, *is to fet the mind a-going*; and, even when we think nothing of it, to affift it in its labour." After which he obferves, that " the ancients never attempt to lead us into knowledge, by a continued chain of rea-foning; on the contrary, they write in fuch a manner, as to force us to think for ourfelves." And, previous to this, he remarks, " that there are certain truths acquired by a long exercife of reafon, both in *particular,* and likewife in thofe fubjects that are moft *general,* as much, perhaps, out of the reach of the greateft mathematician, as Sir Ifaac Newton's fpeculations are above the capacity of fome that are now called mathematicians." The truth of this obfer-vation is fufficiently evinced, in Plato's definition of a phi-lofopher (in his Sophifta), " The philofopher (fays he) is the man who fufficiently fees one idea every way extended through many, every one of them lying apart; and many ideas different from one another, externally comprehended under one.—And farther, one idea, throughout all manys, wrapt up in one; and many ideas, every way feparate or difcreet. This is to have the knowledge to difcern how ideas, as they are general, agree and difagree." Now, he who thinks that a perception of this kind may be acquired

* Notes to Letters on Mind, page 83.

by

by barely reading an accurate difcourfe on the nature of ideas, compofed in intelligible terms, without, at the fame time, employing a long courfe of profound meditation, and patient thought, knows but little the difficulty of the tafk, and until he changes his opinion will never be the wifer. But the folly and prefumption of men, with refpect to this fublime philofophy, is really unpardonable; for there are very few who conceive that much previous inftruction is requifite to its acquifition; but almoft every man decides peremptorily on the moft abftract fpeculations, and reckons himfelf fufficient for the moft profound inveftigations. In the fciences and arts they are willing to proceed to perfection by gradual advances; but they confider philofophy as eafy, of inftant accefs, and haftily approach to her embraces with an affured confidence of fuccefs. Though, like unhappy Ixion, through their prefumption, inftead of a goddefs, they grafp nothing but an empty cloud. Plato was fo fenfible of this truth, that, in his feventh epiftle to Dion, he exprefsly affirms, that he neither has written, nor ever will write explicitly concerning thefe fublime fpeculations; " For a thing of this kind (fays he) cannot be expreffed by words, like other difciplines, but by a lafting familiarity, and conjunction of life, with this *divine object*, a bright light *

on

* This bright light is no other than that of ideas themfelves; which, when it is once enkindled, or rather re-kindled in the foul, becomes the general ftandard, and criterion of truth. He who poffeffes this, is no longer the flave of opinion; puzzled with doubts, and loft in the uncertainties of conjecture. Here the fountain of evidence is alone to be found.—This is the true light, whofe fplendors can alone difpel the darknefs of ignorance, and procure for the foul undecaying good, and fubftantial felicity. Of this I am certain, from my own experience; and happy is he who acquires this invaluable treafure. But let the reader beware of mixing the extravagancies of modern enthufiafm with this exalted illumination. For this light is alone brought into the mind by fcience, patient reflection, and unwearied meditation: it is not produced by any violent agitation of fpirits, or extafy of imagination; for it is far fuperior to the energies of thefe: but it is tranquil and fteady, intellectual and divine. Avicenna, the Arabian, was well acquainted with this light, as is evident from the beautiful defcription he gives of it, in the elegant introduction of Ebn Tophail, to the Life of Hai Ebn Yokdhan. " When a man's

on a fudden, as it were leaping from a fire, will illumi-
nate the foul, and there preferve and nourifh its fplendor.
He adds, that a publication of fuch concerns, is alone ufe-
ful to a few of mankind, who from fome fmall veftiges
previoufly demonftrated, are fufficiently fagacious to their
invention. But it will fill others partly with a bafe con-
tempt, and partly with a rafh and vain confidence, as if
they had now learned fome very excellent things." He
then fubjoins the following inftance of the difficulty attend-
ing fuch an undertaking : " There are three things (fays
he), from which fcience muft neceffarily be produced ;
but the fourth is fcience itfelf. And it is requifite to
eftablifh the fifth as that which is the object of knowledge,
and has a true exiftence. One of thefe is the name of a
thing ; the fecond its definition ; the third the refemblance ;
the fourth fcience. Now take each of thefe, defiring to
learn what we have lately afferted, and think concerning
them all, in a fimilar manner. A circle is called fome-
thing, whofe *name* we have juft expreffed. After this
follows its definition, compofed from nouns and verbs. For
that which every where is equally diftant from the ex-
tremes to the middle, is the definition of that which we
fignify by the name of a round, and a circumference, and

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defires (fays he) are confiderably elevated, and he is competently well exercifed in thefe fpecu-
lations, there will appear to him fome fmall glimmerings of the truth, as it were flafhes of
lightning, very delightful, which juft fhine upon him, and then become extinct. Then the
more he exercifes himfelf, the oftener will he perceive them, till at laft he will become fo well
acquainted with them, that they will occur to him fpontaneoufly, without any exercife at all ;
and then as foon as he perceives any thing, he applies himfelf to the divine effence, fo as to
retain fome impreffion of it ; then fomething occurs to him on a fudden, whereby he begins to
difcern the *truth* in every thing ; till through frequent exercife he at laft attains to a perfect
tranquillity ; and that which ufed to appear to him only by fits and ftarts, becomes habitual,
and that which was only a glimmering before, a conftant light ; and he obtains a conftant and
fteady knowledge." He who defires to know more concerning this, and a ftill brighter light,
that arifing from an union with the fupreme, muft confult the eighth book of Plotinus' fifth
Ennead, and the 7th and 9th of the fixth, and his book on the Beautiful, of which I have
publifhed a tranflation.

a circle.

a circle. But the third is the circle which may be painted, or blotted out, which may be made by a wheel, or deſtroyed. None of which affections, the circle itſelf, which each of theſe reſpects, ſuffers, as being of a different nature. But the fourth is ſcience, and intellect, and true opinion about theſe. And this again muſt be eſtabliſhed as one whole, which neither ſubſiſts in voice, nor in corporeal figures, but in intellect and intelligence. It is therefore manifeſt, that this fourth is different from the nature itſelf of the circle, and again different from the three we have previouſly mentioned. But among the number of theſe, intellect, by its relation and ſimilitude, proximately adheres to the fifth, while the reſt are more remote from its nature. The ſame may likewiſe be affirmed of a ſtraight and crooked figure, of colour, and of the good, the beautiful, and the juſt. And again, of every body, whether faſhioned by the hand, or the work of nature, whether fire or water, and the reſt of this kind; likewiſe of every animal, and the manners of animals; and of all actions and paſſions. For unleſs, among theſe, ſome one, after a manner, receives that fourth, he will never perfectly participate the ſcience about the fifth." He then proceeds to ſhew in what reſpect each of the preceding four are different from the fifth. " Every circle (ſays he) which by the hands of men is either painted, or faſhioned by a wheel, is plainly contrary to our fifth. For it every where participates of the right-line. But we muſt affirm, that the circle itſelf has neither more nor leſs of any thing whatever; that is, it poſſeſſes in itſelf, nothing of a contrary nature. Beſides, none of theſe are endued with any ſtability of name. For nothing hinders our applying the appellation of ſtraight to that which we now denominate round, and calling the ſtraight by the denomination of the round ; nor will there

be any lefs ftability in thefe, when their names are changed
into the contrary. The fame reafoning is likewife true of
definition, fince it is compofed from nouns and verbs,
which poffefs no ftability. And in a variety of ways, it
may be proved, that no one of thefe four is certain and
firm." Now, this fifth divifion of Plato's entirely refpects
ideas, confidered as flourifhing in intellect; by a con-
junction with which, we acquire true intelligence, and the
perfection of human knowledge. The firft three of the
preceding are obnoxious to various mutations; the fourth
lefs; but the laft is perfectly ftable and invariable. The
three firft are rather converfant about the qualities of
things, about the image and fhadow; the fourth raifes us
to the participation of truth; but the fifth to truth itfelf,
and permanent effence. In the firft degrees almoft all are
converfant; in the fourth a few; in the fifth, all the gods,
but a very fmall part of mankind, as it is afferted in the
Timæus. The four firft may be known, indeed, without
the fifth, confufedly; but from the knowledge of the fifth
they become perfectly manifeft, as effects from the know-
ledge of their caufe. But we cannot, by any means, attain
to the apprehenfion of the fifth, unlefs we have been firft
accurately converfant with the reft; for from our imper-
fect condition we are compelled to rife from difference to
identity, from multitude to unity, and from fhadow to
fubftance. While we inveftigate the knowledge of things,
if we are alone defirous to apprehend their refemblance
(which is the cafe with the multitude) we fhall be placed
in the third degree, and may eafily acquire the object of
our purfuit. But if we fhould fortunately poffefs the true
philofophical genius, which is rare in the extreme, and
afpiring to the fifth degree, fhould, by a happy event, at-
tain to its conjunction, though fuch a contact is clearer and

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more certain than all knowledge; yet it is difficult to ex-
prefs it in words, and to manifeft it to others.   And the
reafon of this is obvious: firft, becaufe words are wanting,
which exactly correfpond to the effence of a thing, fince
thefe are only the fymbols of fhadows.   Secondly, becaufe
we fpeak with thofe, who are alone converfant with fha-
dows, and are on this account derided by them, when they
find that our fifth does not, by any means, accord with
material refemblances, which they confider as the only
realities.

8. And here a queftion very naturally prefents itfelf for
our folution, whether the foul, while united with the
body, is able to perceive ideas, without the affiftance of the
phantafy.   For it feems difficult to apprehend how the
foul, thus depreffed and weighed down with the terrene
mafs, fhould be able to raife herfelf to the fupernal light
of ideas, and become united with their refulgence.   The
opinion of the Peripatetics is well known, that fome phan-
tafm muft always accompany intelligence; but this is de-
nied by the Platonifts, and I think with great reafon.   For
the operations of intellect are not dependent on the phan-
tafy, though the perceptions of the latter proceed from
the energies of the former.   Befides, as Plotinus beauti-
fully obferves, our moft vigorous energies are accompanied
with the leaft animadverfion; and there is no abfurdity in
fuppofing that by increafing the force of intellectual energy,
we may fpeculate free from all imagination; fince the
phantafms attending our conceptions, become weak in pro-
portion as the intellectual fight increafes in vigour.   On
this account, the Platonifts affirm, that the moral virtues
free us from the vehemence of perturbations; but the con-
templative from imagination, and the fenfes.   Hence too,
the fciences may be called living waters; in which the

                                                    wings

wings of the foul being dipt, her feathers, which were either feparated or broken by her lapfe into body, are repaired, and reftored to a refemblance of their former perfeftion. For the wings are the powers of the foul, leading to intelligibles: but the feathers are as well the natural inftincts to good and truth, as reafons inferted in the foul; which either fall off, or are broken by her defcent into body, and conjunction with its ruinous bonds. But thefe are repaired and invigorated by the fciences, which, like living ftreams, flowing from the fountains of ideas, reftore life and perfection to the foul. Hence Plato, in the Phæ-drus, afferts that thefe wings of the foul are increafed by every thing which confers to fupernal elevation; as beauty, wifdom, and the like; and by a convenient metaphor, in the fame dialogue, he confiders the chariot of the fouls lives, her charioteer, and the horfes by which her car is drawn; and laftly, every thing which contributes to the elevation of the foul, and her conjunction with intellect and ideas. We may therefore conclude, that this conjunction is poffible to be effected, though it is rarely obtained; and that it is a flight too arduous and facred for the groveling and fordid; a fplendor too bright for the fenfible eye; and a contact too ineffable to be defcribed by the unftable compofition of words.

But I cannot conclude this fection, without folliciting the reader's attention to a comparifon of the difference between the ancient philofophy, and that invented by Mr. Locke, and the moderns. According to Mr. Locke's fyftem ideas are formed from fenfible particulars, by a kind of mechanical operation; fo that truth is fomething by its nature, pofterior to fenfation, and entirely dependent on it for exiftence. According to Plato, ideas are eternal and immaterial beings, the originals of all fenfible forms, and

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the fountains of all evidence and truth; fo that on this fyftem truth ranks among the firft, and not in the laft of things; and would ftill retain its nature, though the corporeal fenfes were no more. According to Mr. Locke, the foul is a mere *rafa tabula*, an empty recipient, a mechanical blank. According to Plato, fhe is an ever-written tablet, a plenitude of forms, a vital and intellectual energy. On the former fyftem, fhe is on a level with the moft degraded natures, the receptacle of material fpecies, and the fpectator of delufion and non-entity *. Hence, her energies are nothing but fomnolent perceptions, and encumbered cogitations; for all her knowledge terminates in fenfe, and her fcience in paffion. Like a man between fleeping and waking, her vifions are turbid and confufed, and the phantoms of a material night, continually glide before her drowfy eye. But on the latter fyftem, the foul is the connecting medium of an intelligible and fenfible nature, the bright repofitory of all middle forms, and the vigilant eye of all cogitative reafons. Hence fhe is capable of roufing herfelf from the fleep of a corporeal life, and emerging from this dark Cimmerian land, into the regions of light and reality. At firft, indeed, before fhe is excited by fcience, fhe is oppreffed with lethargy, and clouded with oblivion; but in proportion as learning and enquiry ftimulate her dormant powers, fhe wakens from the dreams of ignorance, and opens her eye to the irradiations of wif-

* Left the fuperficial reader fhould think this is nothing more than declamation, let him attend to the following argument. If the foul poffeffes another eye diffetent from that of fenfe (and that fhe does fo, the fciences fufficiently evince), there muft be, in the nature of things, fp cies accommodated to her perception, different from fenfible forms. For if our intellect fpeculates things which have no real fubfiftence, fuch as Mr. Locke's ideas, its condition muft be much more unhappy than that of the fenfitive eye, fince this is co-ordinated to beings; but intellect would fpeculate nothing but illufions. Now, if this be abfurd, and if we poffefs an intellectual eye, which is endued with a vifive power, there muft be forms correfpondent and conjoined with its vifion; forms immoveable, indeed; by a corporeal motion, but moved by an intellectual energy.

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dom.   On Mr. Locke's fyftem, the principles of fcience
and fenfe are the fame, for the energies of both originate
from material forms, on which they are continually em-
ployed.   Hence, fcience is fubject to the flowing and
perifhable nature of particulars; and if body and its attri-
butes were deftroyed, would be nothing but a name.   But
on the fyftem of Plato, they differ as much as delufion and
reality; for here the vital, permanent, and lucid nature of
ideas is the fountain of fcience; and the inert, unftable,
and obfcure nature of fenfible objects, the fource of fen-
fation.   On Mr. Locke's fyftem, body may be modified into
thought, and become an intelligent creature; it may be
fubtilized into life, and fhrink, by its exility, into intel-
lect.   On that of Plato, body can never alter its nature by
modification, however, it may be rarefied and refined, va-
ried by the tranfpofition of its parts, or tortured by the
hand of experiment.   In fhort, the two fyftems may be
aptly reprefented by the two fections of a line, in Plato's
Republic.   In the ancient, you have truth itfelf, and
whatever participates of the brighteft evidence and reality:
in the modern, ignorance, and whatever belongs to obfcu-
rity and fhadow.   The former fills the foul with intelligible
light, breaks her lethargic fetters, and elevates her to the
principle of things; the latter clouds the intellectual eye
of the foul, by increafing her oblivion, ftrengthens her
corporeal bands, and hurries her downwards into the dark
labyrinths of matter.

Nor is it wonderful there fhould be fo great a difference
between the two fyftems, and fo much in favour of the
ancients, if we confider the great advantages thefe ancients
poffeffed over the moderns in every thing which contri-
butes to the advancement of philofophy.   For, in the firft
place, they lived in an age when abftract inveftigations

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were in the greateſt requeſt, and the profeſſors of ſuch purſuits in the higheſt eſtimation. Beſides this, they united the moſt exalted abilities with the moſt unwearied attention and obſtinate perſeverance; they devoted their whole lives to the ſearch of truth; and relinquiſhed every thing which might be an obſtacle to its acquiſition. We may add, likewiſe, the advantages of a language extremely philoſophical; and a freedom from the toil of learning any tongue but their own. Now the reverſe of all this is the portion of the moderns: for in the preſent age, abſtract ſpeculations are ridiculed; and its profeſſors deſpiſed. The purſuit of truth is conſidered as perfectly conſiſtent with ordinary avocations, and is rather proſecuted as a relief from the toils of buſineſs than as a thing deſirable for its own ſake, and of the greateſt dignity and worth. Hence, a few years deſultory application at a college, where language is one of the firſt objects of attention, qualifies a modern for philoſophy, raiſes him above Pythagoras and Plato, and perſuades him, with preſumptuous confidence, to enter the liſts againſt theſe venerable heroes. And laſtly, all modern languages are barbarous with reſpect to the Greek; falling far ſhort of its harmony and energy, its copiouſneſs and propriety. If ſuch then be the true ſtate of the caſe, what judgment muſt we form of men who, with all theſe diſadvantages, philoſophized without the aſſiſtance of the ancients, deſpiſing their works, and being ignorant of their contents? Shall we call it prudence or preſumption, wiſdom or folly? Truth will certainly pronounce the latter; and the general voice of poſterity will confirm her deciſion. There are two egregious inſtances in our own country of this daring preſumption; I mean Bacon and Locke. The former of theſe is celebrated for having deſtroyed the jargon of the ſchoolmen, and brought

experimental enquiries into repute; and for attempting t
inveſtigate cauſes through the immenſity of particula
effects. Hence, he fondly expected, by experiment pile
on experiment, to reach the principle of the univerſe; no
conſidering that his undertaking was as ridiculous as tha
of the giants of old, who attempted to invade the heavens
by placing Offa upon Pelion, and Olympus upon Offa
and ignorant that

> Heaven ſtill, with laughter, the vain toil ſurveys,
> And buries madmen in the heaps they raiſe.

The latter of theſe, Mr. Locke, is applauded for having
without affiſtance from the ancients, explained the nature
and exhibited the genuine theory of human underſtanding
But that this applauſe is falſe, the preceding compariſon be-
tween his and the ancient philoſophy, may evince; and the
variety of other ſelf-taught ſyſtems which, like nocturnal
meteors, blaze for a while, and then vaniſh in obſcurity,
abundantly confirms. Had theſe men, indeed, when they
juſtly derided the barbarous writings of the ſchoolmen,
explored the works of antiquity, penetrated the wiſdom
they contain, and enriched their native language with its
illuſtration, they had doubtleſs been celebrated by the lat-
eſt poſterity: but, deſirous of becoming maſters in phi-
loſophy by intuition, they diſdained the inſtruction of the
ancients, and vainly attempted to ſoar on artificial wings to
the very ſummit of ſcience. They are, however, deſtined,
like Icarus, to a precipitate fall; for the influence of time,
which is continually diſſolving the cement of their plumes,
is likewiſe continually weakening their force, and will at laſt
effect their final ſeparation. And thus much concerning
the doctrine of ideas, and numbers, according to Pythagoras
and Plato.

S. E. C.

SECTION II. *

BUT let us now confider the properties of the demon-ftrative fyllogifm, and endeavour to unravel its intricate web; appointing Ariftotle for our guide in this arduous inveftigation. For an enquiry of this kind is naturally connected with the doctrine of ideas, as it enables us to gain a glimpfe of the univerfals participated in mathematical forms, and to rife to the principles of fcience. It brings us acquainted with the laws which bind demonftration; and teaches us that objects of intellect are alone the objects of fcience, and the fources of truth.

Previous to the acquifition of all learning and ratiocinative difcipline, it is neceffary we fhould poffefs certain natural principles of knowledge, as fubfervient to our future progrefs and attainments. Thus, in every fcience there are fome things which require an immediate affent as foon as propofed; whofe certainty is too evident and illuftrious to ftand in need of any demonftrative proof deduced from that particular fcience which, like ftately pillars, they equally fupport and adorn. Hence we are informed by the geometrician, that a point is that which is deftitute of all parts whatever; but we muft previoufly underftand the meaning of the word *part*. Thus the arithmetician defines an odd number, that which is divided according to unequal parts; but it is neceffary we fhould antecedently know the meaning of the word *unequal*. Thus, too, art as well as fcience operates by antecedent knowledge; and hence the architect,

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* The prefent fection contains an illuftration of almoft all the firft book of Ariftotle's laft Analytics. I have for the moft part followed the accurate and elegant paraphrafe of Themiftius, in the execution of this defign, as the learned reader will perceive: but I have likewife every where added elucidations of my own, and endeavoured to render this valuable work intelligible to the thinking mathematical reader.

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the ftatuary, and the fhipwright, learn the names and the ufe of their refpective implements, previous to the exercife of the materials themfelves.   This is particularly evident in the difcurfive arts of rhetoric and logic; thus the logician reafons by fyllogifm, the rhetorician by induction, and the fophift by digreffions and examples; while each proceeds in an orderly progreffion from principles fimple and evident, to the moft remote and complicated conclufions.

2. The antecedent knowledge of things may be divided into two parts : the one a knowledge of their exiftence, or that they exift; the other a knowledge of the terms expreffive of their exiftence.   Thus, previous to the enquiry why iron is attracted by the magnet, it is neceffary we fhould learn the reality of this attraction, and the general mode of its operation : thus too, in an enquiry concerning the nature of motion and time, we muft be previoufly convinced of their exiftence in the nature of things.   The fecond divifion of antecedent knowledge takes place in fubjects whofe very exiftence admits of a difpute : thus previous to a folution of the queftions, Whether there are any gods or not? Whether there is a providence or not? and the like, it is neceffary we fhould firft underftand the meaning of the terms; fince we in vain inveftigate the nature of any thing while we are ignorant of the meaning of its name; although, on the contrary, we may have a perfect conception of the meaning of fome words, and yet be totally ignorant whether the things they exprefs have a real, or only an imaginary exiftence.   Thus, the meaning of the word *centaur* is well underftood by every one; but its exiftence is queftioned by moft.

3. From hence it will eafily appear, that no fmall difference fubfifts between *learning* and *knowledge*.   He who is about to underftand the truth of any propofition, may

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be faid to poffefs a previous conception of its truth; while, on the contrary, it may happen that he who is in the capacity of a learner, has no antecedent knowledge of the fcience he is about to learn. Thus we attain to the diftinct knowledge of a thing which we formerly knew in a general way; and frequently, things of which we were ignorant are learned and known in the fame inftant.

Of this kind are the things contained under fome general idea, of which we poffefs a previous knowledge: thus, he who already knows that the three interior angles of every triangle are equal to two right, and is as yet ignorant that fome particular figure delineated on paper is a triangle, is no fooner convinced from infpection of its being a triangle, than he immediately learns and knows: he learns it is a triangle; he knows the equality of its angles to two right ones. That it is now a triangle he both fees and learns; but the equality of its angles he previoufly knew in that general and comprehenfive idea, which embraces every particular triangle.

Indeed, a definite knowledge of this triangle requires two conditions: the one, that it is a triangle; and the other, that it has angles equal to two right. The firft we receive from infpection; the fecond is the refult of a fyllogiftic procefs; an operation too refined for the energies of fenfe, and alone the province of *intellect* and *demonftration*. But demonftration without the knowledge of that which is univerfal, cannot fubfift; and fince the propofition is univerfal, that in every triangle the angles are equal to two right, as foon as any figure is acknowledged to be a triangle, it muft neceffarily poffefs this general property.

Hence we infer, that of the triangle delineated on paper, and concealed, we are partly ignorant of this general pro-perty, the equality of its angles (becaufe we are ignorant

of

of its exiftence); and we partly underftand it as included in that univerfal idea we previoufly poffeffed. Hence too, it is evident that actual fcience arifes from a medium between abfolute ignorance and perfect knowledge; and that he who poffeffes the principles of demonftration, poffeffes in capacity the conclufions alfo, however complicated and remote; and that by an evocation of thefe principles from dormant power into energy, we advance from general and abftracted knowledge to that which is fenfible and particular.

4. Two acceptations of knowledge may be admitted; the one common and without any reftriction; the other limited and peculiar. Since all knowledge, whether arifing from accidents, or fupported by neceffary principles, is called fcience. Knowledge, properly fo called, arifes from a poffeffion of that caufe from which a thing derives its exiftence, and by which we infer the neceffity of its exiftence; and this conftitutes fimple and abfolute fcience. Thus too, the definitions of thofe general conceptions and fuppofitions, which from their primary nature are incapable of demonftration, are called fcience. But the fcience which treats of the method of arriving at knowledge, is called demonftration; for every demonftration is a fyllogifm producing fcience. Hence, if in every fyllogifm it is neceffary that the propofitions fhould be the caufe of the conclufion; and to know any thing properly, a knowledge of its caufe is requifite; in the propofitions of demonftration, both thefe conditions are required: that they fhould be effective of the conclufion; and the caufes of the thing demonftrated.

Thus, from the ruins of a ftately edifice, we may juftly infer, that the building was beautiful when entire; and from the fmoke we may collect the exiftence of the fire,

though

though concealed : but the ruins of the edifice are not the cause of its beauty; nor does fire originate from fmoke; but, on the contrary, fmoke is the natural refult of fire : the inference, therefore, is in neither cafe a demonftrative one. Again, fince every caufe is both prior to, and more excellent than its effect, it is neceffary that the propofitions fhould be more peculiar, primary, and excellent than the conclufions. And becaufe we then know a thing properly when we believe it to have a neceffary exiftence, hence it is requifite that the propofitions fhould be true; for if falfe, a falfe conclufion may enfue, fuch as, that the diameter of a fquare is commenfurable with its fide. But if every fcience arifes from antecedent knowledge, demonftration muft be founded on fomething previous; and on this account it is requifite that the propofitions fhould be more known than the conclufions. The neceffary properties, then, of all demonftrative propofitions, are thefe; that they exift as caufes, are primary, more excellent, peculiar, true, and known, than the conclufions. Indeed, every demonftration not only confifts of principles prior to others, but of fuch as are eminently firft; for if the affumed propofitions may be demonftrated by other affumptions, fuch propofitions may, indeed, appear prior to the conclufions, but are by no means entitled to the appellation of firft. But others, on the contrary, which require no demonftration, but are of themfelves probable or manifeft, are defervedly efteemed the firft, the trueft, and the beft. Such indemonftrable truths were called by the ancients, axioms, from their majefty and authority; as the affumptions which conftitute the beft fyllogifms derive all their force and efficacy from thefe.

And on this account, above all others, they merit the title of the principles of demonftration. But here it is

worth

worth obferving, that thefe primary propofitions are not the firft in the order of our conceptions; but firft to na-ture, or in the nature of things. To us, that which is firft is particular, and fubject to fenfible infpection; to nature, that which is univerfal, and far remote from the apprehenfion of fenfe. Demonftration does not fubmit it-felf to the meafure of our ingenuity, but, with invariable rectitude, tends to truth as its ultimate aim; and without ftopping to confider what our limited powers can attain, it alone explores and traces out the nature of a thing, though to us unperceived and unknown.

This demonftrative fyllogifm differs not a little from others, by the above property; the reft can as well educe a true conclufion from falfe premifes, which is frequent among the rhetoricians, as that which is prior from that which is pofterior; fuch as, Is every fyllogifm derived from conjecture?

With refpect to the reft, as we have already confeffed, they may be formed from principles that are true, but not from fuch as are proper and peculiar; as if a phyfician fhould endeavour to prove an orbicular wound the moft difficult to coalefce and heal, becaufe its figure is of all others, the moft capacious; fince the demonftration of this is not the province of the phyfician, but of the geome-trician *alone*.

5. That propofition is called immediate, which has none fuperior to itfelf, and which no demonftration whatever can confirm: fuch as thefe are held together by the em-braces of univerfals. There are fome, indeed, united from that which is fenfible and particular: thus, that the gar-ment is white, is an immediate propofition, but not of that kind whofe principles require to be demonftrative ones; the caufe of which we fhall hereafter inveftigate. Of im-

<div align="right">mediate</div>

mediate propofitions fubfervient to the purpofes of demon-ftration, fome are of fuch a fuperior nature, that all men poffefs a knowledge of them without any previous in-ftruction; and thefe are called axioms, or general notions; for without thefe all knowledge and enquiry is vain. Another fpecies of immediate propofitions is pofition; in-capable of being ftrengthened by demonftration, yet not neceffarily foreknown by the learner, but received from the teacher. With refpect to the genus of pofition, one of its fpecies is definition, and another hypothefis. De-finition is an oration, in which we neither fpeak of the exiftence, nor non-exiftence of a thing; but alone deter-mine its nature and effence. It is common to every hy-pothefis, not to be derived from nature, but to be the entire refult of the art of the preceptor.

It likewife always affirms the exiftence or non-exiftence of its fubject; fuch as, that motion is, and that from no-thing nothing is produced. Thofe which are not fo per-fpicuous are called poftulates, or petitions; as that a circle may be defcribed from any centre, and with any radius; and fuch as thefe are properly hypothefes and poftulates.

6. We have now feen the privilege affigned to the prin-ciples of demonftration:—whether or no our decifion has been juft, the enfuing confiderations will evince. We faid that the affumptions in demonftration were more known than the conclufions,—not indeed without reafon, fince through thefe our knowledge and belief of the conclufion arifes. For univerfally, that quality which is attributed to many different things fo as to be affigned to one through the medium of another, abounds moft in that medium by which it is tranfmitted to the reft.

Thus the fun, through the medium of the moon, illu-minates the earth by night; thus the father loves the

preceptor through the medium of his child. And in the first instance the moon is more lucid than any object it enlightens: in the second, the child posseffes more of the father's regard than his preceptor. If then we affent to the conclusions through our belief of the principles alone, it is neceffary that the principles should be more known, and inherit a greater degree of our affent. Hence, if it be true that the principles are more known than the conclusions, it follows, that either our knowledge of them is derived from demonstration, or that it is fuperior to any demonstrative proof; and after this manner we muft conceive of thofe general felf-evident notions which, on account of their indemonftrable certainty, are defervedly placed at the top of all human fcience.

Thefe propofitions not only poffefs greater credibility than their conclufions, they likewife inherit this property as an acceffion to their dignity and importance; that no contrary propofitions deferve greater belief; for if you give no more affent to any principle than to its contrary, neither can you give more credit to the conclufion deduced from that principle than to its oppofite. Were this the cafe, the doctrine of thefe propofitions would immediately lofe its invariable certainty.

7. There are, indeed, fome who, from erroneoufly applying what we have rightly determined, endeavour to take away the poffibility of demonftration. From the preceding doctrine it appears that the principles are more aptly known than the conclufions. This is not evident to fome, who think nothing can be known by us without a demonftrative procefs; and confequently believe that the moft fimple principles muft derive all their credit from the light of demonftration.

But

But if it be neceffary that all affumptions fhould be de-monftrated by others, and thefe again by others; either the enquiry muft be continued to infinity, (but infinity can never be abfolved), or if, wearied by the immenfe procefs, you at length ftop, you muft doubtlefs leave thofe propo-fitions unknown, whofe demonftration was declined through the fatigue of inveftigation. But how can fcience be de-rived from unknown principles? For he who is ignorant of the principles, cannot underftand the conclufions which flow from thefe as their proper fource, unlefs from an hy-pothefis or fuppofition of their reality.

· This argument of the fophifts is, indeed, fo far true, that he who does not underftand that which is firft in the order of demonftration, muft remain ignorant of that which is laft:—But in this it fails, that all knowledge is demon-ftrative; fince this is an affertion no lefs ridiculous than to maintain that nothing can be known. For as it is manifeft that fome things derive their credit and fupport from others, it is equally obvious that many, by their intrinfic excellence, poffefs indubitable certainty and truth; and command our immediate affent as foon as propofed. They inherit, indeed, a higher degree of evidence than thofe we affent to by the confirmation of others; and thefe are the firft principles of demonftration: propofitions indifputable, immediate, and perfpicuous by that native luftre they always poffefs. By means of thefe, we advance from propofition to propofition, and from fyllogifm to fyllogifm, till we ar-rive at the moft complicated and important conclufions: Others, willing to decline this infinite progreffion, defend the neceffity of a circular or reciprocal demonftration. But this is nothing more than to build error upon error, in order to attain the truth; an attempt no lefs ridiculous than that of the giants of old. For fince, as we fhall hereafter

accu-

accurately prove, demonftration ought to confift from that which is firft, and moft known; and fince it is impoffible: that the fame thing fhould be to itfelf both prior and pofterior: hence we infer the abfurdity of circular demonftration; or thofe fyllogifms in which the conclufions are alternately fubftituted as principles, and the principles as conclufions. It may, indeed, happen, that the fame thing may be both prior and pofterior to the fame; but not at one and the fame time, nor according to the fame mode of exiftence. Thus, what is prior in the order of our conceptions, is pofterior in the order of nature; and what is firft in the arrangement of things, is laft in the progreffions of human underftanding. But demonftration always defires that *firft* which is prior in the order and conftitution of nature. But the folly of fuch a method will more plainly appear from confidering its refult: let us fuppofe every a is b, and every b is c; hence we juftly infer, that every a is c. In like manner, if we prove that every a is b, and by a circular demonftration, that every b is a, the confequence from the preceding is no other than that every a is a; and thus the conclufion terminates in that from which it firft began; a deduction equally ufelefs and ridiculous. However, admitting that, in the firft figure, circular demonftration may be in fome cafes adopted, yet this can but feldom happen from the paucity of reciprocal terms.

But that reciprocal terms are very few, is plain from hence: let any fpecies be affumed, as *man*; whatever is the predicate of man, is either conftitutive of his effence, or expreffive of fome accident belonging to his nature. The fuperior genera and differences compofe his effence, among which no equal predicate can be affigned reciprocable with man, except the ultimate differences which cannot be otherwife than one, i. e. rifibility, which mutually re-
ciprocates,

ciprocates with its fubject; fince every man is rifible,, and whatever is rifible is man. Of accidents fome are common, others peculiar; and the common are far more in number than the peculiar; confequently the predicates which reciprocate with man, are much fewer than thofe which do not reciprocate.

8. It is now neceffary to enumerate the queftions pertaining to demonftration; and for this purpofe, we fhall begin with propofitions, fince from thefe, fyllogifms are formed; and fince every propofition confifts of a fubject and predicate, the modes of predication muft be confidered, and thefe are three which I call *total, effential,* and *univerfal*; a *total* predication takes place when that which is affirmed or denied of one individual is affirmed or denied of every individual comprehended under the fame common fpecies.

Thus, animal is predicated of every man, and it has this farther property befides, that of whatever fubject it is true to affirm man, it is at the fame time true to affirm animal.

Thofe things are faid to be *effentially* predicated; firft, when the predicate is not only total, but conftitutes the effence of the fubject; inftances of this kind are, animal of man; a tree of the plantain; a line of a triangle; for a triangle is that which is contained under three right-lines. But here we muft obferve, that not every total predicate is an effential one; thus, whitenefs is predicated of every fwan, becaufe it is inherent in every fwan, and at every inftant of time; but becaufe whitenefs does not conftitute the effence of a fwan, it is not effentially predicated; and this, firft, is one of the modes of effential predication of the greateft importance in demonftration. The fecond mode is of accidents, in the definition of which their common

mon fubject is applied: thus, a line is essentially inherent in rectitude, because in its geometrical definition, a line is adopted; for rectitude is no other than a measure, equally extended between the points of a line. In the same manner, imparity is contained in number; for what is that which is odd, but a number divided into unequal parts? Thus, virtues are resident in the foul, because, in their definition, either some part of the foul, or some one of its powers, is always applied. The third mode of essential predicates pertains to accidents which are inseparably contained in some particular subject, so as to exclude a prior existence in any other subject; such as colour in superficies. The fourth mode is of things neither contained in another, nor predicated of others; and such are all individuals, as Callias, Socrates, Plato. Causes are likewise said to exist substantially, which operate neither from accident nor fortune.

Thus, digging up the ground for the purposes of agriculture, may be the cause of discovering a treafure, but it is only an accidental one. But the death of Socrates, in despite of vigilance, is not the refult of a fortuitous cause, but of an essential one, viz. the operation of poison.

9. These posterior significations of essential predicates are added more for the sake of ornament than use; but the two former have a necessary existence, since they cannot but exist in the definition of names which predicate the essence of a thing, and in subjects which are so entirely the support of accidents, that they are always applied in their definition. But it is a doubt with some, whether those accidents are necessary, which cannot be defined independent of their common subject? To this we answer, that no such accident can, from its nature, be contained in every individual of any species; for curvature is not contained in

every

every line; nor imparity in every number; from whence we infer, that neither is curvature neceffarily exiftent in a line, nor parity in number. The truth of this is evident from confidering thefe accidents abftracted from their fubjects; for then we fhall perceive that a line may exift without curvature, and number without imparity,

Again, I call that an *univerfal* predicate, which is predicated of a fubject totally and effentially, and confidered as primarily and infeparably inherent in that fubject: for it does not follow that a predicate, which is total, fhould be immediately univerfal; for whitenefs is affirmed of every fwan, and blacknefs of every crow, yet neither univerfally. In like manner, a fubftantial predicate is not confequently an univerfal one; for the third mode of effential predicates, and the two following (inftanced before) cannot be univerfal. Thus, colour, although inherent in fuperficies effentially, is not inherent in every fuperficies, and confequently not univerfally. Thus again, Socrates, Gallias, and Plato, though they exift effentially, are not univerfals, but particulars; and thus, laftly, the drinking of poifon was an effential caufe of the death of Socrates, but not an univerfal one, becaufe Socrates might have died by other means than poifon. If then, we wifh to render an accurate definition of an univerfal predicate, we muft not only fay it is total and effential, but that it is primarily prefent to its fubject and no other. Thus, the poffeffion of angles equal to two right, primarily belongs to a triangle; for this affertion is effentially predicated of triangle, and is inherent in every triangle. This property, therefore, is not univerfally in figure, becaufe it is not the property of every figure, not of a fquare, for inftance; nor as univerfal in a fcalene triangle: for although it is contained in every fcalene, and in every equilateral, and ifof-

celes

celes triangle, yet it is not primarily contained in them, but in triangle itfelf; becaufe thefe feveral figures inherit this property, not from the particular fpecies to which they belong, but from the common genus *triangle*. And thus much concerning *total*, *effential*, and *univerfal* predicates.

10. Concerning that which is univerfal, we are frequently liable to err; often from a belief that our demonftration is univerfal, when it is only particular; and frequently from fuppofing it particular when it is, on the contrary, univerfal. There are three caufes of this miftake; the firft, when we demonftrate any particular property of that which is fingular and individual, as the fun, the earth, or the world. For fince there is but one fun, one earth, and one world, when we demonftrate that the orb of the earth poffeffes the middle place, or that the heavens revolve, we do not then appear to demonftrate that which is univerfal.

To this we anfwer: when we demonftrate an eclipfe of the fun to arife from the oppofition of the moon, we do not confider the fun as one particular luminary, but we deduce this confequence as if many other funs exifted befides the prefent.

Juft as if there were but one fpecies of triangles exifted; for inftance, the *ifofceles*; the equality of its angles at the bafe would not be confidered in the demonftration of the equality of all its angles to two right ones: but its triangularity would be effential, fuppofing every fpecies of triangles but the ifofceles extinct, and no other the fubject of this affection. So when we prove that the fun is greater than the earth, our proof does not arife from confidering it as this particular fun alone, but as fun in general; and by applying our reafoning to every fun, if thoufands befides the prefent fhould enlighten the world. This will appear

still

ftill more evident, if we confider that fuch conclufions muft be univerfal, as they are the refult of an induction of particulars : thus, he who demonftrates that an eclipfe of the fun arifes from the oppofition of the moon between the fun and earth, muft previoufly collect, by induction, that when any luminous body is placed in a right-line with any two others opaque, the lucid body fhall be prevented, in a greater or lefs degree, from enlightening the laft of thefe bodies, by the intervention of the fecond; and by extending this reafoning to the fun and earth, the fyllogifm will run thus :

Every lucid body placed in a right-line with two others opaque, will be eclipfed in refpect of the laft by the intervention of the fecond ;

The fun, or every fun, is a luminous body with thefe conditions ;

And confequently the fun, and fo every fun, will be eclipfed to the earth by the oppofition of the moon.

Hence, in cafes of this kind, we muft ever remember, that we demonftrate no property of them as fingulars, but as that univerfal conceived by the abftraction of the mind.

Another caufe of deception arifes, when many different fpecies agree in one ratio or analogy, yet that in which they agree is namelefs. Thus number, magnitude, and time, differ by the diverfities of fpecies; but agree in this, that as any four comparable numbers correfpond in their proportions to each other, fo that as the firft is to the fecond, fo is the third to the fourth ; or alternately, as the firft to the third, fo is the fecond to the fourth : in a fimilar manner, four magnitudes, or four times, accord in their mutual analogies and proportions. Hence, alternate proportion may be attributed to lines as they are lines, to numbers as they are numbers, and afterwards to *times* and

to bodies, as the demonstration of these is usually separate and singular; when the same property might be proved of all these by one comprehensive demonstration, if the common name of their genus could be obtained: but since this is wanting, and the species are different, we are obliged to confider them separately and apart; and as we are now speaking of that universal demonstration which is properly one, as arising from one first subject; hence none of these obtain an universal demonstration, because this affection of alternate proportion is not restricted to numbers or lines, considered in themselves, but to that common something which is supposed to embrace all these, and is destitute of a proper name. Thus too we may happen to be deceived, should we attempt to prove the equality of three angles to two right, separately, of a scalene, an isosceles, and an equilateral triangle, only with this difference, that in the latter case the deception is not so easy as in the former; since here the name triangle, expressive of their common genus, is assigned. A third cause of error arises from believing that to demonstrate any property inherent after some particular manner in the whole of a thing, is to demonstrate that property universally inherent. Thus, geometry proves * that if a right-line falling upon two right-lines makes the outward angle with the one line a right-angle, and the inward and opposite angle with the other a right one, those two right-lines shall be parallel, or never meet, though infinitely extended. This property agrees to all lines which make right-angles: but they are not primarily equidistant on this account, since, if they do not each make a right-angle, but the two conjointly are equal to two right, .they may still be proved equidistant. This latter demonstration then, is primarily and universally conceived; the other

* See the twenty-eighth proposition of the first book of Euclid's Elements.

whic

which always fuppofes the oppofite angles right ones, does
not conclude univerfally; though it concludes totally of all
lines with fuch conditions: the one may be faid to con-
clude of a greater *all*; the other of a leffer. It is this
greater *all* which the mind embraces when it affents to any
felf-evident truth; or to any of the propofitions of Euclid.
But by what method may we difcover whether our demon-
ftration is of this greater or leffer all? We anfwer, that
general affection which conftitutes univerfal demonftration
is always prefent to that fubject, which when taken away,
the predicate is immediately deftroyed, becaufe the firft of
all its inherent properties.

Thus, for inftance, fome particular fenfible triangle
poffeffes thefe properties:—it confifts of brafs; it is fca-
lene; it is a triangle. The query is, by which of thefe
we have juft now enumerated, this affection of poffeffing
angles equal to two right is predicated of the triangle?
Take away the brafs, do you by this means deftroy the
equality of its angles to two right ones? Certainly not:—
take away its fcalenity, yet this general affection remains:
laftly, take away its triangularity, and then you neceffarily
deftroy the predicate; for no longer can this property re-
main, if it ceafes to be a triangle.

But perhaps fome may object from this reafoning, fuch
a general affection extends to figure, fuperficies, and ex-
tremities, fince, if any of thefe are taken away, the equality
of its angles to two right can no longer remain. It is true,
indeed, that by a feparation of figure, fuperficies, and
terms, from a body, you deftroy all the modes and cir-
cumftances of its being; yet not becaufe thefe are taken
away, but becaufe the triangle, by the feparation of thefe,
is neceffarily deftroyed; for if the triangle could ftill be
preferved without figure, fuperficies, and terms, though thefe

g 2   were

were taken away it would ftill retain angles equal to two right; but this is impoffible. And if all thefe remain, and triangle is taken away, this affection no longer remains. Hence the poffeffion of this equality of three angles to two right, is primarily and univerfally inherent in triangle, fince it is not abolifhed by the abolition of the reft:——fuch as to confift of brafs; to be fcalene, or the like. Neither does it derive its being from the exiftence of the reft alone; as figure, fuperficies, terms ; fince it is not every figure which poffeffes this property, as is evident in fuch as are quadrangular, or multangular. And thus it is preferved by the prefervation of triangle, it is deftroyed by its deftruction.

11. From the principles already eftablifhed, it is plain that demonftration muft confift of fuch propofitions as are univerfal and neceffary. That they muft be univerfal, is evident from the preceding; and that they muft be neceffary, we gather probably from hence; that in the fubverfion of any demonftration we ufe no other arguments than the want of neceffary exiftence in the principles.

We collect their neceffity demonftratively, thus; he who does not know a thing by the proper caufe of its exiftence, cannot poffefs fcience of that thing; but he who collects a neceffary conclufion from a medium not neceffary, does not know it by the proper caufe of its exiftence, and therefore he has no proper fcience concerning it. Thus, if the neceffary conclufion c is a, be demonftrated by the medium B, not neceffary; fuch a medium is not the caufe of the conclufion; for fince the medium does not exift neceffarily, it may be fuppofed not to exift; and at the time when it no longer exifts, the conclufion remains in full force; becaufe, fince neceffary, it is eternal. But an effect cannot exift without a caufe of its exiftence; and

hence

higher in our fpeculations, and attentively confider the properties of demonſtration: one of which is, that predicate which is always found in the conclufion, and which affirms or denies the exiſtence of its fubject: another is, thofe axioms or firſt principles by whofe univerfal embrace demonſtration is fortified; and from whofe original light it derives all its luſtre. The third is the fubject genus, and that nature of which the affections and effential properties are predicated; fuch as magnitude and number. In thefe fubjects we muſt examine when, and in what manner a tranfition in demonſtrations from genus to genus may be allowed. Firſt, it is evident, that when the genera are altogether feparate and difcordant, as in arithmetic and geometry, then the demonſtrations of the one cannot be referred to the other. Thus, it is impoffible that arithmetical proofs can ever be accommodated with propriety to the accidents of magnitudes: but when the genera, as it were, communicate, and the one is contained under the other, then the one may transfer the principles of the other to its own convenience. Thus, optics unites in amicable compact with geometry, which defines all its fuppofitions; fuch as lines that are *right*, angles acute, lines equilateral, and the like. The fame order may be perceived between arithmetic and mufic: thus, the double, fefquialter, and the like, are transferred from arithmetic, from which they take their rife, and are applied to the meafures of harmony.

Thus, medicine frequently derives its proofs from nature, becaufe the human body, with which it is converfant, is comprehended under natural body. From hence it follows, that the geometrician cannot, by any geometrical reafons demonſtrate any truth, abſtracted from lines, fuperficies, and folids; fuch as, that of contraries there is the fame

<div align="right">fcience;</div>

fcience; or that contraries follow each other; nor yet fuch as have an exiftence in lines and fuperficies, but not an effential one, in the fenfe previoufly explained.

Of this kind is the queftion, whether a right-line is the moft beautiful of lines? or whether it is more oppofed to a line perfectly orbicular, or to an arch only. For the confideration of beauty, and the oppofition of contraries, does not belong to geometry, but is alone the province of metaphyfics, or the firft philofophy.

But a queftion here occurs, If it be requifite that the propofitions which conftitute demonftration fhould be peculiar to the fcience they eftablifh, after what manner are we to admit in demonftration thofe axioms which are conceived in the moft common and general terms; fuch as, if from equal things you take away equals, the remainders fhall be equal:——as likewife, of every thing that exifts, either affirmation or negation is true? The folution is this: fuch principles, though common, yet when applied to any particular fcience for the purpofes of demonftration, muft be ufed with a certain limitation. Thus the geometrician applies that general principle, if from equal things, &c. not fimply, but with a reftriction to magnitudes; and the arithmetician univerfally to numbers.

Thus too, that other general propofition:——of every thing, affirmation or negation is true; is fubfervient to every art, but not without accommodation to the particular fcience it is ufed by. Thus number is or is not, and fo of others. It is not then alone fufficient in demonftration that its propofitions are true, nor that they are immediate, or fuch as inherit an evidence more illuftrious than the certainty of proof; but, befides all thefe, it is neceffary they fhould be made peculiar by a limitation of their comprehenfive nature to fome particular fubject. It is on this

account that no one efteems the quadrature of Bryfo *, a geometrical demonftration, fince he ufes a principle which, although

* We are informed by Simplicius, in his Commentary on Ariftotle's third Category of Relation, " that though the quadrature of the circle feems to have been unknown to Ariftotle, yet, according to Jamblichus, it was known to the Pythagoreans, as appears from the fayings and demonftrations of Sextus Pythagoricus, who received (fays he) by fucceffion, the art of demonftration; and after him Archimedes fucceeded, who invented the quadrature by a line, which is called the line of Nicomedes. Likewife, Nicomedes attempted to fquare the circle by a line, which is properly called τιταρτημόριον, or *the quadrature*. And Apollonius, by a certain line, which he calls the fifter of the curve line, fimilar to a cockle, or tortoife, and which is the fame with the *quadratix* of Nicomedes. Alfo Carpus wifhed to fquare the circle, by a certain line, which he calls fimply formed from a twofold motion. And many others, according to Jamblichus, have accomplifhed this undertaking in various ways." Thus far Simplicius. In like manner, Boethius, in his Commentary on the fame part of Ariftotle's Categories (p. 166.) obferves, that the quadrature of the circle was not difcovered in Ariftotle's time, but was found out afterwards; the demonftration of which (fays he) becaufe it is long, muft be omitted in this place. From hence it feems very probable, that the ancient mathematicians applied themfelves folely to fquaring the circle geometrically, without attempting to accomplifh this by an arithmetical calculation. Indeed, nothing can be more ungeometrical than to expect, that if ever the circle be fquared, the fquare to which it is equal muft be commenfurable with other known rectilineal fpaces; for thofe who are fkilled in geometry know that many lines and fpaces may be exhibited with the greateft accuracy, geometrically, though they are incapable of being expreffed arithmetically, without an infinite feries. Agreeable to this, Tacquet well obferves (in lib. ii. Geom. Pract. p. 87.) " Denique admonendi hic funt, qui geometriæ non fatis periti, fibi perfuadent ad quadraturam neceffarium effe, ut ratio lineæ circularis ad rectam, aut circuli ad quadratum in numeris exhibeatur. Is fane error valde craffus eft, et indignus geometrâ, quamvis enim irrationalis effet ea proportio, modo in rectis lineis exhibeatur, reperta erat quadratura." And that this quadrature is poffible geometrically, was not only the opinion of the above mentioned learned and acute geometrician, but likewife of Wallis and Barrow; as may be feen in the Mechanics of the former, p. 517, and in the Mathematical Lectures of the latter, p. 194. But the following difcovery will, I hope, convince the liberal geometrical reader, that the quadrature of the circle may be obtained by means of a circle and right-line only, which we have no method of accomplifhing by any invention of the ancients or moderns. At leaft this method, if known to the ancients, is now loft, and though it has been attempted by many of the moderns, it has not been attended with fuccefs.

In the circle *g o e f*, let *g o* be the quadrantal arch, and the right-line *g x* its tangent. Then conceive that the central point *a* flows uniformly along the radius *a e*, infinitely produced; and that it is endued with an uniform impulfive power. Let it likewife be fuppofed, that during its flux, radii emanate from it on all fides, which enlarge themfelves in proportion to the diftance of the point *a* from its firft fituation. This being admitted, conceive that the point *a* by its impulfive power, through the radii *a n*, *a m*, &c. acting every where equally on the arch *g o*, impells it into its equal tangent arch *g r*. And when, by its uniform motion along the infinite line *a φ*, it has at the fame time arrived at *b*, the centre of the arch *g r*, let it impel in a fimilar manner the arch *g r*, into its equal tangent arch *g s*, by acting every where equally through radii equal to *b r*. Now, if this be conceived to take place infinitely (fince a circular line is capable of infinite remiffion) the arch *g o* will at length be unbent into its equal, the tangent line *g x*; and the extreme point *o*, will defcribe by fuch a motion

of

although true, is entirely common.   Previous to his de-
monſtration he ſuppoſes two ſquares deſcribed, the one

<div align="right">circum-</div>
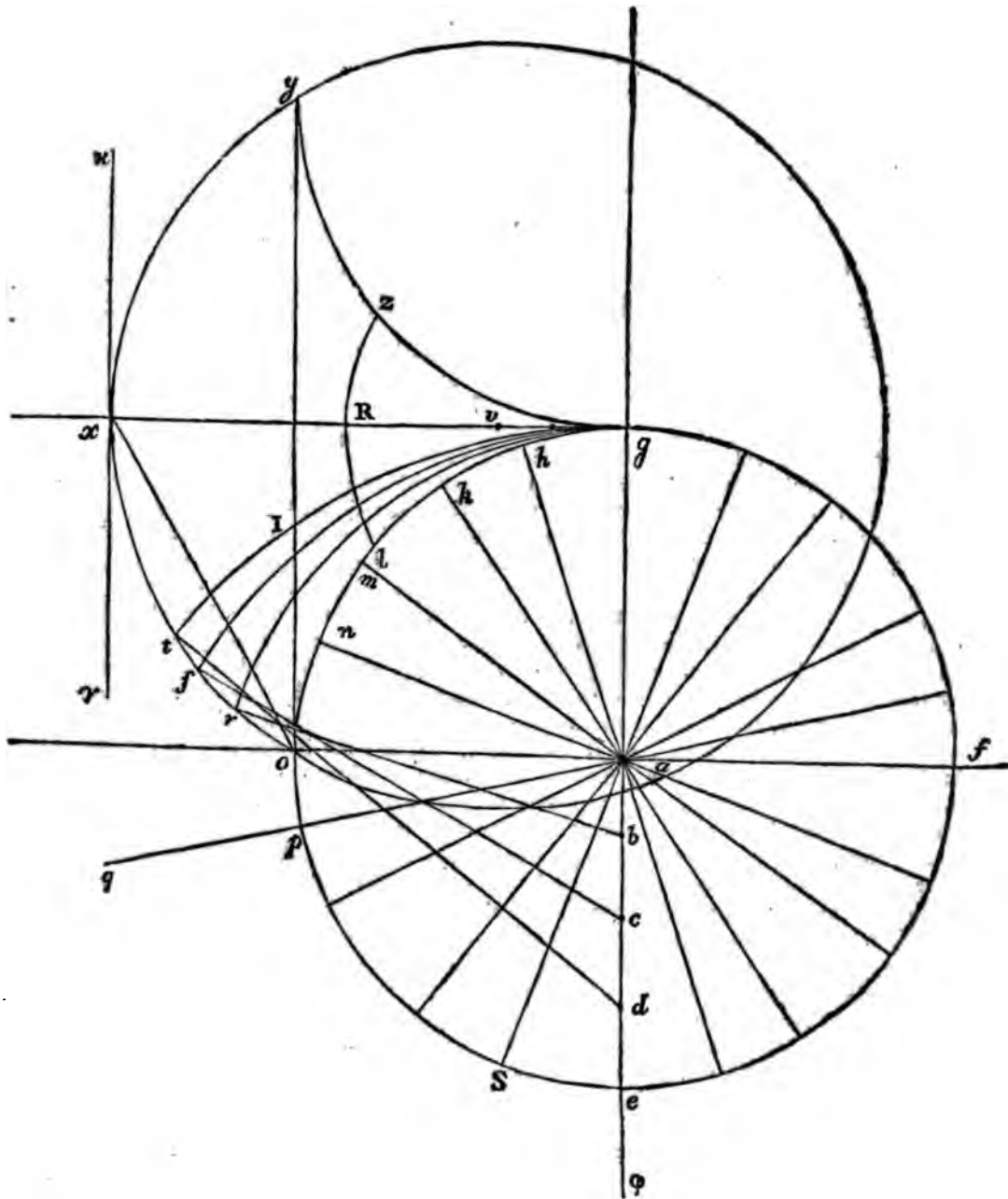
of unbending a circular line *o x*.   For ſince the ſame cauſe, acting every where ſimilarly and
equally, produces every where ſimilar and equal effects; and the arch *g o*, is every where

circumfcribing the circle, which will be consequently greater; the other inscribed, which will be consequently lefs than the given circle. Hence, becaufe the circle is a medium between the two given fquares, let a mean fquare be found between them, which is eafily done from the principles of geometry; this mean fquare, Bryfo affirms, fhall be equal to the given circle. In order to prove this, he reafons after the following manner: thofe things which compared with others without any refpect, are either at the fame time greater, or at the fame time lefs, are equal among themfelves : the circle and the mean fquare are, at the fame time, greater than the internal, and at the fame time lefs than the external fquare; therefore they are

equally remitted or unbent, it will defcribe a line fimilar in every part. Now, on account of the fimplicity of the impulfive motion, fuch a line muft either be ftraight or circular; for there are only three lines every where fimilar, i. e. the right and circular line, and the cylindric helix; but this laft, as Proclus well obferves in his following Commentary on the fourth definition, is not a fimple line, becaufe it is generated by two fimple motions, the rectilineal and circular. But the line which bounds more than two equal tangent arches cannot be a right line, as is well known to all geometricians; it is therefore a circular line. It is likewife evident, that this arch *o x* is concave towards the point *g* : for if not, it would pafs beyond the chord *o x*, which is abfurd. And again, no arch greater than the quadrant can be unbent by this motion : for any one of the radii, as *a p* beyond *g o*, has a tendency from, and not to the tangent *g x*, which laft is neceffary to our hypothefis. Now if we conceive another quadrantal arch of the circle *g o e f*, that is *g y*, touching the former in *g* to be unbent in the fame manner, the arch *x y* fhall be a continuation of the arch *x o*; for if *γ x x* be drawn perpendicular to *x g*, as in the figure, it fhall be a tangent in *x* to the equal arches *y x, x o*; becaufe it cannot fall within either, without making the fine of fome one of the equal arches, equal to the right-line *x g*, which would be abfurd. And hence we may eafily infer, that the centre of the arch *y x o*, is in the tangent line *x g*. Hence too, we have an eafy method of finding a tangent right-line equal to a quadrantal arch : for having the points *y, o* given, it is eafy to find a third point, as *s*; and then the circle paffing through the three points *o, s, y*, fhall cut off the tangent *x g*, equal to the quadrantal arch *g o*. And the point *s* may be fpeedily obtained, by defcribing the arch *g s* with a radius, having to the radius *a g* the proportion of 6 to 4; for then *g s* is the fixth part of its whole circle, and is equal to the arch *g o*. And thus, from this hypothefis, which, I prefume, may be as readily admitted as the increments and decrements of lines in fluxions, the quadrature of the circle may be geometrically obtained; for this is eafily found, when a right-line is difcovered equal to the periphery of a circle. I am well aware the algebraifts will confider it as ufelefs, becaufe it cannot be accommodated to the farrago of an arithmetical calculation; but I hope the lovers of the ancient geometry will deem it deferving an accurate inveftigation; and if they can find no paralogifm in the reafoning, will confider it as a legitimate demonftration.

equal

equal among themfelves. This demonftration can never produce fcience, becaufe it is built only on one common principle, which may with equal propriety be applied to numbers in arithmetic, and to times in natural fcience. It

is defective, therefore, becaufe it affumes no principle peculiar to the nature of the circle alone, but fuch a one as is common to quantity in general.

13. It is likewife evident, that if the propofitions be univerfal, from which the demonftrative fyllogifm confifts, the conclufion muft neceffarily be eternal. For neceffary propofitions are eternal; but from things neceffary and eternal, neceffary and eternal truth muft arife. There is no demonftration, therefore, of corruptible natures, nor any fcience abfolutely, but only by accident; becaufe it is not founded on that which is univerfal. For what confirmation can there be of a conclufion, whofe fubject is diffoluble, and whofe predicate is neither always, nor fimply, but only partially inherent? But as there can be no demonftration, fo likewife there can be no definition of corruptible natures; becaufe definition is either the principle of demonftration, or demonftration differing in the pofition of terms, or it is a certain conclufion of demonftration. It is the beginning of demonftration, when it is either affumed for an immediate propofition, or for a term in the propofition; as if any one fhould prove that man is rifible, becaufe he is a rational animal. And it alone differs in pofition from demonftration, as often as the definition is fuch as contains the caufe of its fubjects exiftence. As the following: an eclipfe of the fun is a concealment of its light, through the interpofition of the moon between that luminary and the earth. For the order of this definition being a little changed, paffes into a demonftration: thus,

> The moon is fubjected and oppofed to the fun:
> That which is fubjected and oppofed, conceals:
> The moon, therefore, being fubjected and oppofed, conceals the fun.

But *that* definition is the conclufion of demonftration,

<div align="right">which</div>

which extends to the material caufe; as in the preceding
inftance, the conclufion affirming that the fubjection and
oppofition of the moon conceals the fun, is a definition of
an eclipfe including the material caufe.

Again, we have already proved that all demonftration con-
fifts of fuch principles as are prior in the nature of things;
and from hence we infer, that it is the bufinefs of no fci-
ence to prove its own principles, fince they can no longer
be called principles if they require confirmation from any
thing prior to themfelves; for, admitting this as neceffary,
an infinite feries of proofs muft enfue. On the contrary,
if this be not neceffary, but things moft known and evident
are admitted, thefe muft be conftituted the principles of
fcience. He who poffeffes a knowledge of thefe, and applies
them as mediums of demonftration, is better fkilled in
fcience, than he who knows only pofterior or mediate pro-
pofitions, and demonftrates from pofterior principles. But
here a doubt arifes whether the firft principles of geometry,
arithmetic, mufic, and of other arts, can ever be demon-
ftrated? Or fhall we allow they are capable of proof, not
by that particular fcience which applies them as principles
or caufes of its conclufions? If fo, this will be the office
of fome fuperior fcience,—which can be no other than the
firft philofophy, to whofe charge the tafk is committed;
and whofe univerfal embrace circumfcribes the whole circle
of fcience, in the fame manner as arithmetic comprehends
mufic, or geometry optics.——This is no other than that
celebrated *wifdom* which merits the appellation of fcience
in a more fimple, as well as in a more eminent degree
than others : not, indeed, that all caufes are within its
reach, but fuch only as are the principal and the beft, be-
caufe no caufe fuperior to them can ever be found. Hence
the difficulty of knowing whether we poffefs fcience or
not,

:not, from the difficulty of underftanding whether it i founded on peculiar or common principles; fince it is ne -ceffary that both thefe fhould be applied in the conftitutio of all real knowledge and fcience.

* Again, axioms differ from poftulates in this:---the demand our affent without any previous folicitation, fron the illuftrious certainty they poffefs. Their truth may indeed, be denied by external fpeech, but never from in ternal connection. He who denies that equal things fhal remain from the fubtraction of equal, diffents, as Euripide fays, with his tongue, and not with his heart. But de monftration depends not on external fpeech, but on intel lectual and internal conviction; and hence, axioms deriv all their authority from intrinfic approbation, and not fron public proclaim. For the prompt decifions of the tongu are frequently diffonant from the fentiments concealed ir the fecret receffes of the heart. Thus the † geometriciar does not fpeculate thofe lines which are the objects of cor·

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* Axioms have a fubfiftence prior to that of magnitudes and mathematical numbers, bu fubordinate to that of ideas; or, in other words, they have a middle fituation between effentia and mathematical magnitude. For of the reafons fubfifting in foul, fome are more fimple an univerfal, and have a greater ambit than others, and on this account approach nearer to in tellect, and are more manifeft and known than fuch as are more particular. But others ar deftitue or all thefe, and receive their completion from more ancient reafons. Hence it i neceffary (fince conceptions are then true, when they are confonant with things themfelves) tha there fhould be fome reafon, in which the axiom afferting, *if from equals you take away equals* &c. is primarily inherent; and which is neither the reafon of magnitude, nor number, no time, but contains all thefe, and every thing in which this axiom is naturally inherent. Vide Syrian. in Arith. Meta. p. 48.

† Geometry, indeed, wifhes to fpeculate the impartible reafons of the foul, but fince fh cannot ufe intellections deftitute of imagination, fhe extends her difcourfes to imaginativ forms, and to *figures* endued with dimenfion, and by this means fpeculates immaterial reafon in thefe; and when imagination is not fufficient for this purpofe, fhe proceeds even to externa matter, in which fhe defcribes the fair variety of her propofitions. But, indeed, even ther the principal defign of geometry is not to apprehend fenfible and external form, but that in terior vital one, refident in the mirror of imagination, which the exterior inanimate form imitates, as far as its imperfect nature will admit. Nor yet is it her principal defign to be converfant with the imaginative form; but when, on account of the imbecility of her intel. lection, fhe cannot receive a form deftitute of imagination, fhe fpeculates the immaterial reafon in the purer form of the phantafy; fo that her principal employment is about univerfal and immaterial forms. Syrian. in Arift. Meta. p. 49.

poreal

poreal fight, but fuch as are exhibited by mental conception, and of which the delineations on paper, or in the duft, are no more than imperfect copies, notes, and refemblances. Thus, when he draws a pedal line which is not pedal, or an equilateral triangle which is not equilateral, we muft pay no regard to the defignations of the pen, but folely attend to the intellection of the mind; for the property demonftrated of fome particular line, is in the conclufion applied to one that is univerfal, and this true line could be no otherwife fignified to the learner than by a material defcription.

The certainty of axioms is, indeed, in a meafure obvious to every one. For what more evident than that nothing exifts of which it is poffible, at the fame time, to affirm and deny any circumftance of being? Indeed, fo illuftrious and indubitable is the light of this axiom, that in any demonftration we are afhamed to affign it the place of an affumption. It would almoft feem prolix and fuperfluous, fince there is nothing more manifeft and certain; and yet there are cafes in which it is neceffary to rank it among affumptions. And thefe take place whenever the intention is to conclude the exiftence of fomething as true, and of its oppofite as falfe. Thus, for inftance, in the demonftration that the world is finite, we affume this principle; and then reafon as follows:

Bound and infinite cannot be at the fame time affirmed and denied of any *body:*

The world is a body:

Therefore the world is not at the fame time finite and infinite.

And in this genus of demonftration, the major propofition ought always to affimilate with the conclufion. But the above axiom is not the only one obvious, for the following

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axioms after the fame manner as other arts;

contrary, they confirm and eftablifh their cer-

though with this difference, that the logician rea-

from probabilities, but the metaphyfician from

higheft certainty and evidence. Befides, we do not

* Syrianus, in his excellent Commentary on Ariftotle's Metaphyfics, (which does not fo much explain Ariftotle, as defend the doctrine of ideas, according to Plato, from the apparent if not real oppofition of Ariftotle to their exiftence), informs us that it is the bufinefs of wifdom, properly fo called, to confider immaterial forms or effences, and their effential accidents. By the method of refolution receiving the principles of being; by a divifive and and definitive method, confidering the effences of all things; but by a demonftrative procefs, concluding concerning the effential properties which fubftances contain. Hence (fays he, becaufe intelligible effences are of the moft fimple nature, they are neither capable of definition nor demonftration, but are perceived by a fimple vifion and energy of intellect alone. But middle effences, which are demonftrable, exift according to their inherent properties : fince, in the moft fimple beings, nothing is inherent befides their being. On which account we cannot fay that *this* is their effence, and *that* fomething elfe; and hence they are better than definition and demonftration. But in univerfal reafons, confidered by themfelves, and adorning a fenfible nature, effential accidents fupervene; and hence demonftration is converfant with thefe. But in material fpecies, individuals, and fenfibles, fuch things as are properly accidents are perceived by the imagination, and are prefent and abfent without the corruption of their fubjects. And thefe again being worfe than demonftrable accidents, are apprehended by figns, not indeed by a wife man, confidered as wife, but perhaps by phyficians, natural philofophers, and all of this kind.

rank

rank logic in the order of the fciences, becaufe it is deftitute of fome determinate genus or fubject, as it is neither converfant about lines, nor numbers, nor proportions. And its chief concern is about apparent properties, and not fuch as are effential to a fubject.

Hence, in logical difquifitions, we confidently employ interrogations, as equally fubfervient to the affirmation or negation of an opinion :---a method utterly impracticable, if we only employed thofe principles which are univerfally acknowledged ; fince it is impoffible of the fame thing to prove contrary properties,---as of the foul, that it is mortal and immortal; but he who demonftrates, affumes one definite part of a queftion, becaufe his purpofe is not to interrogate, but to trace out the latent paths of truth. And hence, if any one affirms that the foul is moved, and immediately after denies it, he is no longer a fubject worthy the exercife of our difcurfive and reafoning powers.

Again, it may fo happen, that the fame fcience at one time confiders *why* a thing is, at another only explains its exiftence, or that it exifts, without confidering the caufe. Thus, the fyllogifm which concludes by mediate propofitions, demonftrates without affigning the proper caufe: but that which determines by immediate ones, in a great meafure explains the caufe or reafon of exiftence. Thus, he who infers that trees do not breathe becaufe they are not animals, reafons from a mediate and fecondary caufe, becaufe there are many animals, fuch as infects, which exift without breathing : but he who infers this from their want of lungs, demonftrates from the immediate and primary caufe.

Thus, the following fyllogifm is a mediate one, or fuch as requires one or more mediums to eftablifh its certainty :

Every thing that is not an animal does not breathe;

A tree is not an animal;

Therefore a tree does not breathe.

lowing poffeffes equal certainty; that of every thing which
exifts, either affirmation or negation is true.   This axiom
is of great ufe in demonftrations leading to an abfurdity;
for he who demonftrates the impoffibility of any oppofite
affertion, neceffarily eftablifhes his own.   Hence it is we
affirm that the diameter of a fquare is either commenfur-
able or incommenfurable with its fide; and this general
principle is accommodated, and, as it were, defcends into
its proper matter as often as that which it poffeffes of
univerfal is contracted to a certain genus; for, as we have
previoufly obferved, common principles are not admitted
in demonftration without any reftriction; but then only
when their general nature is limited to fome particular
fubject, by which they become peculiar and appofite.

14. * Wifdom, or the firft philofophy and logic, agree
in not ufing axioms after the fame manner as other arts;
but on the contrary, they confirm and eftablifh their cer-
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a wife man, confidered as wife, but perhaps by phyficians, natural philofophers, and all of
this kind.

rank

Here the major propofition is evidently mediate, becaufe we are ftill to feek why that which is not an animal does not breathe, which the following immediate fyllogifm folves.

>Every thing that is not endued with lungs does not breathe ;

>Every thing that is not an animal is not endued with lungs ; ergo,

>Every thing that is not an animal does not breathe.

Again, the fame fcience may demonftrate the exiftence of a thing, or that it exifts, and the caufe of fuch exiftence, as often as it affigns two immediate reafons; but the one from the proper caufe, the other only from a fign. Thus, he who demonftrates the increafe of the moon, from the plenitude of her orb, infers the caufe of fuch increafe; but on the contrary, he who collects the plenitude of her orb, from her increafe, reafons only from a fign, and can alone declare its exiftence. And, indeed, it often happens that the *caufe* and *fign* reciprocate, fo that as from the fign we advance to the caufe, demonftration from the caufe fre- quently recurs to the *fign*. Thus, from the breadth and firmnefs of the bafis, we collect the permanent duration of the pyramid; and from its extended exiftence we infer the ftrength of its fupport. Whenever, then, the argument originates from a fign, it gives evidence to the conclufion, as from fomething more known than its caufe. When it begins from the caufe, it proceeds from that which is firft in the order of nature, to that which is laft, and reafons as from the proper principle of the thing.

Sometimes the *caufe* and *fign* do not reciprocate. Thus, although wherever there is fmoke, we infer the exiftence of fire; yet we cannot infer, that wherever there is fire fmoke exifts. Thus, from the palace and the picture we collect the exiftence of the architect and painter; but the laft may exift without the firft;—the living architect with-

out

out the actual palace; and the living painter without the energies of his art. And thus it is that the caufe is illuftrated by its fign; but not always the fign by its caufe.

Hence then, as all caufes do not reciprocate with their effects; fo neither is it always caufes and effects which do reciprocate: becaufe a multitude of figns, mutually inferring each other, may accompany a certain caufe. Thus, the figns which attend the caufes of a fever, are a quick pulfation of the artery, and an intenfe heat: and thefe figns mutually affert each other; but no fyllogifm can be compofed from either expreffing the *why*, but only fimply *that* the other exifts.

15. We now propofe to confider the mode in which the two preceding demonftrations are diftributed in different fciences. When fciences then are fo related, that the one is dependent on the other, as optics on geometry, navigation on aftronomy, and mufic compofed by the arbitration of the ear, on that which confifts in the knowledge of mathematical proportions: in this cafe, the demonftration of fimple exiftence, or *that* they *exift*, pertains to the fcience of fenfibles; but the demonftration *why* they *exift* to the fcience which is fpeculative and mathematical.

Thus the mathematician fpeculates the caufes of a certain fenfible effect, without confidering its actual exiftence; for the contemplation of univerfals excludes the knowledge of particulars; and he whofe intellectual eye is fixed on that which is general and comprehenfive, will think but little of that which is fenfible and fingular. Thus, by mathematics we may learn the refponfive harmony of the *laft* *chord*, and its confonance with the *mean*; but we cannot perceive this concord, if unaccuftomed to the practice of the mufical art. In fine, thofe fciences which are more of a mathematical nature, I mean fuch as are more amply

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converfant with the infpection of things, confidering their forms abftracted from every material fubject, always de-. monftrate the *why*; and fuch is geometry in refpect of optics. Thus geometry confiders only fuch things as are peculiar to right-lines, independent of every fenfible con-. nection. For the geometrician does not inveftigate a right-line as contained in ftone or brafs; but confiders it as entirely detached and unconnected with any object of fenfe.

On the contrary, optics receives a right-line juft as it is perceived in a rule, or engraved in brafs. And, indeed, in treating of fome particulars, natural fcience has the fame relation to optics, as optics to geometry. Thus, in confidering the reafon of the appearance of the rainbow, the natural philofopher defines the bow to be an image refracted from a certain cloud againft the fun; but why it is endued with fuch a form, and feen with fuch a colour, muft be affigned by him who is fkilled in optics. There are, again, fciences, one of which is not fubordinate to the other, becaufe founded on principles totally different; yet, in fome particulars they agree with the preceding. Thus, to know that an orbicular wound is the moft diffi- cult of cure, belongs to the phyfician; but to know *why*, to the geometrician.

16. Of all fyllogiftic figures, the firft is the beft adapted to fcience, fince the arithmetician, geometrician, and laftly all thofe who demonftrate any effect from its proper caufe, fabricate their reafonings according to this figure. For the middle figure is feldom ufed, becaufe only adapted to a few occafions: and fince the knowledge of the *why* is of all others the moft important, which is alone obtained by this figure: hence, in the purfuit of fcience, it is always preferred before the reft. Befides, it is equally acommo- dated to the knowledge of final caufes; to which it alone

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tends : for it compofes definitions from words univerfal, and affirmative. In the fecond figure, a complex negative is conceived; and in the laft, a particular one. Add to this, that mediate propofitions are no other ways reducible to immediate ones than by this figure, in which the mediate propofition tends, by a continued feries, to that which is immediate. But the fecond does not conclude affirmatively, nor the laft univerfally; from whence it appears, that a mediate propofition can never become immediate by thefe figures : not that all affirmative propofitions are immediate ones, fince fome negatives are of this kind; for all propofitions are equally immediate, which cannot be confirmed by fyllogifm; and fuch are thofe negatives, of whofe terms it is impoffible any genus can be affirmed. Thus the propofition, *no fubftance is quality*, is an immediate negative of this kind, whofe terms are two of the moft univerfal genera of things.

Again, as we have frequently affirmed that he who demonftrates, always affumes fuch things as are effentially predicated; but that he who argues dialectically or topically, not always, but generally affumes fuch as are accidentally predicated, and which appear more probable and known than fuch as are effentially inherent; it is proper we fhould define what is meant by accidental predication; or fomething predicated by means of another. Indeed, the term has a diffufe fignification : for, firft, a body is faid to be white by fomething elfe, becaufe by its fuperficies; and in this manner vines are white, becaufe their branches are white. Thus, if accident be predicated of accident, it is, by means of another; as when we fay the mufician is fair; for the being a mufician is an accident of man, and the being fair of the mufician : and man is the fubject of each. The predicate of fubftance is equally accidental, when not

included

included in the number of things fubftantially inherent; as when we affirm of any particular man that he is red, or black. But the predication is especially accidental, as often as, by perverting the order of nature, fubftance is predicated of accident; as when we fay fomething white is an animal: for this affertion differs from that other, *animal is white.* In the latter, the fubject *animal* is neither inherent in another, nor fubfifts by another, but has an effential exiftence. In the former, what is affumed as a fubject derives its exiftence from that of which it is the accident. It is only dialectically, therefore, that we can argue from predicates as probable and known without any diftinction: but in demonftration, all that are prepofterous and accidental muft be carefully avoided, excepting fuch accidents as being effentially in a fubject, admit of an effential predication; and fome of thefe we have enumerated before.

17. We are now entering on a difquifition neither ignoble nor ufelefs: it is this, whether the number of things predicated effentially of a fubject is finite, or whether things in a continued feries run on to infinity. For inftance, let us fuppofe fome ultimate fubject, which is not the predicate of any thing befides; and let c reprefent fuch a fubject, of which b is the firft and immediate predicate; and in the fame manner d of b, and e of d: the query is, Whether or not this extraction muft neceffarily ftop, or will admit of an immenfe progreffion, fo that f may be predicated of e, and g of f, and fo on infinitely; the power of the predicates, which fupplies the common identity, ftill remaining inexhauftible and undiminifhed? The fecond query is this, Suppofing fome general fubject, which we call a, of fuch a nature as to be no longer the fubject of any farther predication, but to be itfelf the fupreme and primary predicate; and fuppofing that it is immediately

inherent

inherent in *f*, and *f* in *e*, and *e* in *g*, whether or not the procefs muft ftop, or extend to infinity, and no fubjeft be found which is not diredtly predicable of another? There is a remarkable difference in the two confiderations; for, in the former we enquire whether any ultimate fubjeft can fupply an infinite afcent of predicates; in the latter, whether any firft predicate can exift in an infinite defcending feries of fubjects. The third queftion is, fuppofing two extremes conftituted from a firft predicate and laft fubjeft, whether it is poffible an infinite number of mediums can intervene? And this is no other than to enquire whether demonftrations admit of an infinite progreffion, fo that whatever is affumed in proof of another, muft be proved itfelf? Or whether it is not more agreeable to truth, that there fhould be fome immediate propofitions and ultimate terms, whofe difcovery may give refpite to enquiry, and ftay the elaborate procefs of demonftration? The fame queftion occurs in negatives. But that fome of thefe are immediate, the inftance lately alledged fufficiently evinces. The folution of this enquiry is not fo difficult in fubjeds which mutually reciprocate; for in thefe, when the ultimate fubjeft is given, no one can doubt the exiftence of their primary predicate; nor when the primary predicate is admitted, can there be any doubt of the exiftence of fome ultimate fubjeft. For, in things which mutually reciprocate, whatever is enquired of the one, is immediately queftioned of the other; and wherever there is a laft fubjeft, there muft be a firft predicate; for by the converfion of the ultimate fubjeft you effeft the primary predicate.

Previous to the difcuffion of the firft queftion, it is neceffary to know that infinite intermediates cannot intervene between two finite terms in an afcending and defcending feries of predications. I call the feries afcending which

rifes

rifes to univerfals; but defcending, which, by a contrary procefs, ftops at particulars. Thus, if any one admits that *a* is fome firft predicate, and *g* fome ultimate fubject, and fhould contend, that between thefe terms there may be infinite mediums, he contradicts himfelf; fince he who begins from *a* in a defcending progreffion, will never, by this means, arrive at *g*; and he who departs from *g* in an afcending feries, can never finally rife to *a*. So that the extremes can be no longer finite, as the hypothefis admitted. Indeed, the abfurdity of fuch a fuppofition is the fame as to contend that between one and ten, an infinity of numbers may exift; which is evidently impoffible, becaufe the difcrete nature of numbers excludes their actual exiftence in infinitum, between any finite limits; fince they can only become infinite from their actual exiftence and precedence, and not from any dormant power or capacity they poffefs: for between any two given numbers there is nothing fimilar to number in capacity, which can ever become number in energy; as in quantity continuous between any two points there are always parts in capacity, which, whenever a proper agent is at hand, become immediately actual. In like manner, he who admits the terms finite, but believes that the mediums are infinite, afferts what is impoffible, fince thefe logical predications are of the fame difcrete nature with numbers themfelves. Thus all the predicates which can exift between Socrates and fubftance, muft exift actually, or not at all; for furely between thefe two terms, or periods, no predicate in capacity can ever be fuppofed to fubfift. If it be urged, that the capacity of receiving thefe predicates exifts between Socrates and fubftance, ftill we reply, it is not that kind of capacity in which thefe predicates can retain the moft fhadowy exiftence; out of which they can ever be called forth into

energy,

energy, as from some latent retreat; or into which they can finally retire, when energy is no more. And hence we conclude it impoffible that infinite mediums can exift between any finite terms.

18. It now remains that we prove, firft, by probable arguments, and then by fuch as are demonftrative, that the extremes in any feries of predications are finite; and that an infinite progreffion is impoffible, not only in fub-ftantial predicates, but in fuch as are accidental. For every thing predicated of another is either effentially or acciden-tally inherent; and is predicated in a natural or prepofterous order. It is predicated according to nature, when accident is declared of fubftance; contrary to nature, when fubftance of accident. That effential predicates are finite, appears from hence, becaufe a contrary hypothefis excludes the exiftence of definition, by admitting that all things are contained in fome fuperior genus, and acknowledge fome farther definition; fince it is impoffible that the definitions of genus can ever be circumfcribed, while there is a conti-nual fupply of other genera, which can never be known without definition; for thus we fhall never obtain either a beginning or an end. But to define all things is not poffible, becaufe infinity can never be abfolved by the moft unwea-ried progreffion. Predications then, of this kind, are always circumfcribed by a certain number of terms, which prevent their infinite procefs, and caufe all the ftrength of demonftration, and all the certainty of human knowledge. The fame may be proved in accidents; for fuch as are pre-dicated of fubftance, are either predicated as qualities or quantities, as relatives, or as actions and paffions; as ex-preffive of fome habit, or fignificant of fome place; or as connected with fome time. Thus we fay the wood is white, the triangle is fcalene; whitenefs being accidental

to the wood, and fcalenity to the triangle. It is therefore certain, that every accident is predicated of fubftance; and it is no lefs certain that the predicates of fubftance are finite, fince they are all included in the ten univerfal genera of things.

19. We have hitherto defended the impoffibility of an infinite progreffion of logical predicates and fubjects, in a demonftrative procefs, by fuch arguments as are dialectical and common : it now remains that we adopt fuch as are peculiar and certain. Demonftrations, then, are derived from affections effentially inherent in a fubject; and thefe are either fuch as take place in definitions of a fubject, as multitude and quantity, are effentially predicated of number; or, fecondly, accidents which are defined from their fubjects, as imparity by number. But the predication cannot, in either cafe, be extended to infinity. For it is not neceffary that in the fame manner that imparity is predicated of number, fomething elfe, fuppofe c, fhould be predicated of imparity; and fo imparity be contained in its definition, fimilar to number in the definition of imparity. For in predications of this kind, the terms are always affumed more contracted than their fubject; and at length, by a continued proceffion, muft terminate in an indivifible. Thus, as imparity is more contracted than number, c muft be more contracted than imparity. Hence, thefe predications either finally ftop, for the reafons we have affigned; or becaufe whatever is predicated of imparity, is neceffarily predicated of number; fo that one thing as number would be actually contained in the definition of an infinity of things; and fo actual infinity muft enfue, which is abfurd. Laftly, whatever is faid to refide in the terms, muft be allowed to refide in the fubject; fo number muft be applied in the definition of every affection;

and

and an infinite number of properties will be effentially in-
herent in number; and number will inherit infinite de-
finitions. But affections effentially refident in a fubject
cannot be infinite, becaufe it is neceffary they fhould exift
in energy. Thus, imparity cannot exift potentially in
number; nor reafon in man; nor rotundity in a circle,
becaufe wherever thefe fubjects have an actual being, it is
neceffary thefe effential attributes fhould be actually inhe-
rent. Again, in the definitions of a fubject, an infinite
procefs is impoffible, becaufe from fuch an hypothefis no-
thing could ever be defined; and thus it appears that neither
can demonftrations be infinitely extended, nor every thing
admit of demonftration, an opinion we have already noticed
in the beginning of this fection: for if neither univerfally,
nor in every propofition a middle term can be affumed, but
as foon as we arrive at immediate propofitions, the labour
of inveftigation is finifhed, the poffibility of demonftrating
every thing can no longer be defended; fince it is proved
above, that by limiting the extremes, an infinite number
of mediums is neceffarily excluded.

And thus, by taking away infinity from the reafoning
art, we have given a fupport to fcience, which the moft
vigorous efforts of fubtle fophiftry can never finally fubvert.
We have fet bounds to that reftlefs fpirit of enquiry which
wanders uncontrouled in the mind unenlightened by fcience,
by every where circumfcribing its progrefs within the limits
of that which is moft particular, and moft univerfal, a firft
predicate, and an ultimate fubject: and finally, by affert-
ing that all the evidence of human knowledge refults from
the luftre of primary and immediate principles, we have
held up a fteady and permanent light, ever fufficient to
direct our fteps through the dark mazes of ignorance and
error, into the bright paths of certainty and truth.

20. Let us next confider whether univerfal demonftration is preferable to particular, or not. And firft, in favour of particulars we may fay that their evidence is more exquifite and certain than that of univerfals. Thus, the knowledge, from infpection, that Callias is a rational animal, is fuperior to that acquired by a reafoning procefs which infers his rationality, becaufe every man is a rational animal. By particular demonftration a thing is known as it is, by univerfal only in common. Befides, particulars poffefs fome folidity, univerfals none: and the demonftration of things which have a real exiftence, is more excellent than that of things which have none. And there are no errors more frequent than thofe about univerfals; demonftration confidering them as things entirely abftracted from fingulars. On the contrary, particulars are ufurped by the fight, grafped, as it were, by the hand, and the general fubject of every fenfe; fo that concerning thefe, demonftration affirms nothing falfe or inconftant. But thefe reafons, however plaufible, are eafily confuted. And, firft, the term effential is more clofely connected with univerfals than particulars. Thus the poffeffion of three angles equal to two right, is an affection more effential to the triangle itfelf, than to one equilateral or fcalene. Add too, that in the demonftration of univerfals we always infer fome property of a fubject from its fimple exiftence, on becaufe it is fuch a fubject. Again, many affections are contained in fingulars affumed from no particular nature, but from that which is univerfal; as rationality in Socrates, which is not inferred from his exiftence as Socrates, but from his exiftence as man. Farther, that demonftration is the more excellent which is derived from the better caufe: but an univerfal caufe is more extended and excellent than a particular one; fince the arduous inveftigation

of

of the *why* in any fubject is ftopt by the arrival at univer-
fals. Thus, if we defire to know why the exterior angles
of a triangle are equal to four right ones, and it is an-
fwered, becaufe the triangle is ifofceles; we again afk, But
why becaufe ifofceles? And if it be replied, becaufe it is a
triangle, we may again enquire, But why becaufe a triangle?
To which we finally anfwer, becaufe a triangle is a right-
lined figure; and here our enquiry refts at that univerfal
idea which embraces every preceding particular one, and
is contained in no other more general and comprehenfive
than itfelf. Add too, that the demonftration of particulars
is almoft the demonftration of infinites; of univerfals, the
demonftration of finites.—We add farther, *that* demon-
ftration is the beft, which furnifhes the mind with the moft
ample knowledge; and this is alone the province of univer-
fals. Again, the principles of fcience become immediate
only in proportion as the demonftration becomes univerfal;
and he who knows univerfals, knows particulars in capacity:
but we cannot infer, that he who has the beft knowledge
of particulars, knows any thing of univerfals. Laftly, that
which is univerfal, is the province of intellect and reafon,
particulars are the offspring of fenfe; and hence we con-
clude that univerfal demonftration exceeds particular both
in dignity and excellence, and is firft in the nature of things,
although laft in the progreffions of the reafoning power.

Again, That affirmative demonftration is fuperior to
negative, appears from hence: the affirmative does not
require the affiftance of the negative; but the negative
cannot exift without the affirmative; on which account, the
demonftration compofed from negatives alone, is incapable
of producing real evidence and conviction. Befides, affirma-
tion exceeds negation both in priority and fimplicity of
exiftence.

Again,

Again, the demonstration which concludes *directly*, is better than that which confirms a proposition by evincing the abfurdity of its contrary. The firft proceeding in a regular order, eftablifhes, by a natural deduction, the truth which was firft advanced. The fecond taking a wider circuit, yet with the fame intentions produces a conclufion quite oppofite to its apparent defign. The one may be compared to the open attack of a valiant and fkilful foldier, who expects the conqueft of his enemy from ftrength and courage alone: the progrefs of the other refembles the fame foldier, uniting force with ftratagem, and advancing, by an irregular march, which his foe miftakes for a retreat, but finds the fecret caufe of his deftruction. The firft is fimple and impromifcuous, as compofed from propofitions alone: the fecond is compound and mifcellaneous, calling in hypothefis to its affiftance.

21. One fcience is faid to be prior to, and more certain than another in many refpects;——when the one reafons from primary caufes, but the other from fuch as are fecondary:——when the one may be ranked in the genera of intelligibles and univerfals; but the other in the genera of fenfibles and particulars. And fuch is the relation of arithmetic to mufic; of geometry to optics; and laftly, of every fuperior to every fubordinate fcience. Again, this happens when the one reafons from fimple principles, the other from fuch as are complex and connected; on which account arithmetic feems to poffefs greater certainty than geometry. For the principle of arithmetic is unity; but of geometry a point; and unity is without pofition, with which a point is always connected. And in this manner geometry inherits greater evidence than aftronomy; for the one confiders body fimply, the other as connected with a circular motion. The fcience is called one which contemplates

templates actions belonging to one genus: the genus is one which poffeffes the fame firft principles; and hence geometry and ftereometry form one fcience. On the contrary, the fciences are called different which have different principles, fuch as geometry and optics; the latter of which does not originate from the principles of the former.

Again, the fame thing may admit of many demonftrations, and may be known from many mediums: at one time from the application of fuch as are congenial: at another, from thofe of a different order or genus. From congenials, as when we demonftrate that the plantain is a fubftance, firft; by the medium of a tree, and then by the medium of a plant, thus:

   Every tree is a fubftance;
   The plantain is a tree:
   Therefore the plantain is a fubftance. And again,
   Every plant is a fubftance:
   The plantain is a plant:
   Therefore the plantain is a fubftance.

We demonftrate, from mediums, of a following order or genus, as when we prove man to be a fubftance, at one one time from his being rational, at another from his being a biped; and thefe mediums, in part, mutually contain each other.

22. Fortuitous events can never, in any fcience, become the fubject of demonftration; fince they are neither limited by neceffity, nor admit the arrangement of fyllogifm. Indeed, fo far from obtaining a neceffary, they do not poffefs a frequent exiftence, but every fyllogifm is compofed from one or other of thefe.

Again, fcience is not the bufinefs of fenfe, fince that which is univerfal is the object of perception in particulars themfelves. For the object of fight is colour in general,

and not this particular colour: the object of hearing is found in general, and not any particular found; and on this account we fee or hear not only this or that colour or found, but likewife every other which falls under the cognizance of thefe fenfes. Hence, if it were poffible for any one to difcern by his fight, the equality of the three angles of fome particular triangle to two right, he would not by this means poffefs a demonftration of the conclufion which affirms this to be the property of every triangle; but his knowledge would extend no farther than the triangle he infpects. Thus too, if we could perceive an eclipfe of the moon to arife from the interpofition of the earth, we could not univerfally conclude that this is the caufe of every eclipfe, but only of the particular one we behold. For the explication of caufes extends to univerfals; and comprehends not only the knowledge of one particular defect of the moon, but fimply of every eclipfe; fince the interpofition of the earth is not fo much the caufe of any prefent eclipfe, as of all which can poffibly exift in every age. Whenever, then, the caufe is univerfal, the knowledge of any effect deduced from fuch a caufe is, in every refpect, fuperior to the evidence arifing from the perceptions of fenfe. It is likewife more excellent than the apprehenfion which fubfifts independent of the proper caufe; as if any one fhould give abfolute credit to the propofition, that the three angles of a triangle are equal to two right, without a previous conviction that the external angle of a triangle, is equal to the two interior oppofite ones; and without applying this laft propofition as the caufe of the firft. The comprehenfion, then, which is conjoined with the proper caufe, far exceeds the ftrongeft evidence of fenfe.

But perhaps it may be faid that fcience confifts in fenfe, becaufe the fcience of any particular, fails from a defect of

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the fenfe by which it is apprehended. To this we reply, that fcience, indeed, is not acquired without the affiftance of fenfe, but it does not follow from hence, that *to perceive* is *to know*; becaufe the object of fcience is that which is univerfal; but of fenfe, that which is particular. Thus, if we could fee light penetrating the pores of glafs (on the atomical hypothefis) the caufe why it illuminates would be manifeft from fenfible infpection as the means, and from the univerfal apprehenfion of fcience, by which we fhould underftand this to be univerfally true.

Again, the principles of all fciences cannot be the fame neither confidered as remote or proximate. Not confidered as proximate, becaufe the principles always correfpond to the demonftrated conclufions; but thefe are not the fame, fince they are often generically different; and confequently the propofitions from which they refult muft be derived from difcordant genera. But propofitions confift of fuch things as effentially exift; and hence we infer, that the principles of geometry are effentially diftinguifhed from thofe of arithmetic, that they cannot admit of reciprocal accommodation, fo that the one may be predicated, or become the fubject of the other, and that the one can never be fubfervient as a medium to the other. Again, common and firft principles are not applied in every fcience; fuch as this, that every thing muft either be affirmed or denied. Nor can any thing be proved by their affiftance alone, but as often as thefe are required in demonftration, other principles more proximate and peculiar to the given propofition, muft always be adopted. Again, axioms univerfally conceived, cannot be affumed in fyllogifm, but they muft be contracted, as it were, to fome fubject genus. Of this kind is that common axiom, that as often as any four quantities are proportionable, by permutation, or changing

the order of the terms, the fame ratio will refult. For the arts apply this axiom in a reftricted fenfe; geometry, by confidering the relatives as four magnitudes, and arithmetic as four numbers; but the natural philofopher, by adapting the comparifon to four motions, or four times. Befides, if the principles of all fciences were the fame, it is neceffary they fhould be comprehended by fome certain number, fimilar to the limitation of the elements: but every fcience is capable of immenfe increafe from the many different modes of amplification the conclufions will admit; and confequently it is requifite to eftablifh a correfpondent number of proper principles; for fuch as are common cannot be alone fufficient. Laftly, if the fame principles accord with every fcience, it follows, that any thing may be demonftrated from fuch principles: but the certainty of geometrical conclufions cannot be eftablifhed from the principles of mufic; and from hence it follows, that although the principles of every fcience are not the fame, they do not poffefs an entire diverfity, nor yet an abfolute affinity of nature.

23. There is a remarkable difference between fcience and opinion. Whatever is the fubject of fcience muft have a neceffary exiftence; on the contrary, opinion is converfant with things liable to mutation and decay. Again, as fcience depends on neceffary propofitions for fupport, fo opinion on fuch as poffefs only a poffibility of exiftence; and fo there is one mode of approbation in fubjects of opinion, and another in thofe of fcience. Hence fcience is diftinguifhed from opinion by two difcriminations, the one arifing from their fubjects, the other from the mode of approbation. That opinion is converfant with things poffible or contingent, we may learn from hence; contingencies cannot belong to fcience, becaufe their exiftence is

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not neceffary ; nor to intellect, or that principle of fcience by which its terms are known; nor to the apprehenfion or belief of immediate propofitions, called indemonftrable fcience. Hence, if every habit by which truth is known, is either fcience, or intellect, or opinion, it remains that opinion alone confifts of things which are, indeed, true; but not neceffary. It is, therefore, inconftant and unftable, from the mutable nature of its fubjects. Befides, no one thinks he poffeffes an opinion of things which he believes to have a neceffary exiftence, fo that they cannot be otherwife than they are; but to fuch conviction he properly gives the name of knowledge, and to its contrary the name of opinion.

. Again, the fame thing from the fame propofitions may at one time become the fubject of knowledge, at another, of opinion; and this happens according to the different formation of the fyllogifm which the propofitions compofe; whether reafoning from the proper caufe it explains the *why*, or only fimply declares a thing exifts. Hence a doubt arifes why opinions of this kind may not be called fcience, fince both the fubjects and propofitions are the fame? The folution is obvious. If it is believed that the propofitions cannot be otherwife than they are, or that they have a neceffary exiftence, fuch an affent of the mind is not opinion, but fcience; becaufe things which inherit an effential exiftence are the ornaments of fcience alone. On the contrary, if we are convinced that the propofitions are true, but at the fame time not neceffary, fuch conviction is not fcience, but opinion. Hence, it is impoffible that fcience and opinion can be the fame, fince they vary in their definition and mode of approbation, and in a different manner demand our belief. Similar to this, although it may happen that of the fame thing a true and a falfe opinion may arife, it

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will not therefore follow, that true and falfe opinions are
the fame. For that which is firm and conftant can never
be the fame with that which is mutable and frail; and that
which is always true muft be effentially different from that
which may be changed into falfe. By the power of habit
indeed in different men, the fame•thing may be compre-
hended by opinion and fcience. Thus it was opinion in
Epicurus when he faid that the fun was eclipfed by the
moon paffing under its orb, becaufe he thought it might
otherwife happen, and that the moon might be interpofed
without obfcuring the light of the fun. It was fcience in
Hipparchus, becaufe he knew it as a neceffary event. But
in the fame mind, at the fame time, and of the fame
thing, it is impoffible that fcience and opinion can exift.
And thus much concerning the difference of the two.

24. Laftly, fagacity is an acute and fudden apprehenfion
of the medium, or proper caufe of a certain effect : as if
any one, beholding the moon, fhould in a moment con-
jecture the caufe of the part oppofite to the fun being
lucid, and the other parts obfcure, becaufe fhe derives her
fplendor from the fun. Hence he is univerfally called acute
and fagacious, who, from the afpect or hearing of the ex-
tremes, can readily perceive the medium which exifts
between them : as the term imports a certain revolution
of the conclufion into its firft propofitions, and, as it were,
a fwift comprehenfion and continuation of the medium.

SECTION III.

IN the enfuing Commentaries, the foul is confidered as immaterial; and as poffeffing a middle nature between intelligibles and fenfibles; but that this important affertion may not remain without proof, the following demonftrative arguments, derived from the Platonic philofophy, are offered to the reader's confideration. And firft, that the foul is an unextended, and confequently immaterial effence, may be thus proved. If, after the manner of magnitude, it confifted of continuous parts, it would be impoffible that any one part could be fentient from the paffion of another; but the foul, for inftance, which is fituated in the finger, would be fenfible of paffion, as if detached from foul in the other parts of the body, and exifting by itfelf; for the foul, from this hypothefis, would be fentient by parts, and not confidered as a whole. Befides, there muft be many fouls governing each part of us, different among themfelves, and endued with their own peculiar energies. For whatever may be faid of continuity, is to no purpofe, unlefs it conduces to unity of fenfation; fo that the hypothefis which fuppofes that the fenfations gradually arrive at the principal part of the foul by a certain continued fucceffion, is not to be admitted, fince it may be reafonably afked, How is the principal part to be peculiarly diftinguifhed? By what rule of quantity can the parts be difcerned, by what difference are they to be diftinguifhed, where the quantity is one, and the bulk continuous? Befides, is the principal part alone, or are the other parts, fentient? If the principal part alone be perceptive, the foul can then alone be fentient when a fenfible paffion meets with this prin-

cipal

cipal part, situated in its peculiar seat; but if a sensible passion falls upon any other part of the soul destitute of sense, it is impossible that such a part should be able to transmit the same passion to the principal, or be at all sentient. For how can that which is void of sense, receive passion, and convey it entire to a sentient part? Besides, if passion accedes to the principal, it either falls on one of its parts, and so either one part alone will be sentient, and the rest without sensation, and consequently superfluous, or there must be innumerable and dissimilar sensations; for if the sensation of each of the parts singly, is the same with the aggregate of them all, of what use is a multitude of parts? But if the sensations are various, a man may say, as it were, I am primarily sentient in this place, and secondarily in another; and every sentient part besides the first, will be ignorant where sensation is most powerful: or perhaps (from such an hypothesis) every part of the soul will be equally deceived, each part thinking the passion to arise in the place where it is situated. But if not the principal part alone, but every part of the soul be sentient, a principal part is superfluous for the purpose of sensation; and if the soul be divisible like magnitude, how is it able to recognize, as belonging to one subject, the qualities which flow, as it were, through many senses, as through the eyes and ears? For that part of the soul which is perceptive through the eyes, distinguishes nothing but colours; that which energises through the ears, nothing but sounds; and that which acts through the medium of the touch, nothing but the surfaces of bodies: what is it then which perceives all these properties of bodies united in one subject, or what is that which perceives any thing as a certain one? For unless the intentions of the senses, and of sensible objects, were collected together in one, the soul could

never

never be able to judge of the peculiar and different proper‑ ties of bodies. Hence it is neceſſary that the ſoul ſhould be, as it were, a general centre; that the ſeveral ſenſes ſhould be extended on all ſides to this, ſimilar to lines verging from the circumference of a circle to the centre; and that a power of this kind, comprehending all things, ſhould be truly one. For if the ſoul was any thing divi‑ ſible, and the intentions of the ſenſes reached the ſoul, and ended in its eſſence like the extremities of various lines, they muſt either again concur in one and the ſame as a medium, or have different ſituations, in ſuch a manner that each ſenſe may perceive different from one another: as if, for inſtance, the ſenſe of ſight ſhould perceive the form of Socrates, and the ſenſe of hearing recognize his voice; that eſſence which pronounces the whole to be one perſon, that of Socrates, muſt be ſomething different from each of the ſenſes. Hence it is neceſſary that the ſoul ſhould be an indiviſible eſſence; for if ſhe poſſeſſed mag‑ nitude, ſhe muſt be divided along with every ſenſible object ſhe perceives; ſo that one part of the ſoul would perceive a certain part of a ſenſible object, and we ſhould poſſeſs no ſentient power capable of perceiving the whole, or of pro‑ nouncing any thing one. Thus, in the perception of a man, conſidered as one, how is it poſſible that the ſoul can be divided, ſo that the perception of a part ſhall be the ſame with the perception of the whole. But if we ſuppoſe the ſoul diviſible in all her perceptions, ſince it is impoſſible ſhe ſhould be co-extended with every ſentient object, in how many parts is the diviſion to be made? Is the ſoul to be diſtributed into the ſame number of parts as the ſenſible object ſhe perceives, ſo that every part of the ſoul may perceive the ſame part of the object? Or ſhall we ſay that the parts of the ſoul have no ſenſation of the parts of the

2   object

object? But this would be abfurd and impoffible.. If every part then of the foul perceived every part of a magnitude as a whole, fince magnitude is divifible to infinity, and fince, on this hypothefis, there muft be innumerable fenfations of every fenfible object, there muft be innumerable images, as it were, of the fame thing in our principal part. Befides, if that which perceives is corporeal, it will not be poffible for it to perceive in any other manner than as if certain images were impreffed from a feal in wax, or in brafs, or in any other fenfible fubftance. But if the images of fenfation exift as in humid bodies (which is moft probable), they will certainly be confounded like images in water; nor can there be any memory, the image departing with its forming fubftance. And if we fuppofe the figures to remain like impreffions in folid bodies, either it will not be poffible for others to fucceed while the former endure; and thus, fenfations of other things cannot take place; or if others fucceed, the former images muft be immediately deftroyed, and memory be no more. So that if we allow it poffible to remember, and, befides this, to perceive other things, without any hindrance from former impreffions, it is impoffible that the foul fhould be corporeal. Since the foul, therefore, is an unextended, indivifible, and immaterial fubftance, it is confequently incorruptible and immortal; for every thing capable of diffolution and difperfion is either corporeal and compofite, or exifts in fome fubject from which it is infeparable. And indeed, whatever may be diffolved is corruptible, as being compounded from many. But whatever naturally fubfifts in fomething different from itfelf, when it is feparated from its fubject, immediately vanifhes into non-entity. But the foul, as we have proved, is incorporeal; it is likewife removed from every fubject, and naturally reverts to itfelf, and is therefore immortal and incorruptible.

2. Let

2. Let us now confider how, and on what account the foul is faid to be of a middle nature, and to be the receptacle of all middle energies, both vital and gnoftic. Since, then, there is a long gradation of beings, proceeding from the firft being, even to formlefs matter, which is nothing more than the dark fhadow of effence, it is requifite to enquire what the properties are of the firft and laft beings, and what the condition is of the middle orders; for thus we fhall know where the effence of the foul ought to be placed. The properties of intelligible natures, therefore, are as follows: true being, eternal, indivifible, immoveable, total, perfect, full of effence, replete with life, free, moving all things, fimilitude, prefiding over all things, and at the fame time feparated from all; for each of thefe properties appears in intelligibles, according to the proceffions of being. But the properties of fenfible natures, different from thefe by the greateft interval, are fuch as, not-true-being, temporal according to effence, partible, moveable, particular, indigent of another, always replete with fubfiftence, living by participation, moved by another, diffimilitude, and occupying place by parts. But the middle properties of thefe are, not-true-being, an effence better than non-being, and inferior to true-being, according to effence eternal, but according to its energies extended with time, indivifible according to its divine part, but divifible according to the various proceffions of reafons, felf-motive, governing things moved by another, but fubordinate to fuch as are immoveable, bearing before itfelf a particular nature, together with its totality; (for, becaufe it contains in itfelf all reafons, it is after a manner a whole, but becaufe it is diminifhed and fallen, ends in parts, and fuffers a tranfition of its energy, it muft be efteemed a particular nature): and again, perfecting itfelf, yet, neverthelefs,

perfected by natures prior to its own; filling itself with power and strength, and at the same time filled by others: living from itself, and receiving life from others, being more divine, indeed, than things which live only by participation, but inferior to things primarily vital; moving other things, and itself moved by others; at the same time similar and diffimilar; and feparated, at the same time, from laft natures, and co-ordinated with them. Such then, being the properties of the firft, middle, and laft orders, let us confider where we ought to place the foul, whether in the firft order, or in thofe which retain the laft place.: but if we eftablifh it among the firft, it muft be true being, every way eternal and immoveable, and it muft confequently poffefs every thing which we have attributed to the firft beings; add too, that on this hypothefis we can no longer attribute to the foul a power of felf-motion, nor the difcurfive proceffions of reafoning, nor a variety of other particulars, which manifeftly belong to the foul. But can we place it with propriety among the laft of beings? The leaft of all: for on this hypothefis we fhall make it alone moved by others, divifible, compofite, and alone poffeffing perfection from others, the oppofite of which is evident in all our fouls; fince they move and perfect themfelves, and are led wherever they pleafe. Since then it is not poffible to place the foul either in the firft, or laft order of beings, it is requifite to affign it a middle place, in imitation of its divine caufe Rhea (according to the theologifts), who is the conciliating band of the two parents Saturn and Jupiter, and is reported, from her prolific bofom to produce the life of the foul. But though the foul is thus the extremity of intelligible, and the principle of fenfible natures, we muft not conceive it to be fuch a principle or extremity as a point in a line, for it is not in both the

natures

natures it terminates, like a point in both the sections of a line; but it is to be called the extremity of intelligibles, becaufe it appears after an intelligible effence, and the principle of fenfibles, as being abftracted from them, and the fource of their motion. And thus it will preferve to us a certain proportion, that as the natures which are moved by others, are to thofe which are moved by themfelves, fo are thefe laft to immoveable natures; and hence it will obtain the condition of a bond, on account of its peculiar mediocrity, unfolding, indeed, united caufes, but reducing the diffipated powers of fenfibles into *one*, and being contained by an immoveable and perpetually abiding caufe; but containing itfelf the generation, which, moved by another, is fubject to continual mutation. It is likewife intelligible, if we regard generated natures; but generated, if we compare it with intelligibles; and thus it exhibits in its middle nature both extremes, imitating alfo, by this means, (according to the Greek theologifts) its divine caufe, for it is faid to be on both fides refulgent, ἀμφιφαὴς, and to be endued with two faces ἀμφιπρόσωπος, and to receive in its bofom the proceffions of intelligible natures. It is likewife faid to be replenifhed with intellectual life, and to be the fountain of the ever-running ftreams of corporeal life, and to contain in itfelf the centre of the proceffions of all beings. On this account it is, with great propriety, affirmed to be generated, and at the fame time without generation. For true being, according to the Platonifts, is without generation, becaufe it has an infinite power of being totally prefent at the fame time: and body is faid to be generated, becaufe it always poffeffes in itfelf an infinite flowing power, which it cannot at once totally receive. The foul, therefore, becaufe it is incorporeal, abiding in itfelf, has an infinite power of being, and this total with refpect to its

effence,

effence, and immortal without generation; but according to parts it may be confidered in infinite production. For it has not the fame total infinity ever prefent, or there would be the fame infinity of the whole and part, of the perfect and imperfect, of the contained and containing, which is impoffible. But neither is it poffible that the whole of its effence fhould be in the act of perpetual pro-duction, any more than that a part of it fhould be eternal being, left the part fhould be more worthy and better than the whole. Hence the ὑποϛασις, or fubfiftence of the foul, is at the fame time of infinite power, and is generated in infinitum; for by this means it participates of being, and obtains the firft place among generated natures; while body alone, both with refpect to its whole and parts, is obnoxious to a perpetual generation.

3. But let us now enquire from what genera Plato com-pofes this nature of the foul, which contains in itfelf the bond of all beings; previous to which it will be requifite to explain what thefe genera are, and from whence they originate. Of the fpecies, then, exifting in the intelligible world, or the divine intellect, which contains in itfelf the caufes of all pofterior natures, fome are moft general, ex-tending themfelves to the univerfality of things; but others are more particular, like the moft fpecial fpecies, and others fubfifting between thefe, expand themfelves, indeed, to a multitude of things, but not to all, according to the divi-fion of the Elean gueft in the Sophifta. For man is pro-duced from the ideal man, and horfe from the ideal horfe, in the intelligible world; but the fimilitude which is found in man and horfe, and other animals, is produced from likenefs itfelf, or the ideal fimilitude, as diffimilitude from unlikenefs itfelf; but the famenefs and difference which are found in all beings, proceed from the famenefs and

difference

difference which subfift in ever-vital energy and perfection, in the fupreme intellect, or the ideal world. Now, as among the sciences some are efpecially univerfal, fo in intelligible caufes fome are perfectly particular, prefiding alone over the proper and peculiar number of one fpecies,; but others extend themfelues to a multitude, fuch as equality, likenefs, totality, (for the whole confidered as a whole is not common to all things, fince the part is not a whole) ; but others, again, expand themfelves to all things, as all beings participate of thefe, confidered as beings, and not confidered as vital or animated, or poffeffing any other property exclufive of the denomination of being. Becaufe, therefore, *being* is the firft, the caufes of *being* obtain the moft univerfal order among genera; and thefe are five in number, as follows, *effence, famenefs, difference, motion,* and *ftation.* For every being is endued with effence; is united to itfelf; is by itfelf, or its own famenefs, feparated from others; proceeds from itfelf, and its own ftate and principle, and no lefs appears to participate of a certain *abiding,* in preferving its own proper fpecies. All things, therefore, whether intelligibles or fenfibles, or fubfifting between both, depend on thefe genera for their exiftence. For without the being of *effence,* nothing could fubfift; in like manner, without *famenefs* every whole would be diffipated, and divided from itfelf; and *difference* being taken away, all things would be one alone, and multitude be deftroyed. But without *motion* and *ftation,* all things would either be inefficacious and dead, or, lofing their proper ftate and ftability, would end in non-entity.

4. Such then being the middle nature of the foul, Plato, with great propriety, in the Phædrus, and in his tenth book of laws, defines it to be number moving itfelf; which definition he received from Philolaus, and Philolaus from
<div align="right">Pythagoras.</div>

Pythagoras. For fince mathematical fpecies have a middle fubfiftence, as is proved in the following Commentaries, they are of all things moft accommodated to the nature of the foul. Hence Plato, in imitation of Pythagoras, fome-times explains the foul by number, as in the prefent in-ftance; and fometimes by figure and magnitude, as in the Timæus; while he confiders in the foul the interfection of lines, and a twofold circle. For fince mathematical forms are feparated from the flux and inconftancy of matter, they participate of a certain, exact, fure, and exquifite condi-tion, by means of which they eminently confer to the ele-vation of our ingenuity, and the explication of latent concerns; and, on this account, as they pertain to numbers, we may fay, preferving the analogy, that there are five orders of numbers, *the divine, the effential, the animative, the natural,* and *the mathematic.* The firft of thefe is uni-form, the fecond immoveable, the third felf-motive, the fourth moved by another, and the laft the image of the others, and their external meafure. The divine number is confidered eminently in the deity, as in the principle of all things; the effential belongs to intellect through ideas, and is called *effence, unity,* and *the firft being*; the animative number belongs to the foul, through the medium of her inherent reafons; the natural to phyfical concerns, through the feeds of nature; and laftly, the mathematical belongs to opinion, as it is nothing more than the image of effential number, formed by the energies of the rational foul. The foul, therefore, is number, not limited by quantity, and mathematical, but animative; it is number, not indeed numbering, but numbered, generating and converted into itfelf. Hence too, becaufe harmony arifes from number, the foul is called harmony; not, indeed, a harmony of the parts of the body, nor the harmonic quantity which

6.   fubfifts

ſubſiſts in ſound or in voice; but a harmony ariſing from its eſſential numbers, placed in its inherent reaſons, and in the genera which conſtitute its nature. It is this harmony which produces, as from its proper cauſe, the harmony of the corporeal parts, the rhythm of motions, and the melody of voices and ſounds. It is this which produces that delight in the ſoul from ſenſible harmony, which ſufficiently indicates it to be ſomething familiar and domeſtic to her nature. From hence it may be inferred, that Plato is not inconſiſtent with himſelf when, in the Phædo, he denies, and in the Timæus affirms, the ſoul to be harmony; for he denies that it is a harmony of a definite quantity, or ſuch as ariſes from the parts of the body; but he aſſerts it to be a harmony in the manner already explained. It may likewiſe be inferred, that Plato is ignorantly accuſed by a many, for affirming that the ſoul is harmony, or number; for they only regard vulgar mathematical number, and ſenſible harmony; while Plato, far more elevated, diſcourſes of intelligible numbers, and ideal harmony, ſubſiſting in immaterial energy and perfection.

5. And here it is neceſſary to conſider what number, in a particular manner belongs to the ſoul; for various numbers, differently conſidered, accord with her ſelf-motive nature. In the firſt place, union and unity may be conſidered in the ſoul, as in her proper degree ſhe participates of divine unity; and likewiſe with relation to her totality, for ſhe is one certain whole. And becauſe *a whole* may be conſidered in a triple reſpect, one before the parts, another riſing from the aggregate of parts, and a third ſubſiſting in the ſingle parts; the ſoul is a whole in each of theſe reſpects. Thus ſhe is a whole prior to the parts, while ſhe is conſidered as diviſible into them, in an incorporeal manner; ſhe is a whole riſing from parts, while aſſuming the

parts

parts in the firſt place, we conſider how her nature is fa-
bricated from their conjunction; and ſhe is a whole in the
ſingle parts, ſince ſhe is total in the whole, and in every
part. Beſides, the duad belongs to the ſoul, becauſe ſhe
contains in her nature bound and infinite, ſameneſs and
difference; and laſtly, a converſion to intelligibles and ſen-
ſibles. And, indeed, the duad conjoined with unity, very
properly accords with the ſoul; for to intellect above ſoul,
unity particularly belongs; to body beneath ſoul, the in-
finite alone; and to ſoul ſituated in the middle, duality
properly agrees, being, as it were, infinite, connected with
unity. Again, the ternary number is attributed to the
ſoul, as well on account of her beginning, middle, and
end, as becauſe ſhe abides in herſelf, proceeds to inferiors,
and returns to ſupernal natures. Beſides, ſhe flows from
the one, recedes from him, and is reflected into the one
when ſhe acquires her proper perfection. Laſtly, as Pro-
clus obſerves, the nature of the ſoul is divided into eſſence,
power, and energy; ſo that ſhe may be ſaid to rejoice in
the ternary number, and to be replete with its perfection.
But the quaternary number belongs to the ſoul ſo far as
ſhe is connected with matter, which is tempered with four
qualities, and four elements; and ſhe is endued with four
principal faculties, nutrition, ſenſation, local motion, and
intellection. But to omit other numbers, and their confor-
mity with the ſoul, the quinary, and ſeptenary numbers
are eſpecially attributed to the ſoul. The quinary, becauſe
the ſoul is compoſed from the five genera of things, we
have previouſly explained; and becauſe five particulars
merit a principal conſideration in the ſoul; firſt, her eſſence;
ſecondly, the harmony of her reaſons; thirdly, the ſpecies
ariſing from the concord of her parts or reaſons; fourthly,
her virtue; and, laſtly, her energies: and on this account,

<div align="right">Proclus.</div>

Proclus obferves, the confideration of the foul ought to receive a quintuple diftribution. Befides, as the foul confifts from a divifible and indivifible nature; fo the quinary is compofed from the firft even and the firft odd number. Laftly, as the foul is the connecting medium of the univerfe; fo the quinary obtains the middle fituation in univerfal number, that is in the decad. But the feptenary number belongs to the foul, becaufe, as Plato fhews, in his Timæus, all harmonical reafons are contained in the feven numbers, 1, 2, 3, 4, 9, 8, 27; and from thefe the foul is compofed. Hence Proclus elegantly obferves, that the feptenary number is dedicated to Apollo, the parent of all harmony; becaufe in one, two, and four, from which the feptenary refults, the firft bifdiapafon is found. Befides, the writers on harmony affirm that all the difference of voices proceeds as far as to the feventh degree. So that Plato ufes, with great propriety, the feptenary number for the compofition of the foul. Again, in thefe numbers of the foul, every medium is found in a convenient proportion; in thefe the geometric medium is contained, correfponding to the right inftitution of laws; of which Plato, in his Republic, fays, that by this cities are properly governed: there is found too, an harmonical medium, which is the fimilitude of juftice: and laftly, we may difcover an arithmetical medium, which is called the illuftrious fymbol of peace. After this manner, therefore, Plato, with a wonderful fecundity of fignifications, affirms that the foul is compofed from numbers. He likewife confiders the figures fhe contains, I mean the circle and triangle; becaufe as the foul is the firft nature which verges to body, fo thefe are the firft of all figures, as well rectilinear as curvilinear. Befides, an orbiculation agrees to the foul, through intellect; but progreffion and rectitude according to her

own proper nature; and on this account fhe contains both a ftraight and circular figure.   I omit other correfpondencies of a circle and triangle with the foul, as they are exhibited in the following Commentaries; and particularly by Proclus, in the third book of his Commentaries on the Timæus: it is fufficient to the defign of this Differtation, juft to have mentioned this analogy, that the nature of the numbers and figures may appear, which are confidered by Plato in the compofition of the foul.

6. Let us now pafs from contemplating the nature of the foul, to a furvey of its various gradations of knowledge, and the means by which it acquires the illuminations of fcience; as this is a fpeculation perfectly effential to a full comprehenfion of the enfuing Commentaries.   Accordinging to Plato, then, in the fixth and feventh books of his Republic *, there are four degrees of the internal cognitions of our foul; imagination, or affimilation; faith, cogitation (διανοία); and laftly, fcience or wifdom.   The two firft degrees conjoined conftitute opinion; but the two laft equally joined produce intelligence in its large acceptation. I fay in its large acceptation, becaufe the word intelligence is confidered by the Platonifts in a triple refpect.   Firft, as it rifes from opinion and fcience; as Plato afferts, in the feventh book of his Republic.   Secondly, as it paffes into the fame with fcience; for thus, in the end of the fixth book, he confiders intelligence and fcience as the fame. Laftly, as it is diftinguifhed from fcience, and intelligible from that which is the object of cognition: thus, fcience regards the effential reafons of the foul; but intelligence elevates us to ideas, and this is intelligence in its proper acceptation.   The diftinction of thefe four degrees, efpecially depends on the diftinction of things with which the

* See Note to Chap. i. Book i. of the enfuing Commentaries.

foul

foul is converfant. For thefe four degrees of things are, the image of a fenfible object, the fenfible object itfelf, the image of an intelligible object, and the intelligible itfelf. Imagination or affimilation regards the image of the fenfible object; which image is nothing more than the fhadow or refemblance appearing in water, or other lucid and polifhed bodies. Faith is converfant with fenfible. objects; and thefe are animals, plants, and every thing fubject to the energies of fenfe. From the junction of imagination and faith, opinion is produced. Cogitation is converfant with the image of an intelligible object; which is nothing more than a certain univerfal, collected from fenfibles, related to the reafons of things exifting in the foul, and conftituted by their affiftance; but not elevated to ideas, and refolved into their lucid nature. Thofe who are converfant with this image of an intelligible object, ufe hypothefes; which do not elevate us to principles, which are reafons and ideas, but bring us down to fubordinate objects. Laftly, fcience, confidered as the fame with intelligence, is converfant with that which is intelligible, or the effence of things; and of this kind are reafons pertaining to the foul, and ideas to intellect. Hence, as *intelligible* is to *fenfible* in fplendor and truth, and form to its image; fuch is the relation of the fuperior to the inferior degrees of cognition. And as thefe four degrees may be refolved into five, by feparating intelligence from fcience; fo they are reduced by Plato into two principal degrees; fo far as the two firft are converfant about generation; but the two laft about effence. And thus much for the firft particular propofed.

It now remains that we inveftigate the mode in which fcience is produced in the foul, according to the doctrine of Plato. For this purpofe, I think it will be neceffary to

confider

confider the rational foul, in whofe nature all reafons cor-
refponding to ideas, have been inferted from eternity, re-
ceived into the human body, as into the plain of oblivion;
bordering on the river of negligence, that is placed near to
the flux of humours; which producing in its nature vari-
ous kinds of perturbations, are the caufes of its felf-oblivion
and neglect. Hence the foul, thus conftituted, as it were,
fleeping and intoxicated; fleeping before it is roufed; in-
toxicated before it is purified; begins from external fenfible
objects to be moved and excited, and to feek with avidity,
the knowledge congenial to its nature. The foul now, en-
quiring after truth and the fciences, is firft converfant with
the external images of things, in which the glimmering
light of fimilitude to truth, prefents itfelf to the view; then
it diftinguifhes thefe images among themfelves by its rea-
foning power; and if they agree in any particular, collects
them into one. Thus, being employed in feparating into
many, things united, and reducing many into one, it ad-
vances from fhade to fubftance, and is elevated from fimi-
litude to truth itfelf; and thus apprehends the effence of a
thing free from every foreign or contrary quality, fhining
in *reafon* and *idea*. Afterwards, from this contact, efpe-
cially falutary to itfelf, it experiences an ineffable joy, as
from a return into its proper nature, and beft difpofition;
and fo great is its exultation, that it neglects and defpifes
the fhadows which it formerly purfued. Then the foul
truly knows, that while a man regards corporeal natures,
he is employed in refemblances; and that though he may
efteem himfelf knowing in many things, he knows nothing
in reality; but is then alone elevated to the fublime degree
of fcience, when he arrives at ideas. Hence it appears,
that there are four fubordinate difpofitions of mankind in
order to fcience. For, in the firft place, children, as new

           guefts

S E C T I O N IV.

WE are informed by Proclus, in the enfuing Commen-
taries, that the end of geometry, and, indeed, of
mathematics in general, is to be referred to the energies
of intellect; and that it is degraded when made fubfervient
to the common utilities of a mere animal life. But as the
very oppofite to this is the prevailing opinion of the prefent
age, let us examine the truth of this doctrine, and attend
to the arguments which the Platonic philofophy affords in
its defence. For if we can prove that this affertion of
Proclus is fupported by the ftrongeft evidence, we fhall
vindicate the dignity of true geometry, reftore it to its
ancient efteem in the minds of the liberal, and fhew how
much it is perverted by applying it to contrary purpofes.

In order to this, I fhall endeavour to prove the following
pofition, that things valuable for their own fakes, are pre-
ferable to fuch as refer to fomething elfe. Now, this may
be demonftrated, by confidering that every natural pro-
duction was made with reference to fome end, as is evident
from an induction of particulars; and if this be the cafe,
it may be fafely inferred, that every thing exifts for the
fake of the end. But that for the fake of which any be-
ing fubfifts is the beft of all; and the end, according to
nature, is that which is perfected the laft of all, from the
birth of any being. Hence the human body receives its
end or perfection firft, but the foul laft. And hence the
foul is pofterior to the body, in the accomplifhment of its
nature; and its ultimate perfection is wifdom. It is on
this account that old age alone purfues and defires the goods
of prudence and wifdom. Hence, wifdom is a certain end
to us according to nature; and to be wife, is the extreme

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or final caufe for which we were produced. It was, there-
fore, beautifully faid by Pythagoras, that man was confti-
tuted by divinity, that he might know, and contemplate.
If then wifdom be the end of our nature, to be endued
with wifdom muft be the beft of all. So that other things
are to be performed for the fake of the good which this
contains. But to enquire in every fcience fomething be-
fides this, and to require that it fhould be ufeful, is alone
the employment of one ignorant of the great difference
between the moft illuftrious goods, and things neceffary.
For they differ, indeed, widely; fince things are to be call-
ed neceffaries, which are the objects of defire for the fake
of others, and without which it is impoffible to live. But
thofe concerns alone are properly good, which are loved by
themfelves, though nothing elfe fhould fall to the lot of
their poffeffor; for one thing is not to be defired for the
fake of another infinitely, but it is requifite to ftop at
fome limited object of defire, of which it would be ridicu-
lous to require any utility abftracted from itfelf. But you
will afk, What is the emolument of contemplative wifdom,
what the good it confers on its poffeffor? What if we fhould
fay (for fuch is the truth of the cafe) that it tranfports us
by intellect and cogitation, to regions fimilar to the fortu-
nate iflands; for utility and neceffity are ftrangers to thofe.
happy and liberal realms. And if this be admitted, ought
we not to blufh, that having it in our power to become
inhabitants of the fortunate iflands, we neglect the purfuit,
through a fordid enquiry after what is ufeful and profitable.
according to vulgar eftimation? The rewards of fcience,
therefore, are not to be reprehended, nor is it a trifling
good which refults from its acquifition. Befides, as men
travel to the mountain Olympus for its fpectacle alone,
preferring a view of its lofty fummit to much wealth; and

as

as many other fpectacles are defired for their own fakes, and valued beyond gold, in like manner the fpeculation of the univerfe is to be prized above every thing which appears ufeful to the purpofes of life : for it is furely fhameful that we fhould eagerly frequent the theatre, and the race, for the fake of the delight afforded to our corporeal fight, and fhould look for no farther utility in thefe than the pleafure they produce; and yet fhould be fo fordidly ftupid as to think that the nature of things, and truth itfelf are not to be fpeculated without fome farther reward than the fincere delight their contemplation affords.

It is on this account that the apprehenfion of truth is compared to corporeal vifion; for the fight is the moft liberal of all the fenfes, as is confirmed by the general teftimony of mankind. Hence, the fight of the fun and moon, and the glorious fpectacle of the ftars is defired by the moft illiterate as well as the moft knowing, for the delight fuch vifions afford; while, on the contrary, the defires of the other fenfes are for the moft part directed to fomething farther than the mere objects of their energy. Thus, even the fenfe of hearing, which is the next in dignity to the fight, is not always defirable for its own fake; for light is the general object of fight, and found that of hearing; but it is evident that light is more univerfally defired than found, fince all light, when not exceffive, is always pleafing, but this is by no means the cafe with every kind of found. Hence it is, that all contemplation is fo delightful, and this in proportion as it becomes abftracted from fenfible objects; for the moft beautiful forms do not produce genuine delight, until they are ftrongly reprefented in the phantafy, as is evident in the paffion of love; fince the faireft face then alone caufes love when it prefents itfelf clearly to the inward eye of thought, in the mirror of

imagi-

imagination, accompanied with living elegance, and a re-
fiftlefs energy of form.

Indeed, fo liberal and fo exalted an employment is con-
templation, that Plotinus, with his ufual profundity, proves
that the univerfe fubfifts for its fake; that all the productions
of nature originate from this; and that even actions them-
felves are undertaken with a view to the enjoyment of
after-fpeculation. May we not, therefore, fay that the
fportfman follows the chace for the fake of a fubfequent
review of his favourite purfuit? That the glutton for this
rejoices in the meal; and even the mifer in his wealth?
And that converfation is alone follicited, that it may recal
paft images to the foul? In fhort, contemplation is the firft
fpring of action, and its only end; fince we are firft in-
cited to any external object by fpeculating its image in the
phantafy: and our fubfequent conduct tends, without
ceafing, to the energy of reflection; for deftroy prior and
pofterior contemplation, and action is no more.

Now if this be the cafe, and if geometry is a fpeculative
fcience (I mean the geometry of the ancients), it is both
defirable for its own fake, and for ftill higher contempla-
tions, the vifions of intellect, to which it is ultimately
fubfervient. For, when ftudied with this view, it opens the
eye of the foul to fpectacles of perfect reality, and purifies
it from the darknefs of material oblivion. Away then, ye
fordid vulgar, who are perpetually demanding the utility
of abftract fpeculations, and who are impatient to bring
down and debafe the nobleft energies, to the moft grovel-
ing purpofes; ignorant of that mighty principle of action,
which influences every part of the univerfe, and through
which even divifion and difcord tend as much as poffible to
union and confent; ignorant that from the depravity of
your nature, and the blindnefs of your inward eye, you

are incapable of fpeculating the fubftance of reality, and are therefore eagerly gazing on its fhadow: and laftly, unconfcious that this is the point about which you are continually making excentric revolutions, miftaking the circumference for the centre, motion for reft, and a departure from good for a tendency to felicity.

It was for the fake of this moft exalted and liberal contemplation that Heraclitus yielded his right of fucceffion to a throne, to his brother; and that Anaxagoras neglected his patrimony, efteeming one drop of genuine wifdom preferable to whole tuns of riches. Led by a defire of this, as by fome guiding ftar, Pythagoras travelled into Egypt, and cheerfully encountered the greateft difficulties, and maintained the moft obftinate perfeverance, until at length he happily penetrated the depths of Egyptian wifdom, and brought into Greece a treafury of truth for future fpeculation. But thefe were happy days; this was the period deftined to the reign of *true philofophy*, and to the advancement of the human foul to the greateft perfection its union with this terrene body can admit. For in our times, the voice of wifdom is no longer heard in the filence of facred folitude; but *folly* ufurping her place, has filled every quarter with the barbarous and deafning clamours of defpicable fectaries; while the brutal hand of commerce has blinded the liberal eye of divine contemplation. For unfortunately, the circle of time, as it produces continual variations, at length reverfes the objects of purfuit; and hence, that which was once defervedly firft, becomes at length, by a degraded revolution, the laft in the general efteem.

2. If geometry, therefore, be both valuable for its own fake, and for its fubferviency to the moft exalted contemplations, there can be no doubt but that the great perfection

fection to which this fcience was brought by the Greeks, was entirely owing to their deep conviction of this important truth. Euclid, we are informed by Proclus, in this work, was of the Platonic fect; and Archimedes is reported, by Plutarch, in his Life of Marcellus, to have poffeffed fuch elevated fentiments of the intrinfic dignity of geometry, that he confidered it perverted and degraded, when fubfervient to mechanical operations; though, at the requeft of king Hiero, he fabricated fuch admirable engines for the defence of Syracufe. From this fource alone, the great accuracy and elegance of their demonftrations was derived, which have been fo defervedly applauded by the greateft modern mathematicians, and the warmeft advocates for the farrago of algebraic calculation. Algebra, indeed, or as it is called, *fpecious analyfis,* is the modern fubftitute for the perfect method adopted by the ancients in geometrical demonftrations; and this folely, becaufe it is capable of being applied with greater facility to the common purpofes of life. Hence, hypothefes have been eagerly admitted in geometry, which the ancients would have blufhed to own: I mean the multiplications and divifions of lines and fpaces as if they were numbers, and confidering geometry and arithmetic as fciences perfectly the fame. But we have fortunately the teftimony of the firft mathematicians among the moderns againft the unlawfulnefs of this ungeometrical invafion. And to begin with the great fir Ifaac Newton, in his Univerfal Arithmetic *: " Equations (fays he) are expreffions of arithmetical computation, and properly have no place in geometry, except fo far as quantities truly geometrical (that is, lines, furfaces, folids, and proportions), may be faid to be fome equal to others. *Multiplications, divifions, and fuch fort of computations, are newly received*

* Page 227.

into

into geometry, and that unwarily, and contrary to the firſt deſign of this ſcience. For whoever confiders the conſtruction of problems by a right-line and a circle, found out by the firſt geometricians, will eafily perceive that geometry was invented that we might expeditioufly avoid, by drawing lines, the *tedioufneſs of computation. Therefore, theſe two ſciences ought not to be confounded.* The ancients fo induſtri-oufly diſtinguiſhed them from one another, that they never introduced arithmetical terms into geometry. *And the mo-derns, by confounding both, have loſt the ſimplicity in which all the elegancy of geometry confiſts.*" And in another part *
of the fame work he obferves, that " *the modern geometers are too fond of the fpeculation of equations.*" To this very high authority we may add that of Dr. Halley, in the preface to his tranflation of Apollonius de Sectione Rationis; for which work he conceived fo great an eſteem, that he was at the pains to learn Arabic in order to accompliſh its tranflation into Latin †: " This method, fays he, (of Apollonius) contends with fpecious algebra in facility, but far excels it in evidence and elegance of demonftrations; as will be abundantly manifeſt if any one compares this doc-trine of Apollonius de Sectione Rationis, with the algebraic analyſis of the fame problem, which the moſt illuſtrious Wallis exhibits in the fecond volume of his mathematical works, cap. liv. p. 220." And in the conclufion of his preface, he obferves ‡, " that it is one thing to give the refolution of a problem fome how or other, which may be

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* Page 250.

† Methodus hæc cum algebrâ fpeciofâ facilitate contendit, evidentiâ vero et demonſtrationum elegantiâ eam longe fuperare videtur : ut abunde conſtabit, fi quis conferat hanc Apollonii doc-trinam *de Sectione Rationis* cum ejufdem Problematis Analyfi Algebraicâ, quam exhibuit cla-riffimus Wallifius, tom. ii. Operum Math. cap. liv. p. 220.

‡ Verum perpendendum eſt, aliud eſſe problema aliqualiter refolutum dare, quod modis variis plerumque fieri poteſt, aliud methodo elegantiffimâ ipfum efficere; Analyfi breviffimâ et fimul perfpicuâ, Synthefi concinnâ et minime operofâ.

accom-

accomplifhed by various ways, but another to effect this by the moft elegant method; by an analyfis the fhorteft, and at the fame time perfpicuous; by a fynthefis elegant, and by no means operofe." And Dr. Barrow, notwith-ftanding he was fo great an advocate for the identity of arithmetic and geometry, exprefsly afferts *, that *algebra is no fcience*. To thefe authorities we may add Simfon and Lawfon, who, fenfible of the fuperior fkill of the ancients, both in analyfis and fynthefis, have made laudable attempts to reftore the *Greek geometry* to its priftine purity and per-fection.

Again, the greateft men of the prefent times have been of opinion, that algebra was not unknown to the ancients; and if this be true, their filence refpecting it is a fufficient proof of their difapprobation. Indeed, if we confider it when applied to geometry, as an art alone fubfervient to the facility of practice, as conveying no evidence, and pof-feffing no elegance of demonftration, we fhall not wonder at its being unnoticed by the ancients, with whom practice was ever confidered as fubfervient to fpeculation; and in whofe writings elegance of theory and accuracy of reafon-ing are found perpetually united.

3. But the lives of the firft cultivators of this fcience (I mean the Egyptian priefts) as well as of the Pythagoreans and Platonifts, by whom it afterwards received fuch im-provements, fufficiently evince that this fcience advanced to perfection from an intellectual theory as its fource, and from being referred to contemplation as its end; and this will be evident, by attending to the follo̓wing hiftory of the Egyptian priefts, as preferved to us by Porphyry, in his excellent work on abftinence †; a tranflation of which will not, I prefume, be unacceptable to the philofophical

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* In his Mathematical Lectures, p. 44.　　　† Lib. iv.

reader.

reader, " Chæremon, the Stoic (fays he) explaining the rites of the Egyptian priefts, who, he fays, are accounted philofophers by the Egyptians, relates, that they choofe a place beft adapted to the ftudy and performance of facred rites; fo that a defire of contemplation is excited by only frequenting thofe receffes which are dedicated to their ufe, and which procure fafety to the priefts, on account of that reverence of the divinity, whofe facred myfteries they perform; fo that all poffible honour is paid to thefe philofophers, in the fame manner as to fome facred animals. But he fays they live entirely folitary, except at particular times, when they mix with others in fuch affemblies as are ufually held, and in public feafts ; and that on all other occafions they are fcarcely to be approached. For he who defires to converfe with them muft firft purify himfelf, and abftain from a multitude of things after the manner of thefe Egyptian priefts. He adds, that thefe men, renouncing every other occupation, and all human affairs, give themfelves entirely, through the whole of life, to the contemplation of divine concerns, and to enquiring into the divine will: by the latter of thefe employments procuring to themfelves honour, fecurity, and the eftimation of piety ; by contemplation, tracing out the latent paths of fcience; and by both thefe occupations united, accuftoming themfelves to manners truly occult, and *worthy of antiquity*. For to dwell always on divine knowledge, and be difpofed for divine infpiration, removes a man beyond all immoderate defires, calms the paffions of the foul, and raifes her intellectual eye to the perception of that which is real and true. But they ftudied tenuity of aliment, and frugality in their apparel, and cultivated temperance and patience, together with juftice and equity, in all their concerns. Indeed, a folitary life rendered them perfectly venerable ; for during that period

which

which they call the time of purification, they fcarcely mixed with the affociates of their own order, or faw any one of them, except him who was converfant with them in that exercife of purity, on account of neceffary ufes. But they by no means concerned themfelves with thofe who were unemployed in the bufinefs of purification. The remaining part of their time they converfed familiarly with thofe fimilar to themfelves; but they lived feparate and apart from thofe who were eftranged from their ceremonies and manner of living. He adds, they are always feen employed among the refemblances of the gods, either carrying their images, or preceding them in their accuftomed proceffions, and difpofing them with gravity of deportment, and in a graceful order. In all which operations they did not indicate any pride of difpofition; but exhibited fome particular natural reafon. But their gravity was confpicuous from their habit; for when they walked, their pace was equable, and their afpect fo perfectly fteady, that they refrained from winking whenever they pleafed. Their rifibility too, extended no farther than to a fmile. But their hands were always contained within their garments; and as there were many orders of priefts, every one carried about him fome remarkable fymbol of the order he was allotted in facred concerns. Their fuftenance too was flender and fimple; and with refpect to wine, fome of them entirely refrained from it; and others drank it very fparingly, affirming that it hurt the nerves, was an impediment to the invention of things, and an incentive to venereal defires. They alfo abftained from many other things, never ufing bread in exercifes of purity; and if they ate it at other times, it was firft cut in pieces, and mingled with hyffop. But they abftained, for the moft part, from oil, and when they ufed it mixt with olives, it was only in

6   fmall

fmall quantities, and as much as was fufficient to mitigate the tafte of the herbs.

In the mean time, it was not lawful for any one to tafte of the aliment, whether folid or fluid, which was brought into Egypt from foreign parts. They likewife abftained from the fifh which Egypt produced; and from all quadrupeds having folid or many fiffured hoofs; from fuch as were without horns; and from all carniverous birds: but many of them abftained entirely from animal food. And at thofe times when they all rendered themfelves pure, they did not even eat an egg. But when the time drew near in which they were to celebrate fome facred rites, or feftival, they employed many days in previous preparation, fome of them fetting apart forty-two days, others a greater length of time than this; and others again a fhorter; but never lefs than feven days; abftaining, during this period, from all animals, and from all leguminous and oily nutriment, but efpecially from venereal congrefs. Every day, they wafhed themfelves three times in cold water; after rifing from bed, before dinner, and when they betook themfelves to reft. And if they happened to be polluted in their fleep, they immediately purified their bodies in a bath. They made cold water too fubfervient to the purpofes of purification at other times, but not fo often as the bath. Their beds were compofed from the branches of palm, which they called βαΐς, *bais*. A piece of wood, of a femi-cylindrical form, and well planed, ferved them for a pillow. But through the whole of life, they were exercifed in the endurance of hunger and thirft, and accuftomed to a paucity and fimplicity of nutriment.

But as a teftimony of their temperance, though they neither ufed the exercife of walking nor riding, yet they lived free from difeafe, and were moderately ftrong. For, indeed,

indeed, they endured great labour in their sacred ceremonies, and performed many services exceeding the common strength of men. They divided the night between observations of the celestial bodies, and offices of purity; but the day was destined by them to the cultivation of the divinities, whom they worshiped with hymns each day three or four times; in the morning and evening, when the sun is at his meridian, and when he is setting. But the rest of their time they were occupied in arithmetical and geometrical speculations, always laborious and inventing, and continually employed in the investigation of things. In winter nights also, they were diligent in the same employments, and were ever vigilant to literary studies; since they were not solicitous about external concerns, and were freed from the base dominion of intemperate desires. Their unwearied and assiduous labour, therefore, is an argument of their great patience; and their continence is sufficiently indicated by their privation of desire. Besides this, it was esteemed very impious to sail from Egypt, as they were particularly careful in abstaining from the manners and luxuries of foreign nations; so that to leave Egypt was alone lawful to those who were compelled to it by state necessities. But they discoursed much concerning a retention of their native manners; and if any priest was judged to have transgressed the laws in the least particular, he was expelled the college. Besides, the true method of philosophizing was preserved in Commentaries and Diaries, by the prophets and ministers of sacred concerns: but the remaining multitude of priests, *Pastophori*, or priests of Isis and Osiris, governors of temples, and servants of the gods, studied purity, yet not so exactly, nor with so great continence as those we have mentioned. And thus much is related of the Egyptians, by a man who is equally a lover of truth, and of accurate

diligence, and who is deeply fkilled in the Stoic philo-
fophy."

4. But the lives of the Pythagoreans and Platonifts, who
carried this divine fcience to its ultimate perfection, no lefs
eminently evince the truth of our pofition. For, as Por-
phyry informs us, in the fame invaluable treatife *, " fome
of the ancient Pythagoreans, and wife men, inhabited the
moft defert places ; and others retired into temples, from
which the multitude and every tumult were expelled. But
Plato was willing to fix his academy in a place not only fo-
litary, and remote from the city, but, as they report, infa-
lubrious. Others, again, have not fpared their eyes, through
a defire of more perfectly enjoying that blifsful contempla-
tion, from which they wifhed never to be feparated."
After this, he prefents us with a defcription from Plato †
of thofe intellectual men, by whom the world has been
enlightened with the fublimeft wifdom and truth : " For
it was not falfly, or in vain (fays he), that a certain philo-
fopher, fpeaking of contemplative men, affirms, that fuch
as thefe are ignorant, from their early youth, of the way
which leads to the forum, or in what place the court or
fenate-houfe is fituated, or any public council of the ftate.
They neither fee nor hear the laws, whether decreed or
promulgated, or written ; and with refpect to the factions
and contentions of their companions for magiftracy, for
affemblies and fplendid entertainments, luxurious eating
and minftrels, they do not even think of thefe as in a
dream. Such an one knows no more of the evil which
has happened to fome one of his anceftors, whether male
or female, or any thing belonging to them, than how
many pitchers of water are contained in the fea. Nor does
he abftain from things of this nature for the fake of acquir-

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* Lib. i. p. 30.   † In Theæteto.

ing

ing fame; but in reality, his body alone abides in the city, and wanders about from place to place, but his intellect esteeming all thefe as of fmall importance, or rather as non-entities, he defpifes them, and, according to Pindar, " from thefe on every fide he foars:" by no means applying himfelf to things which are near him, and to fenfible concerns."

If fuch then were the lives of the men who brought this contemplative fcience to its prefent perfection, and who are to this day our mafters in geometry; if fuch were the exalted fentiments they entertained of its dignity and worth, what greater proof can we require of its being valuable for its own fake, and as fubfervient to the energies of intellect? We have ample evidence too, of its being degraded when brought down to the common purpofes of life, in the example of thofe who, with this view, have difguifed it with the dark and fordid involutions of algebraic calculation; for it was folely to facilitate practice, that this barbarous invafion has been admitted by the moderns. Let me then be permitted to perfuade the few who ftudy geometry in its ancient purity, and who confider the ruins of Grecian literature on this, as well as on every other fcience, the models of perfection, to enter with avidity on the ftudy of the enfuing Commentaries, and endeavour to fathom the depth of our profound and elegant philofopher: for by this means they may happily obtain the end of all true fcience, the purification of the foul; and be able to draw the light of perfect wifdom, from the undecaying and inexhauftible fountain of good.

But if it fhould be afked in what thefe energies of intellect confift, to which all fcience ultimately refers? I anfwer, in the contemplation of true being, or thofe ideal and divine forms, with which the intelligible world is replete.

Now

Now this great end is not to be accomplished without pre-vious difcipline, a long exercife of the reafoning power, and a continued feries of philofophic endurance. For this end, when attained, is no other than the enjoyment of that felicity congenial to the foul previous to her immerfion in body. But, for the further information of the liberal reader on this important fubject, the following paraphrafes from Porphyry and Proclus are fubjoined; the former in-ftructing us in the various purifications neceffary to this end; and the latter exhibiting the gradations by which we may rife to the fpeculation of reality, and (leaving all *mul-titude* behind) afcend to the divinely folitary principle of things, the ineffable *One*.

5. " In the firft place, then (fays Porphyry [*]) my rea-fons are not addreffed to thofe who are occupied in illiberal arts, nor to thofe engaged in corporeal exercifes, neither to foldiers nor failors, neither to rhetoricians nor to thofe who have undertaken the duties of an active life. But I write to the man continually employed in thinking what he is, from whence he comes, and whither he ought to tend : and who, with refpect to every thing pertaining to food, and other offices of life, is entirely changed from thofe who propofe to themfelves a different manner of liv-ing ; for to a man of this kind alone is my prefent difcourfe addreffed. Indeed, in this common ftate of exiftence, one and the fame mode of perfuafion cannot be addreffed to the fleeper, who, if it was poffible, would conciliate to him-felf perpetual fleep, and who, for this purpofe, feeks on every fide for foporiferous incentives, as to him who ftudies continually to drive away fleep, and to difpofe every thing about him to vigilance and intellectual activity. But to the former, it is neceffary to advife intoxication, furfeiting,

[*] In his moft excellent work on Abftinence, lib. i. p. 22, &c.

and

and fatiety, and to recommend a dark houfe; and, as the poets fay, *a bed luxurious, broad, and foft*. Such a one fhould chufe whatever tends to produce ftupor, and give birth to indolence and oblivion, whether confifting of odours, ointments, or medicaments which are accuftomed to be eat or drank. But it is neceffary that the intellectual man fhould ufe fober drink, unmixed with the lethargic fumes of wine; nutriment flender, and almoft approaching to fafting; a lucid houfe, receiving a fubtle air and wind; that he fhould be continually agitated with cares and griefs; and laftly, that he prepares for himfelf a fmall and hard bed, while thus employed in purifying his foul from the ftains contracted by corporeal involution. But whether we are born for this exalted purpofe, I mean for vigilant intellectual energies, allowing as fmall a part of our life as poffible to fleep; (fince we do not exift in a place where fouls perpetually vigilant abide), or whether we are deftined to a contrary purpofe, I mean, to fleep and oblivion, would be foreign from our defign to explain; and would require a longer demonftration than the limits of our work will admit.

But whoever once cautioufly furmizes the delufions of our life in the prefent world, and the inchantments of this material houfe in which we are employed, and who perceives himfelf naturally adapted to vigilant energies; laftly, who apprehends the foporiferous nature of the place in which he acts, to fuch a one we would prefcribe a diet congruous to his fufpicion of this fallacious abode, and to the knowledge he poffeffes of himfelf; in the mean time, advifing him to bid a long farewel to the fleeper, ftretched on his couch, as on the lap of oblivion. Neverthelefs, we fhould be careful left, as thofe who behold the bleer-eyed, contract a fimilar defect, and as we gape when prefent with thofe who are gaping, fo we fhould be filled with

drowfinefs

drowfinefs and fleep, when the place in which we refide is cold, and adapted to fill the eyes with watery humours, from its abounding with marfhes and vapours, which incline their inhabitants to heavinefs and fleep. If then, legiflators had compofed the laws with a view to the utility of the ftate, and had referred thefe to a contemplative and intellectual life as their end, we ought to fubmit to their inftitutions, and acquiefce in the diet they have prefcribed for our fubfiftence. But if they, only regarding that life which is according to nature, and is called of the middle kind, ordain fuch things as the vulgar admit, who only eftimate good and evil as they refpect the body, why fhould any one, adducing thefe laws, weary himfelf in endeavouring to fubvert a life which is far more excellent than every law written and compofed for the fake of the vulgar, and which follows a law not written, but divinely delivered? For fuch is the truth of the cafe.

That contemplation which procures us felicity, is not a mafs of difcourfes, and a multitude of difciplines; or, as fome may think, confifting from hence; nor does it receive any increafe from a quantity of words. For if this was the cafe, nothing could hinder thofe from being happy, who comprehend all difciplines, and *are accurately fkilled in a variety of languages.* But the whole circle of the fciences cannot by any means accomplifh this blifsful contemplation, nor even thofe difciplines which are converfant with true and fubftantial being, unlefs there is alfo a conformation of our nature and life to this divine end. For fince there are, as they fay, three ends of living, if we regard the particular objects to which mankind tend, the end with us is to follow the contemplation of *true being*, promoting, as much as poffible, by an acquifition of this kind, an intimate union of the contemplating individual

<div align="right">with</div>

with the object of contemplation. For, in nothing elfe befides *true being*, is it poffible for the foul to return to its priftine felicity; nor can this be effected by any other conjunction. But intellect is *true being itfelf:* fo that the proper end is to live according to intellect. And on this account, exoteric difcourfes and difciplines, retarding the purgation of the foul, are far from filling up the meafure of our felicity. If then, felicity was defined by the comprehenfion of words or fciences, they who do not pay a proper attention to the kind and quantity of their food, nor to any thing elfe pertaining to their prefent exiftence, might obtain this end: but fince it is requifite to change our life, and to be pure both in fpeech and action, let us confider what difcourfes and what works may render us partakers of this moft neceffary means of acquiring fubftantial felicity.

Are, then, thofe things which feparate us from fenfible objects, and from the affections which they excite, and which lead to a life intellectual, and void of imagination and paffion, are thefe the means we are in purfuit of? So that every thing contrary is foreign from our purpofe, and worthy to be rejected? And in fuch proportion as it draws us afide from intellect? Indeed, I think it is confonant to truth, that we fhould eagerly contend where intellect leads; for in this material abode, we are fimilar to thofe who enter or depart from a foreign region, not only in cafting afide our native manners and cuftoms, but from the long ufe of a ftrange country, we are imbued with affections, manners, and laws foreign from our natural and true region, and with a ftrong propenfity to thefe unnatural habits. Such an one, therefore, fhould not only think earneftly of the way, however long and laborious, by which he may return to his own, but that he may meet with a

more

more favourable reception from his proper kindred, fhould alfo meditate by what means he may diveft himfelf of every thing alien from his true country, which he has contracted; and in what manner he may beft recal to his memory, thofe habits and difpofitions without which he cannot be admitted by his own, and which, from long dif-ufe, have departed from his foul. In like manner, it is requifite, if we wifh to return to fuch things as are truly our own, and proper to man confidered as a rational foul, to lay afide whatever we have affociated to ourfelves from a mortal nature, together with all that propenfity to mate-rial connections, by which the foul is allured, and defcends into the obfcure regions of fenfe; but to be mindful of that bleffed and eternal effence intellect, our true father, and haftening our return to the contemplation of the un-coloured light of *good*, to take efpecial care of thefe two things; one, that we diveft ourfelves (as of foreign gar-ments) of every thing mortal and material; the other how we may return with fafety, fince thus, afcending to our native land, we are different from ourfelves before we de-fcended into mortality. For we were formerly intellectual natures; and even now we are effences purified from every ftain contracted by fenfe, and from that part which is def-titute of reafon: but we are complicated with fenfible connections, on account of our impotence and infirmity, which is the caufe that we cannot always be converfant with intellectual concerns; but with mundane affairs we can be prefent with frequency and eafe: for all our energetic powers are ftupified and clouded with oblivion, through body and fenfe; the foul not remaining in an intellectual ftate; (as the earth when badly affected, though good fruit is depofited in its bofom, produces nothing but weeds); and this, through the improbity of the foul, which does not,

6   indeed,

indeed, deftroy its effence, while it acquires brutality; but by fuch an acceffion it becomes complicated with a perifh-ing nature, is bound in the dark folds of matter, and is drawn afide from its proper ftate, into one that is foreign and bafe.

So that it is highly requifite to ftudy, if we are folicitous of returning to our priftine ftate of felicity, how to depart from fenfe and imagination, and her attendant brutality, and from thofe paffions which are raifed by her phantaftic eye, as much as the neceffity of our nature will permit. For the intellect muft be accurately compofed; and it is proper it fhould obtain a peace and tranquility free from the contentions of that part which is deftitute of reafon, that we may not only hear with attention concerning in-tellect and intelligible objects, but to the utmoft of our ability, may enjoy their contemplation; and thus, being reduced into an incorporeal nature, may truly lead an in-tellectual life, and not in a falfe delufive manner, like thofe who are at the fame time entangled with corporeal con-cerns. We muft, therefore, diveft ourfelves of the various garments of mortality by which our vigour is impeded; as well this vifible and flefhly garment, as that more interior one with which we are invefted contiguous to the fkin. We muft enter the place of conteft naked, and without the incumbrance of drefs, ftriving for the moft glorious of all prizes, the Olympiad of the foul. But the firft requifite, and without which it is not lawful to contend, is, that we ftrip off our garments. And fince our veftments are fome of them exterior, and fome interior, fo with refpect to the denudation of the foul, one procefs is by things more open, another by fuch as are more occult. For inftance, not to eat, or not to accept what is offered, is among things ob-vious and open; but not to defire is more obfcure; fo that

it is here requifite not only to abftain from things improper in deeds, but likewife in defire. For what does it profit to abftain in actions from what is bafe, in the mean time adhering to the caufes which produce fuch actions, as if bound in indiffoluble chains?

But this receding from material affections is brought about partly by force, and partly by perfuafion; and by the affiftance of reafon the affections languifh, and are, as it were, buried in oblivion, or in a certain philofophical death; which is, indeed, the beft mode of defertion, without oppreffing the terrene bandage from which the foul departs. For in things which are the objects of fenfe, a violent devulfion cannot take place without either a laceration of fome part, or at leaft a veftige of feparation. But vice fteals in upon the foul through continual negligence: and careleffnefs is produced by not fufficiently attending to intelligible objects; the affections in the mean time being excited by the drowfy perceptions of fenfe, among which muft be alfo reckoned the fenfations arifing from food. We muft therefore abftain, not lefs than from other things, from fuch food as ufually excites the paffions of our foul. Let us then in this particular enquire a little farther.

There are two fountains, whofe noxious ftreams detain the foul in matter, and with which, as if faturated with lethargic potions, fhe forgets her own proper fpeculations: I mean pleafure and grief, the artificer of which is fenfe and its perceptions, together with the operations attendant on the fenfes, imaginations, opinions, and memory. The paffions, roufed by the energies of thefe, and the irrational part, now fattened with noxious nutriment, draw down the foul, and avert her inclinations from her native love of true being. It is requifite, therefore, that we revolt

from

from thefe to the utmoft of our ability.   But true defections can alone take place by avoiding the paffions and rafh motions produced by the fenfes.   But, fenfation refpects whatever moves the fight, or the hearing, or the tafte, or the fmell.   And fenfe is, as it were, the metropolis of that foreign colony of paffions which refide in the foul, and which muft be expelled by him who wifhes, while connected with body, to become an inhabitant of the royal regions of intellect.   Let us then enquire how much fuel of the paffions enters into us through each of the fenfes; and this either when we behold the fpectacles of horfes in the race, and the labours of the athletic, or the contefts of thofe who twift and bend their bodies in leaping, or when we furvey beautiful women.   For all thefe infnare us, unconfcious of the danger, and fubject to their dominion the irrational appetite, by proffered inchantments of every kind.

For by all fuch inchantments the foul, as if driven into fury, compels the compound man to leap rafhly, and without reafon, and full of the brutal nature to bellow and exclaim.   In the mean time, the perturbation appearing from without, being inflamed by the internal, which was firft of all roufed by fenfe.   But the vehement motions excited by the hearing, arife from certain noifes and founds, from bafe difcourfe, and mixed affemblies; fo that fome, exiled from reafon, behave as if ftruck mad; and others, enervated by effeminate foftnefs, agitate themfelves by a multitude of trifling gefticulations.   And who is ignorant how much the foul is flattened, and infected with material groffnefs, by the ointments and perfumes which unite lovers to each other?   But why is it neceffary to fpeak of the paffions originating from the tafte: in this refpect efpecially, binding the foul in a double bond: one of which is

thickened by the paffions excited by the tafte; the other becomes ftrong and powerful by the different bodies which we receive in food. For as a certain phyfician obferved, thofe are not the only poifons which are prepared by the medical art, but fuch things as we daily receive for food, as well liquid as folid, are to be reckoned among this number; and much greater danger arifes to our life from thefe, than to our bodies from poifons. But the touch does all but tranfmute the foul into body, and excites in it, as in a diffonant body, certain broken and enervated founds. The remembrance, imagination, and cogitation of all thefe raife a collected fwarm of paffions, i, e. of fear, defire, anger, love, emulation, cares, and griefs, they fill the foul with perturbations of this kind, cloud its intellectual eye with oblivion, and bury its divine light in material darknefs.

On which account it is a great undertaking to be purified from all this rout of pollutions; and to beftow much labour in meditating day and night, what meafures we fhall adopt to be freed from thefe bonds, and this becaufe we are complicated with fenfe, from a certain neceffity. From whence, as much as our ability will permit, we ought to recede from thofe places in which we may (perhaps unwillingly), meet with this hoftile rout; and it is requifite we fhould be folicitous not to engage in combat with thefe dangerous foes, left, through too great a confidence of victory and fuccefs, inftead of vigorous contention, we produce only unfkilfulnefs and indolence."

And in the conclufion of the firft book, he adds, " For, indeed, if it be lawful to fpeak freely, and without fear, we can by no other means obtain the true end of a contemplative, intellectual life, but by adhering to the Deity (if I may be allowed the expreffion), as if faftened by a nail, at the fame time being torn away and feparated from

body

in their native inanity, and liften to the inftructions of the divinely elegant Proclus, by which we may afcend to the contemplation of true being, and the ineffable principle of things.

6. * " Pythagoras and Plato command us to fly from the multitude, that we may purfue the moft fimple truth, and apply ourfelves wholly to the contemplation of real being. From the multitude of exterior people drawing us afide in various ways, and deceiving us by fallacious appearances. But much more to fhun the multitude of interior people; for this much more diftracts and deceives. We muft, therefore, fly from the various multitude of affections, the obfcure informations of fenfe, the fhadowy objects of imagination, and the dufky light of opinion. For every multitude of this kind is fo different in itfelf, that its parts are contrary to one another; from whence it is neceffary to betake ourfelves to the fciences, in which multitude has no contrariety. For though affections are contrary to affections, one perception of fenfe to another, imaginations to imaginations, and opinions to opinions, yet no one fcience is found contrary to another. In this multitude, therefore, of propofitions and notions, we may collect into one the number of fciences binding them in one according bond. For they are fo remote from contrariety to each other, that notion is fubfervient to notion, and inferior fciences minifter to fuperior, depending on them for their origin. Above all, it is here neceffary, from many fciences which prefuppofe one, to betake ourfelves to one fcience itfelf, no longer fuppofing another, and in an orderly feries to refer them all to this original one. But after fcience, and its ftudy, it will be neceffary to lay afide compofitions, divi-

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* See the Excerpta of Ficinus from Proclus, on the firft Alcibiades of Plato ; his Latin verfion only of which is extant. Ricini Opera, tom. ii.

fions,

to incorporeal effence; fince every fenfible object poffeffes adventitious unity, is by itfelf fcattered and confufed, and full of formlefs infinity. Hence its good is divifible, and adventitious, diftant and feparated from itfelf, and refiding in a foreign feat. When you have afcended thither, and are placed among incorporeal beings, you will behold above the fluctuating empire of bodies, the fublime animal order, felf-moving, fpontaneoufly energizing in itfelf, and from itfelf poffeffing its own effence, yet multiplied, and anticipating in itfelf a certain apparition or image of the effence divifible about the unftable order of bodies. You will there perceive many habitudes of reafons, various proportions, and according bonds. Likewife the whole and parts, vivid circles, and a multiform variety of powers; together with a perfection of fouls not-eternal, not fubfifting together as a whole, but unfolded by time, gradually departing from their integrity, and converfant with continual circulations. For fuch is the nature of the foul.

But after the multitude belonging to fouls, betake yourfelf to intellect, and the intellectual kingdoms, that you may poffefs the unity of things. There remain in contemplation of a nature ever abiding in eternity, of life ever flourifhing, intelligence ever vigilant, to which no perfection of being is wanting, and which does not defire the chariot of time, for the full energy of its effence. When you have beheld natures of this exalted kind, and have feen by how great an interval they are fuperior to fouls; in the next place enquire whether any multitude is there, and if intellect, fince it is one, is alfo univerfal; and again, fince it is uniform, if not alfo multiform: for you will find it fubfifts after this manner. When, therefore, you have intimately beheld this intellectual multitude, though profoundly indivifible and united, tranfport yourfelf again

to

to another principle, and having confidered, as in a more exalted rank, the *unities* of intellectual effences, in the laft place proceed to unity perfectly feparate and free from all things. And when advanced thus far, lay afide all multitude, and you will at length arrive at the ineffable fountain of *good*. And fince it appears, from thefe various gradations, that the foul then properly obtains perfection, when fhe flies from all external and internal multitude, and the boundlefs variety of the univerfe, we may likewife conclude from hence, that our fouls do not alone collect their knowledge from the obfcure objects of fenfe, nor from things particular and divifible difcover a perfect whole, and a perfect one, but draw forth fcience from their inmoft receffes, and produce accuracy and perfection from whatever in appearances is inaccurate and imperfect. For it is not proper to fuppofe that things falfe and obfcure, fhould be the principal fources of knowledge to the foul; and that things difcordant among themfelves, which require the reafonings and arguments of the foul, and which are ambiguous and confufed, fhould precede fcience which is immutable; nor that things varioufly changed, fhould generate reafons abiding in one; nor that indeterminate beings fhould exift as the caufes of determinate intelligence. It is not, therefore, fit to receive the truth of eternal entities from boundlefs multitude; nor from fenfible objects the judgment of univerfals; nor from things deftitute of reafon, accurate difcrimination of that which is good: but it is proper that the foul, retiring into her immortal effence, fhould there fcrutinize the *good* and the *true*, and the immutable reafons of all things: for the effence of the foul is full of thefe, though they are clouded by oblivion. The foul, therefore, beholding exteriors, enquires after truth, in the mean time poffeffing it in the depths of her effence,

and deferting herfelf, explores the good in the dark regions of matter. Hence, every one in the purfuit of reality ought to begin with the knowledge of himfelf. For, if we conftantly extend our views among the multitude of men, we fhall never difcern the one fpecies man, obfcured by the multitude, and diftracted by the divifion and difcord, and the various mutations of thofe who participate the fpecies. But if we turn our eye inwards, there, remote from perturbation, we fhall behold one reafon and nature of men; fince multitude is an impediment to the converfion of the foul into herfelf. For here variety darkens unity, difference obfcures identity, and diffimilitude clouds fimilitude; fince fpecies are confufed in the folds of matter; and every where that which is excellent is mixed with the bafe." Thus far Proclus; and thus much for our intended Differtation.

L I F E

A N D

COMMENTARIES

O F

P R O C L U S.
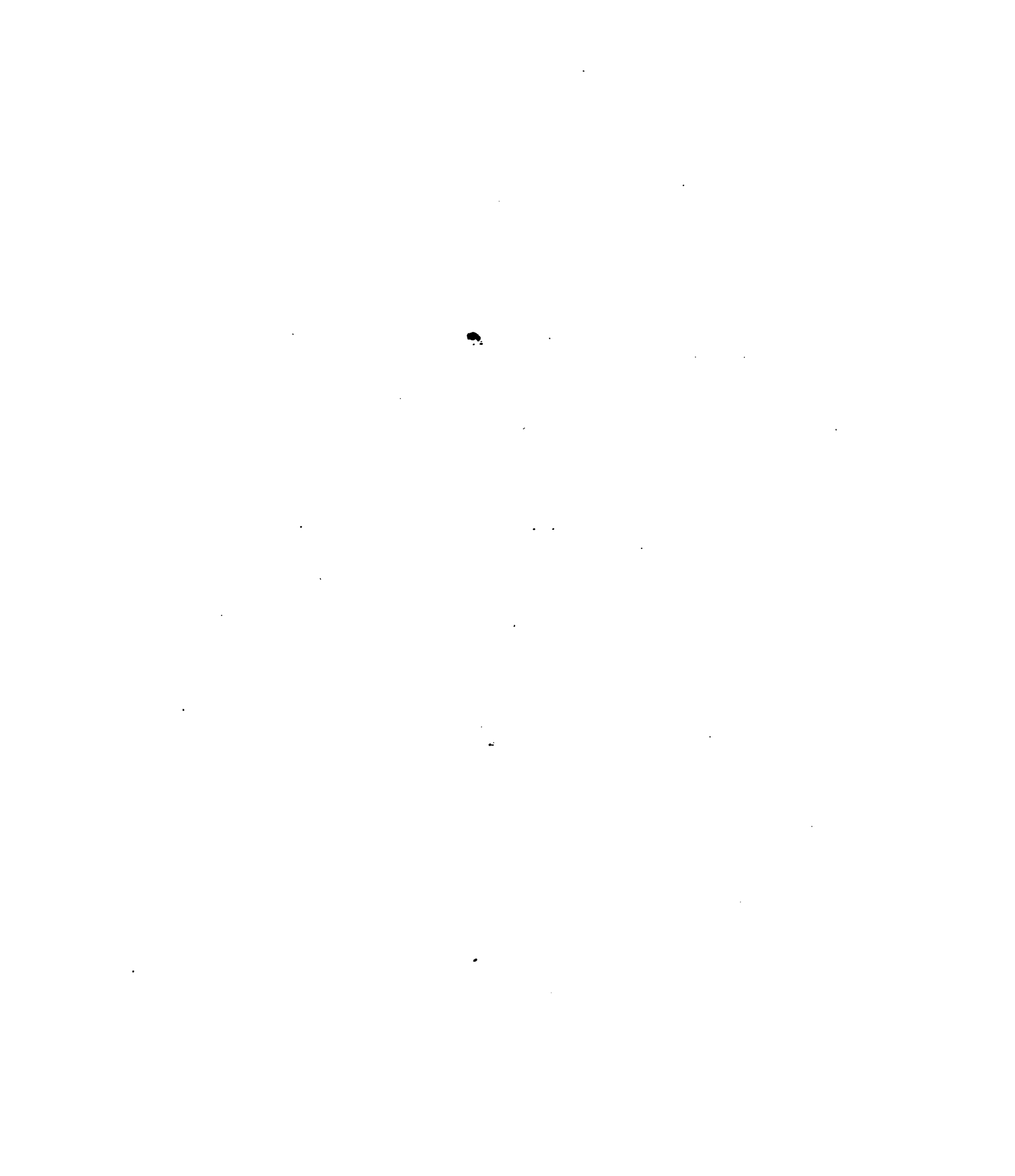
THE

LIFE OF PROCLUS,

BY MARINUS*;

OR,

CONCERNING FELICITY.

WHEN I confider the magnitude of mind, and dignity of character belonging to Proclus, a philofopher of our time, and attend to thofe requifites, and that power of compofition which thofe ought to poffefs who undertake a defcription of his life; and

* Marinus, the author of the enfuing life, was the difciple of Proclus, and his fucceffor in the Athenian fchool. His philofophical writings were not very numerous, and have not been preferved. A commentary afcribed to him, on Euclid's data, is ftill extant; but his moft celebrated work, appears to have been, the prefent life of his mafter. It is indeed in the original elegant and concife; and may be confidered as a very happy fpecimen of philofophical biography. Every liberal mind muft be charmed and elevated with the grandeur and fublimity of character, with which Proclus is prefented to our view. If compared with modern philofophical heroes, he appears to be a being of a fuperior order; and we look back with regret on the glorious period, fo well calculated for the growth of the philofophical genius, and the encouragement of exalted merit. We find in his life, no traces of the common frailties of depraved humanity; no inftances of meannefs, or inftability of conduct: but he is uniformly magnificent, and conftantly good. I am well aware that this account of him will be confidered by many as highly exaggerated; as the refult of weak enthufiafm, blind fuperftition, or grofs deception: but this will never be the perfuafion of thofe, who know by experience what elevation of mind and purity of life the Platonic philofophy is capable of procuring; and who truly underftand the divine truths contained in his works. And the teftimony of the multitude, who meafure the merit of other men's characters by the bafenefs of their own, is furely not to be regarded. I only add, that our Philofopher flourifhed 412 years after Chrift, according to the accurate chronology of Fabricius: and I would recommend thofe who defire a variety of critical information concerning Proclus, to the Prolegomena prefixed by that moft learned man to his excellent Greek and Latin edition of this work, printed at London in 1703.

laftly,

laftly, when I regard my own poverty of diction, I am inclined to believe it more proper to refrain from fuch an undertaking, not to leap over the foffe (according to the proverb), and to decline a difcourfe involved in fo much difficulty and danger. But my fcruples are fomething diminifhed when I confider, on the other hand, that even in temples, thofe who approach to the altars do not all facrifice alike; but that fome are folicitoufly employed in preparing bulls, goats, and other things of a fimilar kind, as not unworthy the beneficence of the Gods to whom thofe altars belong: likewife that they compofe hymns, fome of which are more elegant in verfe, but others in profe; while fome, who are deftitute of all fuch gifts, and facrifice with nothing more perhaps than a cake and a fmall quantity of bread, with frankincenfe, and who finifh their invocations with a fhort addrefs to the particular divinity they adore, are not lefs heard than others. While I thus think with myfelf, I am afraid, according to Ibycus *, left I fhould not offend againft the Gods (for thefe are his words) but againft a wife man, and thus obtain the praife of men.

For I do not think it lawful, that I who was one of his familiars, fhould be filent concerning his life; and fhould not, according to my utmoft ability, relate fuch particulars concerning him as are true, and which perhaps ought to be publifhed in preference to others. And indeed by fuch a neglect I fhall not perhaps obtain the efteem and honour of mankind, who will not entirely afcribe my conduct to the defire of avoiding oftentation, but will fuppofe I avoided fuch a defign from indolence, or fome, more dreadful difeafe of the foul. Incited, therefore, by all thefe confiderations, I have taken upon me to relate fome illuftrious particulars of this philofopher, fince they are almoft infinite, and may be depended on for their undoubted reality.

I fhall begin therefore not according to the ufual manner of writers, who are accuftomed to diftribute their difcourfe into chapters; but I confider that the felicity of this bleffed man ought, with the greateft propriety, to be placed as the foundation of this treatife. For I regard him as the moft happy of thofe men who were celebrated in former ages; I do not fay happy only from the felicity of wifdom, though he

* Plato in Phædro. Meminit et Plutarch. VIII. Sympof. Suidas in μέτοι. Fabricius.

poffeffed

poffeffed this in the higheft degree of all men; nor becaufe he abun-
dantly enjoyed the goods of an animal life; nor again on account of
his fortune, though this belonged to him in a moft eminent degree, for
he was fupplied with a great abundance of all fuch things as are call-
ed external goods: but I call him happy, becaufe his felicity was
perfect, complete in all parts, and compofed from each of the pre-
ceding particulars. Having then in the firft place diftributed * vir-
tues according to their kinds into natural, moral, and political, and
alfo into thofe of a fublimer rank, which are wholly converfant with
purification and contemplation, and are therefore called Cathartic and
Theoretic, and alfo fuch as are denominated Theurgic, by which we
acquire a fimilitude with fome particular divinity ; but omitting fuch
as are fuperior to thefe, as beyond the reach of man, we fhall begin
from fuch as are more natural, and which are firft in the progreffions
of the human foul, though not firft in the nature of things.

This bleffed man, then, whofe praife is the fubject of this treatife,
naturally poffeffed, from the hour of his birth, all thofe phyfical virtues
which fall to the lot of mankind; the traces of which were manifeft
in the lateft period of his life, and appeared to furround and inveft his
body after the manner of a tenacious fhell. In the firft place, he was
endued with a fingular perfection of fenfation, which they denomi-
nate corporeal prudence ; and this was particularly evident in the no-
bler fenfes of feeing and hearing, which are indeed given by the gods
to men for the purpofe of philofophizing, and for the greater conveni-
ence of the animal life; and which remained entire to this divine man
through the whole of his life. Secondly, he poffeffed a ftrength of
body which was not affected by cold, and which was neither weak-
ened nor difturbed by any vicious or negligent diet, nor by any en-
durance of labours, though it was exhaufted day and night, while he
was employed in prayer, in perufing the works of others, in writing
books himfelf, and in converfing with his familiars; all which he per-
formed with fuch expedition, that he appeared to ftudy but one thing
alone. But a power of this kind may with propriety be called forti-
tude of body, from the fingular ftrength employed in fuch exertions.

* For a full account of the diftribution of the virtues according to the Platonifts, confult
the fentences of Porphyry, and the Prolegomena of Fabricius to this work.

B 2   The

The third corporeal virtue with which he was endued was beauty, which, when compared with temperance, the authors of thefe appellations have very properly confidered as poffeffing a fimilitude of nature. For as we confider temperance as confifting in a certain fymphony and confent of the powers of the foul, fo corporeal beauty is underftood to confift in a certain agreement of the organical parts. He was indeed of a moft pleafing afpe6t, not only becaufe he was endued with this excéllent proportion of body, but becaufe the flourifhing condition of his foul beamed through his corporeal frame like a living light, with fplendors too wonderful for language to explain. And indeed he was fo beautiful that no painter could accurately defcribe his refemblance; and all the pi6tures of him which were circulated, although very beautiful, were far fhort of the true beauty of the original. But the fourth corporeal virtue which he poffeffed was health, which they affirm correfponds to juftice in the foul; and that this is a certain juftice in the difpofition of the corporeal parts, as the other in thofe of the foul. For juftice is nothing more than a certain habit, containing the parts of the foul in their proper duty. Hence, that is called health by phyficians, which conciliates the jarring elements of the body into union and confent; and which Proclus poffeffed in fuch perfe6tion, that he affirmed he was not ill above twice or thrice, in the courfe of fo long a life as feventy-five years. But a fufficient proof of this is evident from hence, that, in his laft illnefs, he was entirely ignorant what the diforders were which invaded his body, on account of the great rarity of their incurfions.

Such then were the corporeal goods which Proclus poffeffed, and which may be called the forerunners, and as it were meffengers, of thofe forms into which we have diftributed perfe6t virtue. But the firft powers and progeny of his foul, which he naturally poffeffed, previous to inftru6tion, and thofe parts of virtue with which he was adorned, and which Plato reckons the elements of a philofophic nature *, muft excite the wonder of any one who confiders their excellent quality. For he was remarkable for his memory and ingenuity; he was of a difpofition magnificent, gentle, and friendly; and a compa-

* See the fixth-book of his Republic, and the Epinomis.

nion,

. nion, as it were, of truth, juftice, fortitude and temperance; and his love of truth was fo great, that he never admitted any prudent diffimulation, but hated falfehood vehemently. Indeed it is neceffary that he who profecutes truth, with fo much earneftnefs and fincerity, fhould be extremely defirous of it from his infancy, fince truth is the fource of every good, both to gods and men. But that he defpifed corporeal pleafures, and was an eminent lover of temperance, is fufficiently evident from his great propenfity to difciplines, and his defire of every kind of ftudies; for difpofitions of this kind never fuffer beaftly and illiberal pleafure to dwell in the mind, but are able to excite in the foul, from her own internal operations, fincere pleafure and delight. But it is impoffible to fay how foreign he was from avarice, fo that when a boy he defpifed the wealth of his parents, though very rich, on account of his incredible love towards philofophy. Hence he was far removed from illiberality, and from the care of leffer concerns, as he was moft ftudious of the univerfe, and of every thing divine and human. But from fuch a difpofition of the rational foul, having acquired true magnanimity, he confidered human life as of no account, and, unlike the multitude, viewed nothing dreadful in death. So that he by no means feared all that rout of moleftations which appear terrible to others, and this in confequence of that natural affection which it is proper to call by no other name than that of Fortitude alone. But, from all thefe virtues, I think it muft be evident to thofe who have not experienced his beft of difpofitions, that he loved equity from a boy; that he was juft and mild, and by no means difficult or unjuft in his affociations or contracts. To us indeed he certainly appeared modeft and elegant, neither avaritious nor illiberal, neither arrogant nor timid.

But will it not be fuperfluous to mention the goodnefs and fertility of his ingenuity? Efpecially among thofe who know and who have heard, that he was full of the moft beautiful difciplines, and who are acquainted with the multitude he produced and publifhed to the world, fo that he alone feemed to have drank nothing of the cup of oblivion, as he was endued with a power of memory which was never difturbed, and that which belongs to the oblivious, never happened to him. Befides, he never neglected frefh acquifitions, as if poffeffing a fufficiency

ficiency of difciplines; and as one who is merely delighted with their
ftudy. But he was moft remote from a nature ruftic and horrid, and
averfe from the Mufes, and particularly propenfe to more cultivated
endowments: for on account of his fingular urbanity and feftivity
(without tranfgreffing the bounds of true honefty) in his common af-
fociations, facred feafts, and other actions, he allured and charmed his
familiars, and always difmiffed them more cheerful and pleafed.

His mother, therefore, Marcella, lawfully united to his father
Patricius, both of the Lycian nation, and excelling in birth and virtue,
produced our philofopher, thus endued from the beginning with all
thefe, and other gifts of nature. And * Minerva, the tutelar god-
defs of Byzantium, received him when born, and took care of him
as a midwife, fhe being the caufe of his birth in that city: but after-
wards fhe provided for his well-being, when he was numbered among
boys and young men; for fhe appeared once to him in a dream, and
exhorted him to the ftudy of philofophy, from whence arofe his great
propenfity to this goddefs, as he particularly performed her facred
rites, and cultivated with a greater fury (as I may fay) her inftitu-
tions. Laftly, his parents brought him, when born, into their native
country Zanthus, confecrated to Apollo: and I cannot but think that
this country happened to him by a certain divine providence; as it
was requifite that he, who was to be the prince of all fciences, fhould
be educated under the prefiding deity of the Mufes. Here, being
inftituted in the moft elegant manners, he purfued moral virtues, and
was accuftomed to right conduct, and to a declination of its contrary,
that which is bafe.

But at that time the love of the gods, who had attended him from
his nativity, manifeftly appeared; for being once detained by fome
difeafe of body, and it appearing very difficult, and fcarcely poffible
to cure him, there ftood at his bed a youth of a more than ordinary

* We are informed by Fabricius, that the Platonic Olympiodorus in his MS. Commentary
on the Alcibiades of Plato, divides the orders of the Gods, into ὑπερκόσμιοι, or fuper-mundane,
which are feparate from all connection with body; and into ἐγκόσμιοι, or mundane. And that
of thefe, fome are οὐράνιοι, or celeftial, others αἰθέριοι, or etherial, or πύριοι fiery, others ἀέριο, or
aerial, others ἔνυδροι, or watry, others χθόνιοι, or earthly; and others ὑποταρτάριοι, or fubterra-
nean. But among the terreftrial, fome are κλιμαΐάρχαι, or governors of climates, others πολύχοι,
or rulers over cities; and others laftly κατοικίδιοι, or governors of houfes.

appearance,

appearance, fo that even previous to the declaration of his name, he might be confidered as * Telefphorus or Apollo: but the god proclaiming who he was, and pronouncing his name, touched the head of Proclus (for he ftood reclining his head on Proclus' pillow) and having immediately reftored him to health, vanifhed from his fight. And fuch was the † divine vifion, and the divine benevolence at that time exhibited to our youth.

But having for a fhort fpace of time, in Lycia, applied himfelf to grammar, he went to Alexandria in Egypt, bringing with him very fingular moral virtues, by which he excited towards himfelf the love of the mafters refident in that place. Hence Leonas the rhetorician, who derived (as I think) his lineage from Ifaurus, and was illuftrious among many of that profeffion, who were then at Alexandria, not only made him a partaker of his ftudies, but thought him worthy to become his domeftic, and ordered that he fhould be fupplied with food together with his wife and children, no otherwife than if he had been his ‖ true fon. He likewife took care to procure him the notice of the principal men in Egypt, who being wonderfully delighted with the acumen of the youth's ingenuity, and with the elegance and integrity of his morals, reckoned him among their greateft friends. But he was alfo inftructed by Orion the grammarian, whofe anceftors difcharged the facerdotal office among the Egyptians, and who had made fuch a progrefs in the knowledge of his art, that he compofed elaborate books on this fubject, which he left not without advantage to pofterity. He alfo went to the fchools of the Roman preceptors, and made a great progrefs in that language; for he was at firft led to the ftudy of his father's profeffion, in which he was illuftrious, his employment being the ftudy of law in the royal city. But when it appeared how vehemently the young man was delighted

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* This epithet is likewife afcribed by Onomacritus to the Moon, as may be feen in his hymn to that deity; and the reafon of which we have given in our notes to that hymn.

† Divine vifions, and extraordinary circumftances, may be fairly allowed to happen to fuch exalted geniufes as Proclus; but deferve ridicule when afcribed to the vulgar.

‖ What glorious times! when it was confidered as an extraordinary circumftance for a teacher of rhetoric to treat a noble and wealthy pupil as his domeftic. When we compare them with the prefent, we can only exclaim, *O tempora! O mores!* Philofophy funk in the ruins of ancient Greece and Rome.

with

with the ftudy of rhetoric, as he had not yet touched the writings of the philofophers, he both acquired great glory from his acquifitions, and became the admiration of his fellow pupils and mafters, on account of the elegance of his difcourfe, and his celerity in perceiving; and from his exhibiting more the habit and induftry of the mafter, than that of the fcholar.

But while he yet frequented the rhetorical fchool, the fophift Leonas, made him the companion of his journey to Byzantium: which he undertook for the purpofe of gratifying his friend Theodorus, who was at that time præfect of Alexandria, and who was a man both polite and magnificent, and a lover of philofophy. But Proclus, though a youth, followed his mafter more cheerfully in this journey, left he fhould be compelled to interrupt his ftudies. However, that I may fpeak more truly, a certain good fortune brought him back to the fource of his nativity. For, on his return, his tutelar goddefs exhorted him to philofophy, and to vifit the Athenian fchools. But having firft returned to Alexandria, and bid farewel to rhetoric, and the other arts which he had formerly ftudied, he gave himfelf up to the difcourfes of the philofophers then refident at Alexandria. But he frequented * Olympiodorus, the moft illuftrious of philofophers, for the fake of imbibing the doctrine of Ariftotle; and in mathematical difciplines gave himfelf to Hero †, a religious man, and one who was eminently fkilful in the proper methods of inftitution. But thefe men were fo delighted with the manners of the youth, that Olympiodorus wifhed him to efpoufe his daughter, whom he had taken care to inftruct in philofophy, and Hero committed to him all his religion, and made him his conftant companion. But having, on a certain time, heard Olympiodorus, a man who was endued with a great power of fpeaking, and on account of the celerity of his fpeech, and the gravity of his fubjects, was underftood by very few of his auditors, as he was departing with the difmiffed multitude, he repeated to his companions all that was faid, and almoft verbatim,

* Fabricius rightly obferves, that this Olympiodorus is not the fame with the Philofopher of that name, whofe learned commentaries, on certain books of Plato, are extant in manufcript, in various libraries. As in thefe, not only Proclus himfelf, but Damafcius, who flourifhed long after Proclus, is celebrated.

† Concerning the various mathematicians of this name, fee Fabricius in Bibliotheca Græca.

though

though the difcourfe was copious; as Ulpianus Gazæus, one of his fellow-difciples informed me, who alfo confumed not the leaft part of his life in the ftudy of philofophy. But he likewife learned, with great facility, the writings of Ariftotle, pertaining to rational philofophy *, though the bare reading of them is difficult to thofe who are engaged in the attempt.

Having therefore, at Alexandria, applied himfelf to thefe mafters, and enjoyed their confidence in fuch inftruction as they were able to afford, when upon reading together with them a certain author, they appeared to him not to interpret the mind of the philofopher as they ought; conceiving a contempt for thefe fchools, and at the fame time being mindful of the exhortation which had been divinely fent to him at Byzantium, he went to Athens, attended by the prefiding deities of eloquence and philofophy, and by beneficent dæmons. For that he might preferve the genuine and entire fucceffion † of Plato, he was brought by the gods to the guardian city of philofophy, as the circumftances which happened on his firft entrance into the city, and all the divine excitations manifeflly evince: for they openly prefaged, that this gift was fent from the father Apollo, and was a future fuffrage of his fucceffion confirmed by divine events. For when his veffel drove to the Pyræum, and it was told to the citizens, Nicolaus, who afterwards flourifhed in the rhetorical art, but at that time ftudied under the mafters of Athens, defcended to the fhore as if to an acquaintance, received him for his gueft as if he had been a citizen, and brought him to the city; for Nicolaus was alfo a Lycian. But Proclus, who perceived himfelf weary from his journey, fat down at the temple of Socrates, though he did not yet know, nor had heard that Socrates was worfhipped in that place, and requefted Nicolaus that he would ftay there for a fhort time, and, if poffible, procure him fome water, as he faid that he was exceeding thirfty.

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* The word in the original is λογικὰ, which Fabricius rightly conjectures has in this place a more extenfive fignification than either Logic, or Rhetoric: but I muft beg leave to differ from that great critic, in not tranflating it fimply *philofophical*, as I fhould rather imagine, Marinus intended to confine it to that part of Ariftotle's works, which comprehends only logic, rhetoric, and poetry. For the verb ἐξιμάνθανω, or *to learn*, which Marinus ufes on this occafion, cannot with propriety be applied to the more abftrufe writings of Ariftotle

† Hence Proclus was called, by way of eminence, διάδοχος Πλατώνικος, or the Platonic Succeffor.

Immedi-

Immediately Nicolaus, from that very confecrated place, brought him fome water; for the fountain belonging to the ftatue of Socrates was not far diftant. But while he was drinking, Nicolaus, for the firft time, confidering the circumftance: This is an omen, fays he, becaufe you have fate in the temple of Socrates, and have there firft drank the Attic water. Then Proclus rifing, and having paid due reverence to the place, proceeded to the city. But when he came to the tower, the porter who was prefent at his entrance, and was about to clofe the gates with bars, faid to him, (that I may repeat the words of the man,) "Certainly unlefs you had come, I fhould have clofed the gates." And what omen could be more manifeft, or could lefs require the interpretations of * Polletes or Melampodes, or fuch like diviners?

But Proclus, defpifing the fchools of the rhetoricians, though they very much defired his affociation, as if he had come for that very purpofe, met with the prince of philofophers Syrianus †, the fon of Philoxenus. At that time too, Lachares was in the fame company, a man much converfant in philofophy, and the companion of Syrianus in his ftudy, but in eloquence he was in as great admiration as Homer in the poetic art. He then was, as I have faid, prefent at the fame time. But it was now the evening twilight, and while they were engaged in mutual converfe, the fun fate, and the moon made her firft appearance after her change: wherefore, having faluted the ftranger, they endeavoured to difmifs him, as being a young man, from their company, that they might adore the goddefs apart. But he not having proceeded far, beheld alfo the moon appearing from the fame houfe, and laying afide his fandals, in their prefence faluted the goddefs. Here Lachares, admiring the confidence of the youth,

* Concerning Polletes, fee Suidas; and for Melampodes, confult Fabricius in Bibliotheca Græca.

† This Syranius was indeed a moft excellent philofopher, as we may be convinced from his commentary on the metaphyfics of Ariftotle, a Latin tranflation only of which, by one Hieronimus Bagolinus, was publifhed at Venice in 1558. The Greek is extant, according to Fabricius, in many of the Italian libraries, and in the Johannean library at Hamburg. According to Suidas, he writ a commentary on the whole of Homer in fix books; on Plato's politics, in four books; and on the confent of Orpheus, Pythagoras, and Plato, with the Chaldean Oracles, in ten books. All thefe are unfortunately loft; and the liberal few, are by this means deprived of treafures of wifdom, which another philofophical age, in fome diftant revolution, is alone likely to produce.

<div align="right">faid,</div>

faid, fpeaking to Syrianus, "This is what Plato * divinely affirms of great geniufes; that they either produce great good, or its contrary." And fuch, that I may mention a few out of many, were the fignatures of divine original, which happened to our philofopher, on his firft arrival at Athens.

But Syrianus brought him to the great Plutarch †, the fon of Neftor, who, when he faw the young man, not yet twenty years old, and heard of his love and defire of a philofophic life, he was much delighted, and immediately made him a partaker of his philofophic ftudy, though his age almoft forbade fuch an attempt ; for he was then very old. He therefore read to Proclus his commentary on Ariftotle's books on the foul, and on the Phædo of Plato : and this great man exhorted him to commit to writing what he heard, employing the ambition of the youth as an inftrument, by telling him, that if he completed thofe fcholia, they would be reported as the commentaries of Proclus. And as he loved the youth very much on account of his inclination to the beft ftudies, he called him his fon, and caufed him to refide with him as a domeftic. But after he faw his temperance, with regard to animal food, he exhorted him not to abftain from animals entirely, but to ufe them fo far as was neceffary to the vital energies of the corporeal part. He likewife gave the fame advice to Syrianus, concerning the youth's diet. But he anfwered the old man, as that divine head (Proclus) informed us: " Suffer him with that frugality to learn what I wifh, and then, if he pleafes, he may die." Such was the care of his mafters refpecting him, in every concern. But the old man lived about two years after the arrival of Proclus; and dying, commended the youth to his fucceffor Syrianus, as alfo his grandfon Archiadas. But Syrianus, when he had received Proclus as his pupil, not only much affifted him in learning, but made him his domeftic as to other concerns, and the companion of his philofophic life, having found him fuch an auditor and fucceffor

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* Socrates, in the 6th book of Plato's Republic, fays, that from great geniufes nothing of a middle kind muft be expected ; but either great good, or great evil.

† The reader will pleafe to take notice, that this great man is not the fame with Plutarch the biographer, whofe works are fo well known ; but an Athenian philofopher of a much later period.

as

as he had for a long time fought for; and one who was adapted for the reception of a multitude of difciplines, and divine dogmata.

In a fhorter fpace, therefore, than two years, he read, together with Syrianus, all the works of Ariftotle, i. e. his logic, ethics, politics, phyfics, and theological fcience. But being fufficiently inftruted in thefe, as in certain * proteleia, and fmall myfteries, Syrianus led him to the facred difcipline of Plato, and this by an orderly progreffion, and not † according to the oracle, with a tranfcendent foot. And he was careful that he might furvey with him true myfteries, with the eyes of his foul, free from material darknefs, and with a fpeculation of intellet refined and pure. Hence Proclus was employed night and day in vigilant energies, and in writing compendioufly what he had heard, employing his own judgment in the feletion and order. And in confequence of this unwearied affiduity, he made fo great a progrefs in a fhort time, that by then he was twenty-eight years of age, he compofed a multitude of works; and among the reft his very learned and elegant commentaries on the Timæus. But from an inftitution of this kind, his manners alfo received a greater ornament, fince as he advanced in fcience he accumulated virtue.

But he likewife purfued the civil virtues from Ariftotle's political writings, and Plato's books concerning laws and a republic. How-

* Ariftotle's philofophy, when compared with the difcipline of Plato is, I think, defervedly confidered in this place as bearing the relation of the proteleia to the epopteia in facred myfteries. Now the proteleia, or things previous to perfetion, belong to the initiated, and the myftics; the former of whom were introduced into fome lighter ceremonies only: but the myftics, were permitted to be prefent with certain preliminary and leffer facred concerns. On the other hand, the epoptæ were admitted into the fanctuary of the greater facred rites; and became fpetators of the fymbols, and more interior ceremonies. Ariftotle indeed appears to be every where an enemy to the dotrine of ideas, as underftood by Plato; though they are doubt-lefs the leading ftars of all true philofophy. However, the great excellence of his works, confidered as an introdution to the divine theology of Plato, deferves the moft unbounded com-mendation. Agreeable to this, Damafcius informs us that Ifidorus the philofopher, " when he applied himfelf to the more holy philofophy of Ariftotle, and faw that he trufted more to neceffary reafons than to his own proper fenfe, yet did not entirely employ a divine intelletion, was but little folicitous about his dotrine: but that when he had tafted of Plato's conceptions, he no longer deigned to behold him in the language of Pindar. But hoping he fhould obtain his defired end, if he could penetrate into the fantuary of Plato's mind, he direted to this purpofe the whole courfe of his application." Phot.i Bibliotheca. p. 1034.

† *according to the oracle.*] In the original καλὰ τὸ λόγιον, which I wonder Fabricius fhould tranflate, *quod aiunt*, as it is ufual with the Platonifts, to cite the Zoroaftrian oracles exatly in thefe words, inftances of which may be found in Proclus on Plato's theology; and the very words prove themfelves to be a part of an oracle, when attentively confidered.

ever,

And when he was once very much molested by the improbity of some
violent men, which was both pernicious and dangerous to himself, he
undertook a journey into Afia, which contributed greatly to his own
advantage: for as he was not unfkilful in the more ancient rites of
that place, which he yet preferved, a divine power afforded him this
occafion of departure. . Hence, as he well knew the whole of their
concerns, he taught them more accurately in things pertaining to the
gods, if they happened to have neglected any thing through a long
interval of time. And while he was engaged in all thefe employ-
ments, and lived in a correfpondent manner, he was fo concealed
from the multitude that he even excelled the Pythagoreans, who
keep with unfhaken conftancy this precept of their founder, λάθε
βιώσας, *live concealed.* But having paffed a year only in the parts of
Lydia, he returned to Athens, under the providential protection of
the prefiding goddefs of philofophy. And thus fortitude was per-
fected in our philofopher, firft by nature, then by cuftom, and after-
wards by fcience and the confideration of caufes. Befides this, he
exhibited in another manner his politic habit practically, by com-
pofing letters for noblemen; and by this means procuring good to
entire cities. But of this I have a fufficient teftimony from thofe on
whom they were beftowed, as well Athenians as Argives, and others
of different nations.

But he likewife much promoted and increafed literary ftudies, de-
manding of princes rewards for the preceptors, according to their fe-
veral deferts. Nor did he undertake this rafhly, nor with any inte-
refted views, but he compelled them (as he confidered it a matter of
great moment) to be diligent in their profeffion, interrogating and
difcourfing with them refpecting every particular: for he was a judge
fufficiently inftructed in the employments of them all. And if he
ever found any one negligent in his profeffion, he fharply reproved
him; fo that he appeared very vehement and ambitious, becaufe he
was both willing and able to give a juft determination on every fub-
ject: and he was indeed a lover of glory. But this was not a fault
in him, as in moft, becaufe it alone regarded virtue and goodnefs.
And, perhaps, without an energy of this kind, nothing great and ex-
cellent would ever fubfift in the human mind. But he was in this

refpect

refpeÄ.vehement: this I will not deny. Yet, at the fame time, he
was gentle; for he was eafily pleafed, and demonftrated in a moment
that his anger was as pliable as wax. For, almoft at the fame time,
he was (as I may fay) wholly tranfported in reprehenfion, and with
a defire of becoming fubfervient to their intereft, and that he might
intercede with princes in their names ; being moved with a certain na-
tural conjunÄion of foul, and, as it were, fympathy of grief.

And here, I very opportunely recolleÄ a peculiar example of his
natural fympathy of foul with others : nor do I think the like was
ever related of any other man. For, notwithftanding he was unmar-
ried, and had no children (becaufe he was not defirous of fuch con-
neÄions, but remained free from them all, though many noble and
rich alliances were offered him), yet his care of all his familiars and
friends, and of their wives and children, was as great as if he had
been fome common father, and the caufe of their birth; for he be-
ftowed a fingular attention on the life of each. And whenever any
one of them was detained by any difeafe, he firft earneftly fupplicated
the gods on their behalf with facrifices and hymns; afterwards he gave
a prompt attendance on the fick perfon himfelf, convened the phyfi-
cians, and urged them to make an immediate trial, if they knew of
any thing in their art advantageous to the condition of the difeafed;
and fometimes he produced fome fingular advice himfelf, among the
phyficians; and thus delivered many from imminent dangers. And
the greatnefs of this blefled man's humanity towards his fervants, may
be underftood by thofe who defire it, from his will. But of all his
familiars, he loved Archiadas and his kindred the moft; becaufe, in
the firft place, their fucceffion was derived from the genus of Plutarch
the philofopher; and afterwards on account of that * Pythagoric
friendfhip which he maintained with Archiadas, as he was both the
companion and preceptor of his ftudies. And this other kind of
friendfhip, differing from the two already mentioned, appears to have

* Nothing is more celebrated by the ancients than that ftriÄ friendfhip which fubfifted
among the Pythagoreans ; to the exercife of which they were accuftomed to admonifh each
other, *not to divide the god which they contained*, as Jamblichus relates, lib. i. c. 33. De Vita
Pythagoræ. Indeed, true friendfhip can alone fubfift in fouls, properly enlightened with ge-
nuine wifdom and virtue ; for it then becomes an union of intelleÄs, and muft confequently be
immortal and divine.

been

been the moſt firm and excellent, among theſe illuſtrious men. For Archiadas defired nothing, which was not alſo the wiſh of Proclus; and on the contrary, the defires of Proclus were the conſtant wiſhes of Archiadas.

But having now brought the political virtues, which are inferior to the true ones, to an end, and terminating them in * friendſhip, as their proper bound, we ſhall now paſs to the Cathartic differing from the politic virtues. Indeed, the employment of theſe laſt confiſts in purifying the ſoul, that ſo being liberated from the body as much as they are able to effect, it may regard human concerns, and poſſeſs a certain ſimilitude with divinity; which is the ſoul's beſt and moſt exalted end. Yet they do not all liberate after the ſame manner, but ſome more, and others leſs. Since there are certain political purgations which adorn their poſſeſſors, even while connected with body, and reduce them to a better condition; bringing under the dominion of reaſon, anger and defire, and entirely deſtroying paſſion and every falſe opinion: but the Cathartic virtues, which are ſuperior to theſe, ſeparate entirely from this truly leaden weight of body, and procure an eaſy flight from mundane concerns. And in theſe, indeed, our philoſopher was ſtudiouſly employed during the whole of his life, which was devoted to philoſophy; ſince he both taught by his diſcourſes what they were, and after what manner they were preparatory to felicity, and in a particular manner conformed his life to their inſtitutions; performing every thing which could contribute to the ſeparation of his ſoul, uſing both night and day prayers, luſtrations, and other purifications, as well according to the Orphic as the Chaldaic inſtitutions: and every month he deſcended, with great diligence, to the ſea; and this ſometimes twice or thrice. But he was exerciſed in theſe, not only in the vigour of his age, but alſo towards the cloſe of his life; and theſe cuſtoms he obſerved perpetually, as if they were certain invariable ſtatutes.

But he uſed meat and drink, and other neceſſary pleaſures, only ſo far as was neceſſary to avoid the moleſtations of diſeaſe; for he was in theſe by much the moſt frugal, and particularly loved abſtinence

* Pythagoras, according to Damaſcius, ſaid, that friendſhip was the mother of all the political virtues.

from

from animal food. And if at any time he was invited to eat it more
vehemently, he was so cautious in its use, that he ate it merely after
the manner of a taster. But he purified himself every month by the
sacred rites, in honour of the mother of the gods, celebrated by the
Romans, and prior to them by the Phrygians: he likewise more dili-
gently observed the unfortunate days of the Egyptians than they
themselves; and, besides this, fasted on certain days in a peculiar man-
ner, on account of the lunar appearances *. He likewise instituted
a fast on the last day of the month, not having supped the day before.
But in what a splendid manner, and with what piety, he celebrated
the new moon, and properly observed, with sacrifices, the more il-
lustrious feasts of almost all nations, according to the manner of each
country; and how from these he did not, according to the custom of
many, take occasion of becoming idle and intemperate, but employed
himself in continual prayers, hymns, and the like, his hymns suffi-
ciently evince, which not only celebrate the divinities of the Greeks,
but likewise Marna Gazæus, Esculapius Leonteuchus, Ascalonites,
and Theandrites, another god much venerated by the Arabians; to-
gether with Isis, worshipped by the Philians; and lastly, all the rest
which were the subjects of his devotion. For this sentiment was very
familiar to this most religious man, that it was proper a philosopher
should not be careful in the observance of the rites and institutions of
one particular city, nor of certain nations only, but that he should be
the general priest of the universe. And thus was he pure and holy,
so far as pertains to the virtue of temperance.

* A genuine modern will doubtless consider the whole of Proclus' religious conduct as ridi-
culously superstitious. And so, indeed, at first sight, it appears; but he who has penetrated
the depths of ancient wisdom, will find in it more than meets the vulgar ear. The religion of
the Heathens, has indeed, for many centuries, been the object of ridicule and contempt: yet
the author of the present work is not ashamed to own, that he is a perfect convert to it in every
particular, so far as it was understood and illustrated by the Pythagoric and Platonic philoso-
phers. Indeed the theology of the ancient, as well as of the modern vulgar, was no doubt full
of absurdity; but that of the ancient philosophers, appears to be worthy of the highest com-
mendations, and the most assiduous cultivation. However, the present prevailing opinions,
forbid the defence of such a system; for this must be the business of a more enlightened and
philosophic age. Besides, the author is not forgetful of Porphyry's destiny, whose polemical
writings were suppressed by the decrees of emperors; and whose arguments in defence of his
religion were so very futile and easy of solution, that, as St. Hierom informs us, in his preface
on Daniel, Eusebius answered him in twenty five, and Apollinaris in thirty volumes!

But he declined, as much as poffible, pain: and if it ever happened to him, he bore it with gentlenefs, and diminifhed it with this view, that his beft part might not at the fame time be affected with its moleftations. And the fortitude of his foul in this refpect, was fufficiently evinced in his laft illnefs; for when, at that time, he was oppreffed and tormented with the moft excruciating pains, he endeavoured to the utmoft to mitigate and expel their afflictive invafions. Hence, on fuch occafions, he often commanded us to repeat certain hymns, which when repeated procured him a remiffion and ceffation of pain. And what is more wonderful, he remembered what he heard of thefe, though forgetful of almoft all human concerns, from the diffolution of his corporeal part continually increafing. For when we began to repeat, he fupplied what was unfinifhed of the hymns, together with many of the Orphic verfes; for it was thefe we were then reciting. Nor was he only thus conftant in enduring corporeal evils, but much more fo in external unfortunate events, and fuch as appeared to happen contrary to expectation. So that he would fay, concerning particulars of this kind, *So it is, fuch things are ufual;* which feemed to me, or rather at that time appeared to be, worthy of remembrance, and an evident argument of our philofopher's magnanimity. But befides this, he reftrained anger as much as poffible, fo that it might either remain free from all excitation, or that at leaft reafon might not confent to its indulgence, but the irrational faculty alone, contrary to his will, might be moderately and lightly excited. And with refpect to venereal concerns, he ufed them in the natural way, but fo as that he might not proceed beyond a very moderate and light phantafy, in their indulgence.

And thus the foul of this bleffed man, having collected itfelf from all parts, and retiring into the depths of its effence, departed after a manner from body; while it yet appeared to be contained in its dark receptacle. For he poffeffed a prudence, not like that of a civil nature, which is converfant in the adminiftration of fluctuating particulars, but prudence itfelf, by itfelf fincere, which is engaged in contemplating, and converting itfelf into itfelf, without any longer confenting to a corporeal nature. He likewife poffeffed a temperance free from evil; and which is not even moderately influenced by perturbations,

turbations, but is abstracted from all affections. And lastly, he acquired a fortitude, which does not fear a departure from body. But reason and intellect having obtained in him a perfect dominion, and the inferior powers of his soul no longer opposing themselves to purifying justice, his whole life was adorned with the divine irradiations of genuine virtue.

Our philosopher, therefore, having most happily absolved this form of virtues, advancing now, as it were, by the highest and most mystical step, he ascended to the greatest and most consummate or telestic virtues; employing for this purpose, the felicity of his nature, and a sciential institution. Hence, being now purified, and the victor of his nativity, and despising the vain Thyrsus-bearers, and boasters of wisdom, he happily penetrated into her profound recesses, and enjoyed the contemplation of the truly blessed spectacles she contains. No longer requiring prolix dissertations, or demonstrations, for the purpose of collecting the science of these, but, with a simple vision and energy of intellect, beholding the exemplar of the divine mind, he obtained a virtue which cannot with sufficient propriety be called prudence, but is more properly denominated wisdom, or something, if possible, still more venerable and divine. But the philosopher energizing, according to this virtue, easily comprehended all the theology of the Greeks and Barbarians, and whatever is shadowed over by the figments of fables, and placed it in a clear light, for the use of those who are willing and able to pursue its latent signification. But having interpreted divinely every thing of this kind, and shewing the symphony between them all; at the same time, investigating all the writings of the ancients, whatever he found in them of genuine wisdom, and approved by general consent, this he judiciously applied to use; but if he found any thing of a different and dissonant nature, this he entirely rejected, as vicious and false. And whatever he met with contrary to wisdom, though endued with a friendly appearance, this he vigorously subverted by a diligent examination. Nor did he employ less force and perspicuity in his association with other men. For he was a man laborious to a miracle; as he often, in one day absolved five, and sometimes more lectures; and writ besides, many verses, often to the number of seven hundred. Besides this, he went

to other philofophers, and frequented their company; and again celebrated with them an evening affociation, ceafing from the labour of writing. And all thefe employments he executed in fuch a manner, as not to neglect his nocturnal and vigilant piety to the gods, and affiduoufly fupplicating the fun when rifing, when at his meridian, and when he fets.

But he was the parent of many dogmata, which were never known before, both in phyfics, and in intellectual and more divine concerns. For he firft taught, that there was a kind of fouls *, endued with the power of contemplating many forms at once, which he placed, not without great probability, between intellect, collectively, and as it were with one intuition comprehending every thing, and fouls, which are alone able to direct their vifion to one particular form. And thofe who are willing to perufe his works will meet with a great variety of dogmata, peculiar to him alone; the relation of which I fhall omit, left I fhould give a too great extent to my difcourfe. But he who evolves his writings, will eafily perceive that all we have above related of him is moft true, and much more if he happens to have known him, to have feen his face, and to have heard him interpreting in the moft excellent differtations, and delivering the Platonic and Socratic dogmata in his yearly fchools. Nor did he feem deftitute of divine infpiration; for he produced from his wife mouth, words fimilar to the moft white and thick falling fnow †; fo that his eyes emitted a bright radiance, and the reft of his countenance was refplendent with a divine light. Hence, when on a certain time, one Rufinus, a man of a great name in the republic, who was ftudious of truth, and in other refpects worthy of veneration, came to him when he was teaching and interpreting, he perceived that the head of Proclus was furrounded with a light; and when the philofopher had

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* See Proclus on Plato's Politics, p. 399. Inftit. Theolog. num. 196; and the extracts of Ficinus from Proclus's commentary on the firft Alcibiades, p. 246. &c.

† Alluding to the beautiful defcription given of Ulyffes, in the 3d book of the Iliad, v. 222.

Καὶ ἔπεα νιφάδεσσιν ἐοικότα χειμερίησιν.

Which is thus elegantly paraphrafed by Mr. Pope.

But when he fpeaks, what elocution flows!
Soft as the fleeces of defcending fnows
The copious accents fall, with eafy art;
Melting they fall, and fink into the heart! &c.

finifhed

finifhed his interpretation, Rufinus rifing, adored him; and offered to give a public teftimony, by oath, of the divine vifion which he had obferved. And much gold was offered to him, by the fame Rufinus, on his return from Afia, having efcaped the danger of the war. But Proclus likewife rejected this gift, and was by no means willing to receive it.

But that we may return to our firft defign, having now difcourfed concerning the contemplative wifdom of the philofopher, though in a manner but little fuited to its dignity, it remains that we now fpeak of the juftice pertaining to this kind of virtues. For this, unlike that of which we fpoke before, is not converfant in diftribution, or proportion; but muft be equally removed from the kind of felf-energizing juftice, by which all things are alone directed to the rational foul. For to that, concerning which we now treat, it is alone proper to refer every energy to intellect and the deity, which our philofopher performed in the moft exalted manner. For he fcarcely refted from his diurnal labours, or refrefhed his body with fleep; and perhaps even then was not free from meditation and contemplation. This is certain, that having very fpeedily roufed himfelf from fleep, as from a certain torpor of the foul, he afpired after the morning, the time of prayer; and left the greater part of the night, fhould glide from him without advantage, as he was lying alone in his bed, he either compofed hymns, or examined and fortified thofe dogmata which afterwards, in the day time, he committed to writing.

After a fimilar manner he purfued that temperance which has an affinity with thefe virtues, and which confifts in a converfion of the foul to intellect, fo as not to fuffer itfelf to be touched, nor moved with any other concerns. Laftly, he joined fortitude in alliance with thefe, by a certain perfect method, zealoufly afpiring after that liberty which is ignorant of all paffion, and which he perceived was natural to the divine object of his contemplation. And thus, through the whole of his conduct, he did not lead the life of a man merely good, to which, as Plotinus fays, the political virtues may lead, but leaving this far behind him, he endeavoured to change it for one much more perfect and divine, the life of the gods themfelves; fince, to become fimilar

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to thefe, and not to virtuous men, was the great objeſt of his con-
tention.

And thus he had rendered virtues of this kind familiar to himfelf,
while he frequented the philofopher Syrianus, and evolved and ftudied
the commentaries of the ancients. But he received from the mouth
of his preceptor certain fmall feeds, as it were, of the Orphic and
Chaldaic theology; becaufe he was prevented from hearing the com-
plete interpretation of his mafter on the Orphic verfes. For Syrianus
left to the choice of Proclus and one Domninus *, a philofopher of
the Syrian nation, and who afterwards fucceeded Syrianus, the expo-
fition of the Orphic writings, or the oracles. But they were by no
means unanimous in their choice; for Domninus preferred the inter-
pretation of the Orphic verfes, and Proclus that of the oracles. But
our philofopher did not perfeſt his undertaking, becaufe the death of
the great Syrianus happened not long after. Having therefore, as I
have faid, received the outlines from the mouth of his mafter, he applied
himfelf with the greateft diligence to the written commentaries of Sy-
rianus upon Orpheus; and being affiduoufly nourifhed with the copious
lucubrations of Porphyry and Jamblichus on the oracles, and fimilar
writings of the Chaldeans, he arrived, as much as is poffible to man,
to the top of thofe higheft virtues, which the divine Jamblichus was
accuftomed to call after a truly divine manner, theurgic. He labour-
ed therefore, not without exquifite judgment, in colleſting the expo-
fitions of philofophers prior to his time; and contraſted into one, other
Chaldaic hypothefes, and the moft excellent of the commentaries on
the divine oracles, completing this great work in the fpace of five years;
concerning which this divine vifion appeared to him in his fleep. For
he faw the great Plutarch approach to him, affirming that he fhould
live fo many years as he had compofed tetrads or quaternions on the
oracles. Afterwards, having colleſted the number of thefe, he found
they amounted to feventy. But that this was a divine dream, was
fufficiently evinced by the laft part of his life; for though he lived
feventy-five years, as we have mentioned above, yet he had not the
perfeſt ufe of his powers, in the five laft. But his body, though na-

* Concerning Domninus, fee Photius and Suidas from Damafcius in his Life of Ifidorus.

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turally of the beft conftitution, being debilitated by that hard and fcarcely tolerable kind of diet which he ufed, and by fo many frequent labours and faftings, began to languifh exactly at his feventieth year, fo that he then became much more remifs in all his energies than before. Yet, even at this period, and thus affected, he compofed orations and hymns: he alfo writ fome things, and converfed with his friends; but his ancient vigour was wanting in each. Hence the memory of the dream excited his wonder, and he every where faid, that he had only lived feventy years. But while he laboured under this infirmity of body, a certain youth, named Hegias, rendered him more alert in the bufinefs of interpretation. This young man, who already exhibited from his tender years fuch egregious figns of all the virtues of his anceftors, was one of that golden chain of philofophers, who had formerly appeared to men; and adhered moft diligently to Proclus delivering the Platonic and other theologies. But Proclus, at this advanced period, was not moderately rejoiced in communicating with the young man, his own writings, after he underftood that he made cubital advances in every kind of difciplines. And thus we have briefly difcourfed concerning the apparatus of the philofopher in the Chaldean oracles.

But I, on a certain time, having read with him the Orphic verfes, and heard, among his interpretations, not only the recondite theology which is to be found in Jamblichus and Syrianus, but alfo, among many other divine men, I requefted the philofopher that he would not leave thefe divine verfes alfo without his explanation: but his anfwer was, That he had often thought of writing commentaries on Orpheus, but that he had been ftrongly prohibited in more than one dream. For Syrianus appearing to him in his fleep, had deterred him with threats from the defign. Having therefore employed other machines, I intreated that at leaft he would mark what he principally approved of in the books of his mafter; which when this beft of men had performed, in confequence of my perfuafions, and had noted fome things in the front of each of Syrianus's commentaries, we obtained a collection of all thefe, and by this means fcholia, and commentaries of no fmall bulk; though to accomplifh this on the whole of that divine poetry, and on all the Orphic rhapfiodies, was not the intention of Proclus.

But

But since, as we have said, his incredible study of these concerns, procured him a greater and more perfect degree of theurgic virtue, hence he no longer remained in the contemplative order, nor contented with either of the two-fold properties in divine concerns, exercised intellect and thought alone on the more excellent objects of speculation: but he was careful to obtain another kind of virtues more divine than the former, and separated from the politic mode; for he used the Chaldean assemblies and conferences, and their divine and ineffable concealments. And having comprehended these, he learned the manner of pronouncing and expressing them, with their remaining use, from Asclepigenia, the daughter of Plutarch: for she alone, at that time, preserved the knowledge of the great Orgies, and of the whole theurgic discipline, delivered to her by her father, who received it from Nestor. Besides this, our philosopher also being purified in an orderly manner in the Chaldean lustrations, was an inspector of the lucid hecatic phasmai (or visions) of which he himself makes mention in one of his commentaries. But by moving a certain hecatic sphærula *, he very opportunely brought down showers of rain, and freed Athens from an unseasonable heat. Besides this, by certain phylacteria, or charms, he stopt an earthquake, and had thoroughly tried the energies of a divining tripod, having learned from certain verses concerning its defect. For when he was in his fortieth year, he thought in a dream, that he repeated to himself the following verses:

> High above æther there with radiance bright,
> A pure immortal splendor wings its flight;
> Whose beams divine with vivid force aspire,
> And leap refounding from a fount of fire.

* Nicephorus, in his commentary on Synesius de Infomniis, p. 362. informs us, that the hecatic orb, is a golden sphere, which has a sapphire stone included in its middle part, and through its whole extremity, characters and various figures. He adds, that turning this sphere round, they perform invocations, which they call Jyngæ. Thus too, according to Suidas, the magician Julian of Chaldea, and Arnuphis the Egyptian, brought down showers of rain, by a magical power. And by an artifice of this kind, Empedocles was accustomed to restrain the fury of the winds; on which account he was called ἀλεξάνεμος, or a chaser of winds.

And

And in the beginning of his forty-fecond year, he appeared to himfelf to pronounce thefe verfes, with a loud voice:

> Lo! on my foul a facred fire defcends,
> Whofe vivid pow'r the intellect extends;
> From whence far-beaming through dull bodies night,
> It foars to æther deck'd with ftarry light;
> And with foft murmurs through the azure round,
> The lucid regions of the gods refound.

Befides, he clearly perceived that he belonged to the Mercurial feries; and was perfuaded from a dream that he poffeffed the foul of * Nico-machus the Pythagorean.

Indeed, if I were willing to be prolix, I could relate many theurgi-cal operations of this bleffed man ; from the infinite number of which,

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* No opinion is more celebrated, than that of the metempfychofis of Pythagoras: but per-haps, no doctrine is more generally miftaken. By moft of the prefent day it is exploded as ridi-culous ; and the few who retain fome veneration for its founder, endeavour to deftroy the literal, and to confine it to an allegorical meaning. By fome of the ancients this mutation was limited to fimilar bodies : fo that they conceived the human foul might tranfmigrate into various human bodies, but not into thofe of brutes; and this was the opinion of Hierocles, as may be feen in his comment on the Golden Verfes. But why may not the human foul become connected with fubordinate as well as with fuperior lives, by a tendency of inclination ? Do not fimilars love to be united ; and is there not in all kinds of life, fomething fimilar and common ? Hence, when the affections of the foul verge to a bafer nature, while connected with a human body, thefe affections, on the diffolution of fuch a body, become enveloped as it were, in a brutal nature, and the rational eye, in this cafe, clouded with perturbations, is oppreffed by the irrational energies of the brute, and furveys nothing but the dark phantafms of a degraded imagination. But this doctrine is vindicated by Proclus with his ufual fubtilty, in his admirable commentary on the Timæus, lib. v. p. 329. as follows, "It is ufual, fays he, to enquire how fouls can defcend into brute animals. And fome, indeed, think that there are certain fimilitudes of men to brutes, which they call favage lives: for they by no means think it poffible that the rational effence can become the foul of a favage animal. On the contrary, others allow it may be fent into brutes, becaufe all fouls are of one and the fame kind; fo that they may become wolves and panthers, and ichneumons. But true reafon, indeed, afferts that the human foul may be lodged in brutes, yet in fuch a manner, as that it may obtain its own proper life, and that the degraded foul may, as it were, be carried above it, and be bound to the bafer nature, by a propenfity and fimilitude of affection. And that this is the only mode of infinuation, we have proved by a multitude of reafons, in our commentaries on the Phædrus. But if it is requifite to take notice, that this is the opinion of Plato, we add, that in his politics, he fays, that the foul of Therfites affumed an ape, but not the body of an ape: and in the Phædrus, that the foul defcends into a favage life, but not into a favage body; for life is conjoined with its proper foul. And in this place he fays it is changed into a brutal nature: for a brutal nature is not a brutal body, but a brutal life."

I will only relate one in this place of a very wonderful nature.
Afclepigenia, the daughter of Archiades and Plutarch, but the wife of
Theagenes, from whom I have received many benefits, while she was
yet a girl, and was nourished by her parents, fell into a difeafe,
grievous, and incurable by the phyficians. And Archiades, who had
repofed in her alone the hope of his race, was deeply afflicted with
her condition. But the phyficians defpairing of her recovery, the
father fled to his laft anchor, Proclus the philofopher, which he was
accuftomed to do in affairs of the greateft moment, confidering him as
his beft preferver, and earneftly intreated him to intercede by his
prayers with the god for his daughter, whofe condition was well
known to our philofopher. Proclus therefore, taking with him Peri-
cles * the Lydian, a man well deferving the name of a philofopher,
went to the temple of Efculapius †, that he might pray to the god
for the health of the fick virgin : for the city at that time happily
poffeffed this divinity, and rejoiced in the temple of the faviour god,
which was as yet free from the deftruction of the Chriftians. As foon,
therefore, as Proclus had prayed after the ancient manner, the girl im-
mediately perceived a great change and alleviation of her difeafe; for
the preferving god had eafily reftored her to health. The facred rites
being finifhed, Proclus went to Afclepigenia, and found her delivered
from the moleftations of difeafe, and in a healthy condition. But
this affair, with many others, was performed privately, fo that no
traces of it remained to fucceeding inveftigators; and the houfe in
which he refided greatly affifted him in this defign. For befides his
other fortunes, he enjoyed a moft convenient dwelling, which his
father Syrianus and his grand-father Plutarch (for fo he did not he-
fitate to call him) had once inhabited; and this was fituated near the
temple of Efculapius, together with that of the celebrated Sophocles,
and of Bacchus, which is next to the theatre, and looks towards,
or is at leaft feen from, the lofty towers of Minerva.

But how much Proclus was loved by the philofophic goddefs is abun-
dantly evinced by his philofophic life, which he chofe through her
perfuafions, and that with the great fuccefs we have hitherto defcribed..

* Pericles Lydus, a Stoic philofopher.   † Vide Paufan. lib. i. Atticorum, cap. 21. et 20.

But

But she clearly demonstrated her affection to Proclus, by the following circumstance. When her image, which had been so long dedicated in the parthenon, or temple, was taken away by those * who, without any hesitation, moved out of their places things the most holy, and which ought to be immoveable, there appeared to the philosopher in a dream, a woman of a graceful form, who admonished him to build a temple with great expedition, for, says she, *it pleases Minerva, the presiding deity of philosophy, to dwell with you.* And how familiar he was with Esculapius, besides what we have mentioned above, may be evinced from his perceiving the presence of the same god in his last illness. For while he was between sleeping and waking, he perceived a dragon creeping on his head, which vision occasioned a remission of his disease, and a mitigation of his pain; so that it is probable he would have been restored to perfect health, if his desire and vehement expectation of death had not prevented his recovery, or his no longer bestowing a diligent attention on his body.

But he also related the following circumstances, (worthy of being remembered) but not without tears, through the sympathy of his mind. For, when a young man, he was afraid left he should be infested with the gout, which was the disease of his father, and which loves to descend from parents to their children. Nor was he afraid, as it seems, without reason; for prior to that which we have already related concerning him, he was tormented with pains of this kind, when another extraordinary circumstance happened to this blessed man. In consequence of a certain person's prescription, whom he had consulted, he applied a plaster to his tormented foot, which an unexpected bird flew away with as he was lying on his bed. And this was certainly a divine and salutary symbol to Proclus, and might have persuaded him not to be afraid of this calamity in future. But he, as I have said, was notwithstanding afflicted with the dread of this disease; he therefore supplicated the healing god concerning this, and intreated him to afford him a more evident token of his will: and after this he saw in his sleep (it is bold indeed to conceive such a circumstance in the mind, but we must dare, nevertheless, nor dread to bring truth

* He means the Christians.

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to

to light) Efculapius approaching to him, and accurately contemplating his leg, nor difdaining, through his great philanthropy to embrace his knee. Hence, in confequence of this vifion, Proclus was, through the whole of his life, free from the apprehenfions of this difeafe, and was never more tortured with this kind of pains.

Again, the god who is worfhipped by the Adrotteni, did not lefs confpicuoufly evince his great familiarity with this friend of the gods; for, more than once, the god benevolently prefented himfelf to the fight of Proclus, frequenting his temple. And when he was doubt- ful, and wifhed to know what god or gods refided and were worfhip- ped in that place, becaufe the inhabitants were of different opinions in this affair, fome believing that the temple of Efculapius was there, per- fuaded of this by various figns, as voices are certainly faid to be heard in that place, and a table is confecrated to the gods, and oracles are given concerning the recovery of health, and thofe who approach are miraculoufly delivered from the moft grievous dangers of life. Others, on the contrary, think that the Diofcuri refide in that place, becaufe there have been thofe who faw in the way tending towards Adrotta, two young men of the moft beautiful form, and riding with great ce- lerity on horfeback; at the fame time declaring, that they haftened to the temple. They add befides, that the countenance of thefe was in- deed human, but that they immediately gave evidence of a more di- vine prefence; for when the men were arrived at the temple, the above mentioned youths appeared to them, making no enquiry con- cerning the affair, and occupied in the facred concerns; but prefently after, withdrew themfelves from their fight. As the philofopher, therefore, was in doubt concerning thefe divinities, and did not difcre- dit the relations, having requefted the gods who inhabited that place, that they would condefcend to manifeft who they were; a god ap- peared to him in a dream, and clearly fpoke to him as follows: *What! Haft thou not heard of Jamblichus teaching concerning, and celebrating thofe two, Machaon and Podalirius?* And befides this, the god conde- fcended to afford fo great an inftance of benevolence to this bleffed man, that he ftood after the manner of thofe who beftow encomiums on others in the theatres, and with a clear voice, and compofed habit, extending his right hand, did not hefitate to exclaim, (that I may
relate

relate the words of the god,) *Proclus the ornament of the city*. And what could poffibly be a greater teftimony of this bleffed man's friend-fhip with divinity? But he, indeed, in confequence of a moft remark-able fympathy, by which he was united with the gods, could never refrain from tears when he mentioned this affair to us, and related the divine encomium of himfelf.

Indeed, if I were willing to purfue every particular in like manner, and to relate his familiarity with the Hermetic Pan, together with the great benevolence and manifold affiftance which that divinity conde-fcended to afford Proclus at Athens, and of that perfectly fingular kind of felicity which he obtained from the mother of the gods, and in which he was accuftomed efpecially to triumph and rejoice; I fhould perhaps feem to many readers, to be rafh in my affertions, and to others, the author of things perfectly incredible. For many and great were the daily inftances of this goddefs's benevolence towards him, in words and actions, which are both innumerable and unheard of, and concerning which, I do not at prefent retain an accurate remembrance: But if any one is defirous of knowing how great he was in thefe, he muft evolve his book concerning the mother of the gods, from which he will underftand, that it was not without divine affiftance he pro-duced all the theology of that goddefs into light; as likewife, what-ever is acted or affirmed concerning the fame in the fables of Athis, which he has explained after a philofophical manner, that vulgar ears-might afterwards ceafe to be difturbed, on perceiving the lamenta-tions and other obfcure ceremonies with which her myfteries are ce-lebrated.

After having, therefore, run through the theurgic virtue of Proclus, and its energies, and the happy circumftances which befel him, re-fpecting its operations, and having fhewn, that he did not lefs excel in every kind of virtue, and that he was a man fuch as mortals have not beheld for a long period of time, it remains that we now bring our difcourfe concerning him to a conclufion : for we are not now at the beginning of our narration, nor does the half of the whole remain to us, according to the proverb, but the whole itfelf is now perfect and complete. Since having begun from the felicity of the philofo-pher, and proceeded in its exemplification, our difcourfe now returns

to

to it again. For we have explained the great goods, and providential exertions, which were granted to this moft excellent man from the gods, and have fhewn their prompt attention to his prayers, the divine vifions which he enjoyed, and the help and folicitude which the gods teftified towards him. We have likewife explained his profperous fate, and propitious fortune, his country, parents, the good habit of his body, his mafters, friends, and other external advantages; each of which we have fhewn to have been far greater and more fplendid in him than in other men; and have likewife diligently enumerated fuch things as cannot be reckoned amongft outward allurements, but entirely depended on his will, fuch as are the upright and illuftrious deeds of his foul, according to univerfal virtue. And thus we have fummarily demonftrated that his foul arrived in reality at the fummit of the moft confummate virtue, and was happily eftablifhed in a perfect life, by human and divine goods of every kind.

But that the lovers of more elegant ftudies may be able to conjecture from the pofition of the ftars under which he was born, that the condition of his life, was by no means among the laft or middling claffes, but ranked among the higheft and moft happy orders, we have thought fit to expofe in this place, the following fcheme of his nativity *.

But

☉	16° 26′	♒
☽	17 29	♊
♄	24 23	♉
♃	24 41	♉
♂	29 50	♐
♀	— 23	♓
☿	4 42	♒
Horof.	8 19	♈
Mid. Heaven.	4 4²	♑

☊ or the head of the dragon.

24 33 ♏

The new moon preceding his birth.

8 51 ♒

* Proclus was born in the year of Chrift 412, on the 6th of the Ides of February. But, for the fake of the aftrologers, I have fubjoined the following figure from the Prolegomena of Fabricius to this life: and though I am not fkilled in the art myfelf, I am perfuaded, from the arguments of Plotinus, that it contains many general truths; but when made fubfervient to particulars, is liable to great inaccuracy and error. In fhort, its evidence is wholly of a phyfiognomic nature; for fuch is the admirable order and connection of things, that throughout the univerfe, one thing is fignified by another, and wholes are after a manner contained in their

parts.

But Proclus departed from this corporeal life, in the one hundred and twenty-fourth year after the government of Julian, on the seventeenth day of the Attic Munichion, or the April of the Romans. Nicagoras, the junior, being at that time the Athenian archon. The dead body was taken care of according to the Athenian rites, as he himfelf while living had directed. And if that diligent attention which is due to the deceafed, was ever paid by any one, it was certainly beftowed by this moft bleffed man. For he did not neglect any particular of the accuftomed ceremonies, but every year, on certain days, vifited the fepulchres of the Athenian heroes and philofophers, and befides this facrificed himfelf, and not through the medium of another, to the manes of his friends and familiars. And having thus exhibited to each a proper reverence and honour, proceeding into the Academy, he there, in a certain place, pleafed one by one the manes of his anceftors and kindred; and fhortly after, in another part of the

parts. So that the language of the obfcure and profound Heraclitus is perfectly juft, when he fays, "You muft connect the perfect and the imperfect, the agreeing and the difagreeirg, the confonant and the diffonant, and out of one all things, and out of all things one."

A Scheme of the fituation of the Stars, fuch as it was at Byzantium, when the philofopher Proclus was born.

fame

fame Academy, he fupplicated in common the fouls of all philofophers. And thefe ceremonies being ended, this moft excellent man, having chofen a third place, performed facred rites to all the fouls of deceafed mankind. The dead body of Proclus, therefore, being difpofed of according to his appointment, as we have faid, was carried by his friends and buried in the eaftern part of the fuburbs *, near to Lyca-betus, where alfo the body of his preceptor Syrianus repofes, who, when he was alive, had requefted this of our philofopher, and, in con-fequence of this, had taken care to procure a twofold receptacle in his fepulchre. But when this moft holy man was confulted how he would wifh his funeral to be conducted, left there fhould be any thing indecent, or without a proper decorum, he defired flutes, with which he was threatened in a dream, and nothing befides. An epigram, confifting of four verfes, is infcribed on his tomb, which he compofed himfelf, as follows:

> I Proclus, here the debt of nature paid,
> (My country Lycia) in the duft am laid;
> Great Syrianus form'd my early youth,
> And left me his fucceffor in *the truth.*
> One common tomb, our earthly part contains,
> One place our kindred fouls,—th' ætherial plains.

Nor were prognoftics wanting in the year prior to his deceafe, fuch as an eclipfe † of the fun, fo great that night was produced in the day-time: for the darknefs was fo profound, that the ftars became vifible. And this happened when the fun was in Capricorn, in the eaftern centre. Befides, the writers of Ephemerides obferve that there will be another eclipfe of the fun, at the conclufion of the next year. But fuch like affections of the heavenly bodies are faid to portend events on the earth: to us indeed, the eclipfe perfpicuoufly fignified a priva-tion and, as it were, defection of the illuftrious luminary of philo-fophy.

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* It was formerly the cuftom of almoft all nations, to have their burial places in the fuburbs, and not in the city itfelf.

† This eclipfe happened, according to Fabricius, in A. C. 484. 19 Cal. Feb. at fun-rife.

And

And thus much may fuffice for the life of the philofopher. It now remains for thofe who are willing to undertake the labour, to write concerning his difciples and friends. For it appears that many reforted to him from all parts; among which fome were only his auditors, but others firm adherents to his doctrine; and, on account of philofophy, his familiars. I could likewife wifh that fome one, who is more laborious than myfelf, would give a particular account of his writings. For I have alone delivered thefe particulars of his life, that I might fatisfy my confcience, and might teftify that I religioufly reverence the foul of Proclus, and his good dæmon. I fhall, therefore, add nothing concerning the philofopher's writings, except that I have always heard him prefer his commentaries on the Timæus of Plato beyond all his works. He likewife vehemently approved of his commentaries on the Theætetus. And he ufed frequently to fay, that if he was endued with defpotic power, he would alone preferve, of all the writings of the ancients, the Oracles and the Timæus. *For*, faid he, *I would abolifh all the reft, and remove them from the prefent age, becaufe it happens that many are offended, who undertake to read them rafhly, and without proper inftitution.*

Concern-

Concerning the Published Writings of PROCLUS.

MARINUS having neglected to give us an account of the writings of Proclus, I thought it would not be unacceptable to the reader, to prefent him with a catalogue of his works which are still extant; and the moft perfect relation I am able of fuch as are unfortunately loft.   In the execution of this defign, I fhall follow, for the moft part, the accurate Fabricius in the order, and critical account of his works; but fhall not hefitate to diffent from him in deciding on their philofophical merit: for it is very rare that philology and philofophy are united in the fame perfon, and coalefce in amicable conjunction.

I. *Four Hymns.* In the preceding life we are informed, by Marinus, that Proclus compofed many hymns, in which the divinities, both of Greeks and Barbarians, were celebrated; but unfortunately there are only four preferved, the firft of which is to the Sun, the fecond and third to Venus (which may be found in the Florilegium of Stobus, p. 249. edit. Grotii,) and the fourth to the Mufes.   They are collected by Fabricius, in the eighth volume of his Bibliotheca Græca; and are moft happy fpecimens of philofophical poetry. Indeed, they bear moft evident marks of a mind full of divine light, and agitated with the fury of the Mufes; and poffefs all that elegance of compofition for which the writings of Proclus are fo remarkable.   So that it is very ftrange Gyraldus fhould afcribe them to a Hierophant of Laodicæa, of that name; fince, as Fabricius obferves, Suidas mentions other writings of this prieft, but does not fpeak of him as the author of any hymns.   And if he had, thefe hymns breathe too much of the fpirit and manner of Proclus, to be the production of any other.

2. *Two Books concerning the ufeful Parts of Learning, contained in Grammar.* Three books on this fubject are mentioned by Suidas, and four by Photius; but there are alone extant fragments of two, in the Bibliotheca of Photius; which were publifhed feparate by Andrew Schottus, at the end of the fyntax of Apollonius Alexandrinus, Francof. 1590.

in elevated conceptions, until his mind, like Homer's difcord, reaches the heavens. His ftyle is at the fame time nervous and diffufe, accurate and elegant. We are aftonifhed with the magnificence of his metaphors, delighted with the copious variety of his matter, and filled with a divine light, by the facred truths he unfolds. This great work, however, unfortunately, fcarcely explains a third part of the Timæus; from whence it is probable, as Fabricius well obferves, that feveral books have been loft through the injury of time. It was publifhed in Greek, at Bafil, in the year 1556; and is full of miftakes, as is generally the cafe with the Bafil editions of books, fo that a more correct edition is greatly to be defired, though at prefent not much to be expected. On concluding my account of this ineftimable work, I find my indignation roufed by the following words of Dr. Cudworth, in his Intellectual Syftem, p. 306. " Proclus (fays he) had fome peculiar fancies and whims of his own, and was indeed a confounder of the Platonic theology, and a mingler of much unintelligible ftuff · with it." I muft confefs, (and I am neither afraid, nor afhamed of the declaration,) that I never found any thing in Proclus, but what by patient thought, accompanied with a fincere and vehement thirft after truth, I have been able to fathom. Had Dr. Cudworth been endued with thefe requifites, he would doubtlefs have had equal fuccefs; but without them, the fublimeft truths will certainly appear to be *unintelligible ftuff.* Befides this confideration is not to be omitted, that a modern prieft makes a bad philofopher.

5. *On Plato's Republic.* Thefe commentaries, or rather fragments of commentaries, are extant in Greek, at the end of Proclus on the Timæus. Suidas mentions four books of Proclus on Plato's politics; and fome of Proclus's differtations on thefe books, were found (according to Fabricius) in the library of Lucas Holftenius. The chief defign of this work feems to be the unveiling the theological myfteries concealed under the fables of Homer, and other divine poets; which Proclus has accomplifhed (in my opinion) in a moft wonderful manner. That Homer, indeed, every where abounds with Egyptian learning, is obvious to every one; but few are acquainted with the profound wifdom which his fables conceal. The latent meaning of moft of thefe is unfolded in the prefent invaluable, though imperfect

<div align="right">work;</div>

work; `and he whofe mind is fufficiently enlightened by the ancient philofophy, to comprehend the beauty of thefe illuftrations, will receive an additional delight from the ftudy of Homer, which it is impoffible to exprefs. An epitome of this work was publifhed in Latin, by the learned Gefner, 1542. 8vo. under the following title: Apologiæ quædam pro Homero, et Arte Poetica, Fabularumque aliquot Enarrationes ex commentariis Procli Lycii Diadochi philofophi Platonici in libros Platonis de Rep. in quibus plurimæ de Diis Fabulæ non juxta grammaticorum vulgus hiftoricè, phyficè aut ethicè tractantur, fed Theologicis, ut Gentiles loquuntur, ex prima Philofophia rationibus explanantur.

6. *On the firft Alcibiades of Plato.* Ficinus tranflated parts of this work into Latin, and publifhed them under the title of Procli de Anima ac dæmone, de Sacrificio et Magia, Venice 1497. and 1516. fol. by Aldus ; and in a more fimple form at Lyons. ,Fabricius informs us, that the manufcript commentary of Proclus in Greek, but fcarcely explaining the half part of the Alcibiades, is to be found in various libraries of France, England, and Italy. Alfo at Lyons, among the books of Ifaac Voffius; and at Hamburgh in the Johannean library. From the fpecimen given of this work by Ficinus, it appears, like all Proclus's philofophical writings, to be an invaluable treafury of wifdom ; and nothing certainly, reflects greater difgrace on a nation than fuffering fuch monuments of ancient learning and wifdom to lie concealed in colleges, covered with duft, and never confulted.

7. *Six Books on Plato's Theology.* A moft divine work, in which the philofopher collects into a fyftem the theology difperfed in the writings of Plato, and eftablifhes it by invincible demonftrations. He deduces, in a beautiful and connected feries, all the divine orders, from the retreats of ineffable unity ; every where connects them by proper mediums, and, after leading us through the long gradation of principles, brings us back again to the original from whence they flowed, and to which they conftantly tend. The whole is uncommonly profound and abftrufe ; and it was not before the third reading, that I could fathom the depth it contains. Fabricius obferves, " that it is a fubtle and learned work, but from which, you will fooner learn the opinion of Syrianus and Proclus, concerning the deity and divine

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concerns, than that of Plato. He adds, that it is ufual with the Platonifts, even from Plotinus, to unite to the doctrine of Plato, a thoufand dogmata, foreign from his philofophy, as if Plato, though he did not perceive after this manner, ought certainly fo to perceive." When men miftake their abilities, they always act abfurdly, and often dangeroufly. As a laborious and accurate critic on philological matters, Fabricius merits the higheft commendation fuch attainments can deferve; but when he leaves the beaten road in which nature defigned him to walk, and attempts the tractlefs paths of philofophy, he perpetually ftumbles, and often falls on the ground. The wings of philology, like thofe of the fwallow, were never deftined for a lofty flight:—it muft be the eagle wing of genius, which can alone foar to the fun of philofophy. The Greek and Latin edition of this valuable work, was publifhed at Hamburgh, by Æmilius Portus, 1618, folio.

8. *Theological Inftitutions*; or, as it may be called, *the Elements of Theology.* This admirable work contains two hundred and ten propofitions, difpofed in a fcientific order, and fupported by the firmeft demonftrations. They begin from fuper-effential unity, and proceed gradually through all the beautiful and wonderful progreffions of divine caufes, ending in the felf-moving energies of foul. They poffefs all the accuracy of Euclid, and all the fubtilty and fublimity neceffary to a knowledge of the moft profound theology; and may be confidered as bearing the fame relation to the Pythagoric and Platonic wifdom, as Euclid's Elements, to the moft abftrufe geometry. Patricius, the firft Latin tranflator of this divine work, feems to have been very fenfible of the truth of this obfervation: for he every where carefully diftinguifhes the propofitions from their demonftrations; and adds the word corollary to fuch confequencies as merit that appellation. His edition was publifhed at Ferraria, 1583. quarto, under the title of Theological Elements. The Greek and Latin edition, is fubjoined to Proclus's fix books on Plato's Theology, Hamburgh, 1618. folio.

9. *Two Books concerning Motion.* This ufeful work, collected, as Fabricius obferves, from the third and following books of Ariftotle's phyfics, was publifhed in Greek at Bafil, 1531, and with the Latin verfion of one Juftus Velfius, a phyfician, Bafil, 1545. octavo. It

was

was likewife tranflated by Patricius, and is annexed to his verfion of the Theological Inftitutions.

10. *An Hypotypofis, or Information concerning Aftronomical Hypothefes.* This work, which Fabricius obferves is a compendium of Ptolemy's Almageft, was publifhed in Greek, at Bafil, 1540. quarto; and in Latin by George Valla, folio, 1541. A part of this work, which treats of the ufe of the aftrolabe, Fabricius informs us, is extant in manufcript, in various libraries. The fame accurate critic likewife obferves, that a fmall treatife, infcribed Uranodromus, is extant, under the name of Proclus, in fome libraries, as in that of Vindobona, and of Oxford, among the Barrocian volumes. The comprehenfive variety of Proclus's genius equally demands our admiration and applaufe.

11. *A fmall Treatife concerning the Sphere, or Celeftial Circles.* This little work is an accurate and elegant introduction to aftronomy; and is almoft wholly taken from the Ifagoge of Geminus Rhodius, on the phænomena. The beft editions are the Greek and Latin one publifhed at Paris in 1552, quarto; and that of Bainbridge, profeffor of aftronomy at Oxford, London 1620. quarto.

12. *A Paraphrafe in four Books, on the Quadripartite of Ptolemy.* This elegant work muft, I fhould imagine, be an invaluable treafure to the lovers of aftrology. It was firft publifhed in Greek by Melanfthon; and afterwards in Greek and Latin by Leo Allatius, at Lyons Bat. 1654. octavo.

13. *Four Books, on the firft Book of Euclid's Elements.* For an account of this work, fee the introduction, and the following fheets, in which it fpeaks for itfelf, in an Englifh drefs.

14. *A Commentary on Hefiod's Works and Days.* This work contains a valuable moral explanation of this great poet's meaning; and Fabricius juftly obferves, that he is often affaulted without occafion, by the petulant jeers of that vain man Joh. Tzetzes. The beft edition of this work is that of Daniel Heinfius, Lugd. Bat. 1603. quarto.

15. Fabricius informs us, that in fome manufcripts, as in the Vindobonenfian and Barrocian, a fmall treatife is ufually afcribed to Proclus, entitled *Egyptiac Characters*; and is prefixed to the Epiftles of Phalaris and Brutis, and publifhed under the name of Libanius, in Greek,

Greek, with the verſion of Caſp. Stibilinus, Commelin. 1597, octavo:
But it is doubtful whether Proclus is the genuine author of this work:
from the title, I ſhould ſuppoſe the contrary. And thus much for an
account of thoſe writings of Proclus which have eſcaped the ravages
of time, and have been fortunately expoſed to public inſpection : it
now remains that we relate ſuch ineſtimable works of this philoſopher,
as are yet preſerved in ſhameful concealment ; or are utterly loſt in the
ruins of antiquity.

Concerning the Unpubliſhed Writings of P R O C L U S.

16. *On the Alcibiades of Plato.* See num. 6.

17. *On Plato's Politics.* See num. 5.

18. *On Plato's Parmenides.* A commentary, in ſeven books; the
laſt of which was not completed by Proclus, but by Damaſcius.
From occaſional fragments, which have been publiſhed of this com-
mentary, it appears to be a moſt divine work; and indeed it cannot
be otherwiſe, if we conſider it as the production of one of the greateſt
philoſophers,on the moſt ſublime and profound of all Plato's Dialogues.
It is dedicated to Aſclepiodotus, a phyſician and philoſopher, and is
not only extant in Greek MS. in the library of the German emperor,
according to Lambecius, lib. vii. p. 41. but alſo in Latin, from the
unpubliſhed verſion of one Antonius Hermannus Gogava, as the
ſame Lambecius informs us, p. 41. Four books of this work are
extant in Greek, in the Bodleian library at Oxford; and it is much
to be lamented that Thomſon did not publiſh theſe, inſtead of his
trifling edition of the Parmenides. Fabricius likewiſe informs us,
that Livius Galantes mentions his having found ſix of theſe books
in ſome of the Italian libraries. They are alſo extant in the Medi-
cæan library of the great Etruſcan commander.

19. *On the Cratylus of Plato.* We have already obſerved, in the
diſſertation on the Orphic theology, p. 105. what a great treaſure of
ancient mythology, muſt be contained in this work; but there is little
hope of its ever emerging from the obſcurity of public libraries. It is

<div align="right">extant.</div>

extant in Greek, not only in the Italian libraries, but alfo among the manufcript books of Ifaac Voffius.

20. *Uranodromus.* See above, num. 10.

21. *Ten Doubts concerning Providence, in one Book.* Philoponus mentions this work, in his fecond book againft Proclus on the eternity of the world; and a Latin verfion of it is extant by one William de Morbeka, in the Johannean library of Hamburgh. Extracts from this tranflation are preferved by Fabricius, in his Greek Library; and they are in every refpect worthy of the genius of Proclus.

22. *Concerning Providence and Fate, and that which is in our Power, one Book.* This work is dedicated to one Theodorus, a mechanift; and is extant in the Latin tranflation of the fame Morbeka, in the Greek Library of Fabricius. The tranflation is for the moft part barbarous, but is, however, fufficiently legible to difcover that it is a moft valuable treatife, replete with the ufual elegance, fubtilty, and fublimity of our philofopher.

23. *Concerning the Hypoftafis, or Subfiftence of Evil.* This book is extant in Latin, in the Johannean library; and fragments of it are preferved by Fabricius, in his Greek library. It is to be regretted, that Fabricius did not preferve the whole in that excellent philological work.

Concerning the Loft Writings of PROCLUS.

24. *On the Speech of Diotima, in Plato's Banquet, concerning the Subfiftence of the Beautiful.* Fabricius informs us, that this work is diftributed into many books; and Holftenius obferves, that it is mentioned in a certain fcholium of the Medicean copy of Proclus's commentaries on Plato's politics; but it is unfortunately no where extant.

25. *On the Philebus of Plato;* as may be inferred from the narration of Damafcius in Photius, p. 550; and Suidas in Marinus. For Damafcius relates, that Marinus having compofed a commentary on this dialogue, on fhewing it to Ifidorus for his approbation, that philofopher obferved, *that thofe of his mafter were fufficient;* which words Fabricius, with great propriety, applies to the commentaries of Proclus on the Philebus.

26. *On the Theætetus of Plato.* This work is praifed by Marinus, in the laft chapter of the preceding life; and no doubt with great propriety: for this abftrufe and fublime dialogue would naturally call forth all the divine fire and elegance of our philofopher.

27. *Commentaries on the Enneads of Plotinus.* This work is men-- tioned by Gyraldus, in his fecond dialogue on ancient poets; by Fi-- cinus on Plotinus; by Philip Labbeus, in his account of MS. books, p. 286; and in the notes of Bullialdus to Theo of Smyrna, p. 224. But alfo in a certain note prefixed to an ancient manufcript of Jam-- blichus, on the Egyptian myfteries, to this effect : " The philofopher Proclus, commenting on the Enneads of the great Plotinus, fays, that, it is the divine Jamblichus who anfwers the epiftle of Porphyry." This note is in Greek, in the original, and is (in my opinion) of, itfelf fufficient to prove that fuch a work was once extant, though. now unfortunately loft. How much the want of thefe commentaries, is to be regretted, muft be deeply felt by every lover of the Platonic. philofophy. For the unequalled profundity, and divine myfteries, contained in the writings of Plotinus, could never be more happily illuftrated than by the irradiations of fuch a genius as Proclus.

28. *Lectures on Ariftotle's Book* Περι Ερμηνειας, *or concerning Interprc- tation.* This work, it feems, was never publifhed; but Ammonius. Hermeas, the difciple of Proclus, has inferted in his valuable com-- mentary on this book all that he could retain in his memory of Pro-- clus's lectures.

29. *Hymns*, not a few, fee num. 1.

30. *Chreftomathea.* See num. 3.

31. *On the Mother of the Gods*, one book, mentioned by Marinus, in the preceding Life.

32. *On the Theology of Orpheus.* This work is mentioned by Ma- rinus, in the preceding Life, and by Suidas; and its lofs muft be par- ticularly regretted by all the lovers of recondite theology.

33. *Ten Books, on the Chaldean Oracles.* This moft valuable work is mentioned by Marinus, in the preceding Life, and by Proclus him- felf on Plato's Politics, p. 359. It was doubtlefs not extant at the time when Pfellus and Pletho undertook the illuftration of a few of

thefe.

thefe oracles: at leaft the inconfiderable merit of their commentaries, ftrongly favours this fuppofition.

34. *A Commentary on the whole of Homer*. Suidas. A fpecimen of the great value of this work may be feen in our philofopher's commentaries on Plato's republic. The works of Homer are not only the great fountain of poetry, but likewife of philofophy; and are no lefs admirable for infpiring the fury of the Mufes than for containing the myfteries of the moft recondite theology.

35. *Concerning the Gods, according to Homer*. Had this work been preferved, we fhould doubtlefs have been furnifhed with a defence of the heathen religion, which would have filenced the ignorant clamours of its opponents.

36. *The Symphony or Concord of Orpheus, Pythagoras, and Plato*. Suidas. Proclus, in his publifhed writings, is every where ftudious of reconciling the doctrines of thefe great men, and is always fuccefsful in this undertaking. Indeed, the fame divine genius feems to have irradiated and infpired thefe wonderful heroes, but in different ways: in Orpheus it was accompanied with the fire of the Mufes; in Pythagoras it fhone through the myfterious veil of numbers; and in Plato, combining the preceding modes, it was feen enfhrined in awful majefty of thought, clothed with the graces of poetical diction, and refplendent with ineffable light.

37. *Two Books on the Theurgic Difcipline*. Suidas. How much Proclus excelled in this art, may be feen in the preceding Life.

38. *Concerning the Oppofition of Ariftotle to Plato's Timæus*. This work is mentioned by Proclus in the 2d book of his commentary on the Timæus, p. 226. and feems to have efcaped the notice of the accurate Fabricius. Ariftotle may, no doubt, in many particulars be reconciled with Plato; but it is alfo certain, that in fome he is perfectly different. And thus much for the Life and Writings of Proclus.

COMMENTARIES

OF

PROCLUS.

BOOK I.

CHAP. I.

On the Middle Nature of the Mathematical Effence.

IT is neceffary that the mathematical effence fhould neither be feparated from the firft nor laft genera of things, nor from that which obtains a fimplicity of effence; but that it fhould obtain a middle fituation between fubftances deftitute of parts, fimple, incompofite and indivifible, and fuch as are fubject to partition, and are terminated in manifold compofitions and various divifions. For fince that which fubfifts in its inherent reafons remains perpetually the fame, is firm and durable, and cannot be confuted, it evidently declares it is fuperior to the forms exifting in matter. But that power of progreffion which apprehends, and which befides ufes the dimenfions of fubjects, and prepares different conclufions from different principles, gives it an order inferior to that nature which is allotted an indivifible effence, perfectly conftituted in itfelf. Hence (as it appears to me) * Plato alfo

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* All the ancient theologifts, among whom Plato holds a diftinguifhed rank, affirmed that the foul was of a certain middle nature and condition between intelligibles and fenfibles : agreeable to which doctrine, Plotinus divinely afferts that fhe is placed in the horizon, or in the boundary and ifthmus, as it were, of eternal and mortal natures ; and hence, according to the Magi, fhe is fimilar to the moon, one of whofe parts is lucid, but the other dark. Now the foul, in

con-

alfo divides the knowledge of things which are, into firſt, middle, and laſt ſubſtances. And to indiviſible natures, indeed, he attributes an

conſequence of this middle condition, muſt neceſſarily be the receptacle of all middle energies, both vital and gnoſtic: ſo that her knowledge is inferior to the indiviſible ſimplicity of intellectual comprehenſion, but ſuperior to the impulſive perceptions of ſenſe. Hence the mathematical genera and ſpecies reſide in her eſſence, as in their proper and natural region; for they are entirely of a middle nature, as Proclus proves in this and the ſixth following chapter. But this doctrine of Plato's, originally derived from Brontinus and Archytas, is thus elegantly explained by that philoſopher, in the concluding part of the ſixth book of his Republic. "Socrates, know then, they are, as we ſay, two (the Good itſelf, and the Sun,) and that the one reigns over the intelligible world, but the other over the viſible, not to ſay the heavens, leſt I ſhould deceive you by the name. You comprehend then, theſe two orders of things, I mean the viſible and the intelligible?—Glauco. I do.—Socrates. Continue this diviſion then, as if it were a line divided into two unequal ſegments; and each part again, i. e. the ſenſible and intelligible, divided after a ſimilar manner, and you will have evidence and obſcurity placed by each other. In the viſible ſegment, indeed, one part will contain images. But I call images, in the firſt place, ſhadows; afterwards, the reſemblances of things appearing in water, and in denſe, ſmooth, and lucid bodies, and every thing of this kind, if you apprehend me?—Glauco. I apprehend you. —Socrates. Now conceive that the other ſection comprehends the things, of which theſe images are nothing more than ſimilitudes, ſuch as the animals around us, together with plants, and whatever is the work of nature and art.—Glauco. I conceive it.—Socrates. Do you conſider this ſection then, as divided into true and falſe? And that the hypotheſis of opinion is to the knowledge of ſcience, as a reſemblance to its original?—Glauco. I do, very readily.—Socrates. Now then, conſider how the ſection of the intelligible is to be divided.—Glauco. How?—Socrates. Thus: one ſegment is that which the ſoul enquires after, uſing the former diviſions as images, and compelled to proceed from hypotheſes, not to the principle, but to the concluſion. The other is that which employs the cogitative power of the ſoul, as ſhe proceeds from an hypotheſis to a principle no longer ſuppoſed, and, neglecting images, advances through their *obſcurity* into *the light of* ideas themſelves.—Glauco. I do not, in this, ſufficiently underſtand you.—Socrates. But again, for you will more eaſily underſtand me from what has been already premiſed. I think you are not ignorant, that thoſe who are converſant in geometry, arithmetic, and the like, ſuppoſe even and odd, together with various figures, and the three ſpecies of angles, and other things ſimilar to theſe, according to each method of proceeding. Now, having eſtabliſhed theſe, as hypotheſes ſufficiently known, they conceive that no reaſon is to be required for their poſition: but beginning from theſe, they deſcend through the reſt, and arrive at laſt, at the object of their inveſtigation.—Glauco. This I know perfectly well.—Socrates. This alſo you know, that they uſe viſible forms, and make them the ſubject of their diſcourſe, at the ſame time not directing their intellect to the perception of theſe, but to the *originals* they reſemble; I mean the ſquare itſelf, and the diameter itſelf; and not to the figures they delineate. And thus, other forms, which are repreſented by ſhadows and images in water, are employed by them, merely as reſemblances, while they ſtrive to behold that which can be ſeen by cogitation alone.— Glauco. You ſpeak the truth.—Socrates. This is what I called above a ſpecies of the intelligible, in the inveſtigation of which, the ſoul was compelled to uſe hypotheſes; not aſcending to the principle, as incapable of riſing above hypotheſes, but uſing the images formed from inferior objects, to a ſimilitude of ſuch as are ſuperior, and which are ſo conceived and diſtinguiſhed by opinion, as if they perſpicuouſly contributed to the knowledge of things themſelves —Glauco. I underſtand indeed, that you are ſpeaking of the circumſtances which take place in geometry, and her kindred arts.—Socrates. Underſtand now, that by the other ſection of the intelligible, I mean that which reaſon herſelf reaches, by her power of demonſtrating, when no longer eſteeming hypotheſes for principles, but receiving them in reality for hypotheſes, ſhe uſes them as ſo many ſteps and handles in her aſcent, until ſhe arrives at that which is no longer

intelligence, which, in a collective manner, and by a certain fimple power, divides the objects of intellectual perception; fo that being divefted of matter, and endued with the greateft purity, it apprehends things themfelves, by a certain unifying perception, and excels the other kinds of knowledge. But to divifible effences, and fuch as are allotted the loweft nature, and to all fenfible beings, he attributes opinion, which obtains an obfcure and imperfect truth. But to middle effences (and fuch are mathematical forms), and to things inferior to an indivifible and fuperior to a divifible nature, he attributes cogitation. For this, indeed, is inferior to intellect, and the fupreme fcience dialectic; but is more perfect than opinion, and more certain and pure. For it advances by a difcurfive proceffion, expands the indivifibility of intellect, and unfolds that which was involved in the unity of intellectual apprehenfion: but it collects things which are divided, and brings them back to mind. Hence, as knowledges differ among themfelves, fo the objects of knowledge are diftinguifhed by nature. So that intelligible effences having an uniform fubfiftence,

hypothetical, the principle of the univerfe; and afterwards defcending, holding by ideas which adhere to the principle, fhe arrives at the conclufion, employing nothing fenfible in her progrefs, but proceeding through ideas, and in thefe at laft terminating her defcent.—Glauco. I underftand you, but not fo well as I defire: for you feem to me to propofe a great undertaking. You endeavour, indeed, to determine that the portion of true being and intelligible, which we fpeculate by the fcience of demonftration, is more evident than the difcoveries made by the fciences called arts; becaufe in the firft hypothefes are principles, and their mafters are compelled to employ the eye of cogitation, and not the perceptions of the fenfes. Yet, becaufe they do not afcend to the principle, but invefligate from hypothefes, they feem to you not to have intelligence concerning thefe, though they are intelligible, through the light of the principle. But you feem to me to call the habit of reafoning on geometrical and the like concerns, cogitation, rather than intelligence, as if cogitation held the middle fituation between opinion and intellect. — Socrates. You underftand me fufficiently well. And again: with thefe four proportions take thefe four correfponding affections of the foul: with the higheft intelligence; with the fecond cogitation; againft the third fet opinion; and againft the fourth affimilation, or imagination. Befides this, eftablifh them in the order of alternate proportion, fo that they may partake of evidence, in the fame manner as their correfponding objects participate of reality." I have taken the liberty of tranflating this fine paffage differently from both Petvin and Spers; becaufe they have neglected to give the proper meaning of the word διανοία, or cogitation, the former tranflating it *mind*, and the *eye of the mind*, and by this means confounding it with intellect; and the latter calling it *underftanding*. But it is certain that Plato, in this place, ranks intellect as the firft, on accourt of the fuperior evidence of its perceptions; in the next place, cogitation; in the third, opinion; and in the fourth, imagination. However, the reader will pleafe to remember, that by διανοία, or cogitation, in the prefent work, is underftood that power of the foul which reafons from premifes to conclufions, and whofe fyllogiftic energy, on active fubjects, is called prudence; and on fuch as are fpeculative, fcience. But for further information concerning its nature, fee the differtation prefixed to this work, and the following fifth chapter.

evidently

evidently excel all others. But fenfibles are entirely excelled by primary effences: and mathematical natures, and whatever falls under cogitation, are allotted a middle order: for they are excelled by the divifion of intelligibles; but becaufe deftitute of matter, they are fuperior to fenfible natures; and by a certain fimple power, they are excelled by the firft; but by a certain reafon are more exalted than the laft. Hence they poffefs notions of an intellectual effence, which are more manifeft than fenfibles, but which are, at the fame time, only the images of an intellectual nature; and they imitate divifibly the indivifible, and, in a multiform manner, the uniform exemplars of things. And, that I may fum up the whole in a few words, they are placed in the veftibules or entrances of primary forms, and difclofe their indivifible and prolific fubfiftence collected into one, but they do not yet excel the divifion and compofition of reafons, and an effence accommodated to the obfcurity of images; nor are they capable of paffing beyond the various notions of the foul, endued with a difcurfive power, and of adhering to intellections perfectly fimple, and purified from all material imperfection. After this manner then, is the middle nature of mathematical genera and forms to be underftood; as filling up the medium between effences entirely indivifible, and fuch as are divifible about matter.

CHAP. II.

Concerning the common Principles of Beings, and of the Mathematical Essence, * bound and infinite.

BUT it is neceffary that, confidering the principles of the whole, mathematical effence, we fhould return to thofe general principles, which pervade through and produce all things from themfelves, I mean *bound* and *infinite*. For from thefe two after that caufe of *one*, which can neither be explained, nor entirely comprehended, every other thing, as well as the nature of the mathematical difciplines, is conftituted. In the former, indeed, producing all things colleſtively and feparately; but in thefe proceeding in a convenient meafure, and receiving a progreffion in a becoming order; and in fome, fubfifting among primary, but in others among middle, and in others again among pofterior natures. For intelligible genera, by their fimplicity of power, are the firft participants of *bound* and *infinite:* becaufe, on account of their union and identity, and their firm and ftable exiftence, they are perfeſted by bound: but on account of their divifion into multitude, their copious power of generation, and their divine diverfity and progreffion, they obtain the nature of infinite. But mathematical genera originate, indeed, from bound and infinite, yet not from primary, intelligible, and occult principles only; but alfo from thofe principles which proceed from the firft to a fecondary order, and which are fufficient to produce the middle ornaments of

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* Thefe two principles, *bound* and *infinite*, will doubtlefs be confidered by the unthinking part of mankind, as nothing more than general terms, and not as the moft real of beings. However, an accurate contemplation of the univerfe, will convince every *truly* philofophic mind of their reality. For the heavens themfelves, by the coherence and order of their parts, evince their participation of bound. But by their prolific powers, and the unceafing revolutions of the orbs they contain, they demonftrate their participation of *infinity*. And the finite and perpetually abiding forms with which the world is replete, bear a fimilitude *to bound:* while, on the contrary, the variety of particulars, their never-ceafing mutation, and the conneſtion of more and lefs in the communion of forms, reprefents an image of *infinity*. Add too, that every natural fpecies, by its form is fimilar to *bound*; but by its matter, to *infinity*. For thefe two, form and matter, depend on *bound* and *infinity*, and are their ultimate progreffions. And each of thefe, indeed, participates of unity; but form is the meafure and bound of matter, and is more *one*. But matter is in capacity all things, becaufe it fubfifts by an emanation from the firft capacity, or the *infinite itfelf*.

beings,

beings, and the variety which is alternately found in their natures. Hence, in thefe alfo, the reafons and proportions advance to infinity, but are reftrained and confined by that which is the caufe of bound. For number rifing from the retreats of unity, receives an inceffant increafe, but that which is received as it ftops in its progreffion, is always finite. Magnitude alfo fuffers an infinite divifion, yet all the parts which are divided are bounded, and the particles of the whole exift finite in energy. So that without the being of infinity, all magnitudes would be commenfurable, and no one would be found but what might either be explained by words, or comprehended by reafon (in which indeed geometrical fubjects appear to differ from fuch as are arithmetical;) and numbers would be very little able to evince the prolific power of unity, and all the multiplex and fuper-particular proportions which they contain. For every number changes its proportion, looking back upon, and diligently enquiring after unity, and a reafon prior to itfelf. But *bound* being taken away, the commenfurability and communication of reafons, and one and the fame perpetual effence of forms, together with equality, and whatever regards a better co-ordination, would never appear in mathematical anticipations: nor would there be any fcience of thefe; nor any firm and certain comprehenfions. Hence then, as all other genera of beings require thefe two principles, fo likewife the mathematical effences. But fuch things as are laft in the order of beings, which fubfift in matter, and are formed by the plaftic hand of nature, are manifeftly feen to enjoy thefe two principles effentially. Infinite as the fubject feat of their forms; but bound as that which invefts them with reafons, figures, and forms. And hence it is manifeft that mathematical effences have the fame pre-exiftent principles with all the other genera of beings.

C H A P. III.

What the common Theorems are of the Mathematical Effences.

BUT as we have contemplated the common principles of things, which are diffufed through all the mathematical genera, after the fame manner we muft confider thofe common and fimple theorems, originating from one fcience, which contains all mathematical knowledge in one. And we muft inveftigate how they are capable of according with all numbers, magnitudes and motions. But of this kind are all confiderations refpecting proportions, compofitions, divifions, converfions, and alternate changes: alfo the fpeculation of every kind of reafons, multiplex, fuper-particular, fuper-partient, and the oppofite to thefe: together with the common and univerfal confiderations refpecting equal and unequal, not as converfant in figures, or numbers, or motions, but fo far as each of thefe poffeffes a common nature effentially, and affords a more fimple knowledge of itfelf. But beauty and order are alfo common to all the mathematical difciplines, together with a paffage from things more known, to fuch as are fought for, and a tranfition from thefe to thofe which are called refolutions and compofitions. Befides, a fimilitude and diffimilitude of reafons are by no means abfent from the mathematical genera: for we call fome figures fimilar, and others diffimilar; and the fame with refpect to numbers. And again, all the confiderations which regard powers, agree in like manner to all the mathematical difciplines, as well the powers themfelves, as things fubject to their dominion: which, indeed, Socrates, in the Republic, dedicates to the Mufes, fpeaking things arduous and fublime, becaufe he had embraced things common to all mathematical reafons, in terminated limits, and had determined them in given numbers, in which the meafures both of abundance and fterility appear.

C H A P. IV.

How thefe Common Properties fubfift, and by what Science they are confidered.

BU.T it is requifite to believe, that thefe common properties do not primarily fubfift in many and divided forms, nor originate from things many and the laft: but we ought to place them as things preceding in a certain fimplicity and excellence. For the knowledge of thefe antecedes many knowledges, and fupplies them with principles; and the multitude of fciences fubfift about this, and are referred to it as their fource. Thus the geometrician affirms, that when four magnitudes are proportional, they fhall be alternately proportional; and he demonftrates this from principles peculiar to his fcience, and which the arithmetician never ufes. In like manner, the arithmetician affirms, that when four numbers are proportional, they fhall be fo alternately: and this he evinces from the proper principles of his fcience. For who is he that knows alternate ratio confidered by itfelf, whether it fubfifts in magnitudes or in numbers? And the divifion of compofite magnitudes or numbers, and in like manner, the compofition of fuch as are divided? They are furely not the fciences and cognitions of things divifible: but we have no fcience of things deftitute of matter, and which are affigned a more intellectual contemplation; for the knowledge of thefe is by much prior to fcience, and from thefe the common reafons of many fciences are derived. And there is a gradual afcent in cognitions from things more particular to more univerfal, till we revert to the fcience of that which *is*, confidered *as it is*, abftracted from all fecondary properties. For this fublime fcience does not think it fuitable to its dignity, to contemplate the common properties which are effentially inherent in numbers, and are common to all quantities; but it contemplates the one, and firm effence of all the things which *are*. Hence, it is the moft capacious of all fciences, and from this all the reft affume their own peculiar principles. For the fuperior fciences always afford the firft fuppofitions of demonftrations to fuch as are fubordinate. But that, which is the moft perfect

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of all the fciences, diftributes from itfelf principles to all the reft, to
fome indeed, fuch as are more univerfal, but to others, fuch as are
more particular. Hence, Socrates, in the Theætetus, mingling the
jocofe with the ferious, compares the fciences which refide in us to
doves : but he fays they fly away, fome in flocks, but others fepa-
rate from one another. For fuch, indeed, as are more common and
more capacious, comprehend in themfelves many fuch as are more.
particular: but fuch as being diftributed into forms, touch things fub-
ject to knowledge, are diftant from one another, and can by no means
be copulated together, fince they are excited by different primary
principles. One fcience, therefore, precedes all fciences and difci-
plines, fince it knows the common properties which pervade through
all the genera of beings, and fupplies principles to all the mathemati-
cal fciences. And thus far our doctrine concerning dialectic * is ter-
minated.

C H A P.   V.

*What the Inftrument is, which judges of the Mathematical Genera and
Species.*

LET us now confider what that inftrument is †, adapted to the
judgment of mathematical concerns; and let us appoint Plato
as our guide in this affair, who, in his Republic, divides cognitions
feparately from fuch things as are the objects of knowledge; and
diftributes cognitions in conjunction with things fubject to knowledge.
For of the things which are, fome he ranks among intelligibles, and
others among fenfibles. And of intelligibles, fome are again pure
intelligibles, and others fubject to cogitation. And of fenfibles, fome

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* Of human difciplines, thofe alone deferve to be called fciences which ufe no hypothefes,
which refolve things into their principles, which arc converfant with true being, and elevate us
to ideas themfelves. Dialectic is wholly of this kind (I mean the dialectic of Plato) ; for this
alone ufes no fuppofitions, but, neglecting fhadows and images, raifes us, by a fublime invefliga-
tion, to the principle of the univerfe; and on this account, deferves to be called the very apex
of difciplines. But we muft not imagine, that by the word dialectic here, is meant logic, or
any part of logic, or that method of difputation, by which we fabricate probable reafons ; but
we muft conceive it as fignifying a difcipline, endued with the greateft acutenefs; neglecting
all hypothefes, truly foaring to primary caufes, and ultimately repofing in their contemplation.
Plotinus has given us moft happy fpecimens of this method, in his books *on the genera of being.*

† See note to the firft chapter.

are purely fenfibles, but others conjectural. To intelligibles, indeed, which are the firft of the four genera, he affigns an intelligible knowledge; but to thofe which are fubject to cogitation, he attributes thought: to fenfibles, faith; but to conjecturals, a conjectural or affimilatory power. And he fhews, that the affimilatory power has the fame proportion to fenfe as thought to intelligence. For the conjectural power knows the fpectres of fenfible forms, while they are beheld in water and other bodies, which perfpicuoufly reprefent their image: fince, by their fituation in water, they are after a manner, allotted the laft feat in the gradations of forms, and truly become the refemblances of refemblances. In like manner, thought beholds the images of intelligibles in a degraded ftate, fallen from primary fimple and indivifible forms, into multitude and divifion. Hence, a knowledge of this kind, depends on other more ancient hypothefes; but intelligence arrives at that principle which is no longer fuppofed. If then, mathematical concerns are neither allotted an effence feparate from all divifion and variety, nor that nature which is apprehended by fenfe, which is obnoxious to many mutations, and is in every proportion divifible, it muft be manifeft to every one, that they are effentially fubject to cogitation: but cogitation prefides over thefe as an inftrument adapted to judgment, in the fame manner as fenfe to fenfibles, and the affimilatory power to conjecturals. From whence, indeed, Socrates determines that the knowledge of thefe is more obfcure than the firft fcience, but is more evident than the impulfive apprehenfion of opinion. For in this the mathematical fciences are inferior to intelligence, becaufe they contemplate that which is evolved, and is endued with a power of progreffion; but they are fuperior to opinion, by that ftability of reafons which they contain, and which cannot be confuted. And they originate from fuppofition, through a diminution of the firft fcience; but they contain forms independent of matter, from their poffeffing a knowledge more perfect than that of fenfibles. We have therefore determined an inftrument adapted to the judgment of all mathematical concerns, i. e. cogitation, according to the mind of Plato; which places itfelf indeed above opinion, but is excelled by intelligence.

CHAP.

C H A P.   VI.

Concerning the Effence of Mathematical Genera and Species *.

IT now remains, that we confider what fubfiftence or effence ought
to be affigned to mathematical genera and fpecies? Whether we
muft deduce their origin and fubfiftence from fenfible objeds, or from
abftradion, or from a colledion of fuch things as are difperfed by
parts into one common definition; or muft allow them an exiftence
prior to that of fenfibles, as Plato affirms, and as the progreffion of
univerfal being demonftrates? Firft then, if we affirm that mathema-
tical fpecies are compofed from fenfibles; whilft the foul from mate-
rial triangles or circles, forms in herfelf the trigonic, or circular fpe-
cies, by a kind of fecondary generation; I would afk from whence is
derived the great certainty and accuracy of definitions? For it muft
either proceed from fenfibles, or from the foul herfelf. But from
fenfibles is impoffible, for thefe, in a continual flow of generation and
decay, do not for a moment retain an exad famenefs of being; and
confequently fall far fhort of the exadnefs contained in the definitions
themfelves. It muft therefore proceed from the foul, which, by her
immaterial nature, procures perfedion from the imperfed, accurate

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* I would particularly recommend this chapter to modern mathematicians, moft of whom, I
am afraid, have never confidered whether or not the fubjeds of their fpeculation have any real
fubfiftence: though it is furely an enquiry worthy the earneft attention of every liberal mind.
For if the objeds of mathematical inveftigation are merely imaginary, I mean the point with-
out parts, the line without breadth, &c. the fcience, founded on thefe falfe principles, muft of
courfe be entirely delufive. Indeed, an abfolutely true conclufion, can never flow from an erro-
neous principle, as from its caufe: as the ftream muft always participate of its fource. I mean
fuch a conclufion as is demonftrated by the proper caufe, πλην ὁ διοτι, αλλ' ὁτι, fays Ariftotle,
in his firft Analytics; that is, a fyllogifm from falfe principles will not prove the *why*, but
only fimply *that it is:* indeed it can only fimply prove *that it is*, to him who admits the falfe
propofitions; becaufe he who allows the premifes, cannot deny the conclufion, when the fyl-
logium is properly conftruded. Thus we may fyllogize in the firft figure,

<div style="text-align:center">

Every thing white, is an animal:
Every bird is white:
Therefore, Every bird is an animal.
</div>

And the conclufion will be true, though the major and minor terms are falfe; but then thefe
terms are not the caufes of the conclufion, and we have an inference without a proof. In like
manner, if mathematical fpecies are delufive and fiditious, the conclufions deduced from them
as principles, are merely hypothetical, and not demonftrative.

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fubtilty
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fubtilty from that which is neither accurate nor fubtle, and rekindles the light of ideas from the obfcure and unreal objeds of fenfe.

For where fhall we find, amongft fenfible objeds, an indivifible nature, fuch as that of a point, or a line without the dimenfion of breadth, or a fuperficies without depth, or the ever conftant propor- tion of fides, and exad rectitude of angles ? For my part, I cannot fee where, fince all divifible natures are thus mixed and confufed to- gether, nothing fincere, nothing free from its contrary, but things every where yielding to feparation, as well fuch as are removed by diftance of place, as thofe which are united together. How then fhall we obtain this durable effence for thefe immoveable natures from the ever fluctuating forms of fenfe? For whatever derives its exiftence from moveable beings, muft of neceffity be mutable and frail. And how fhall we gain this perfed accuracy for the ftable fpecies, from the inaccurate and imperfed? For whatever is the caufe of a conception, always immutable, is itfelf much more ftable than its effed. We muft therefore admit the foul to be the generator of thefe mathemati- cal fpecies and reafons. But if fhe contains them in herfelf, as firft exemplars, fhe gives them an effential being, fo that the generations are nothing elfe than propagations of fpecies, which had a prior fub- fiftence in herfelf: and thus we fhall fpeak agreeably to the fentiments of Plato, and difcover the true effence of mathematical entities. But if the foul, though fhe neither poffeffes nor received the mathemati- cal reafons prior to the energies of fenfe, yet fabricates this admirable immaterial building, and generates this fair feries of fpeculations; how can fhe difcern whether her productions are ftable and conftant, or things which the winds may diffipate, and phantoms rather than realities ? What ftandard can fhe apply as the meafure of their truth?. Or how, fince fhe is deftitute of their effence, can fhe generate fuch a variety of reafons ? For from fuch an hypothefis, we make their fubfiftence fortuitous, not tending to any fcientific bound. Mathe- matical fpecies are therefore the genuine offspring of the foul: nor does fhe derive from fenfible objeds the definitions fhe frames, but rather the firft are propagated from the fecond ; they are the energies of foul, which, as it were, pregnant with forms, delivers her imma-

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terial progeny into the dark and fluctuating regions of matter, as evidences of the permanent duration of her species.

Again, if we collect mathematical reasons from externals, why are not demonstrations compofed from fenfibles, better than the demonftrations of univerfal and fimple fpecies? For we fay, in order to the investigation of any thing fought, that the principles and propofitions, fhould be allied to the conclufions. If then, particulars are the caufes of univerfals, and fenfibles the fources of reafoning, why does the boundary of demonftration always refer to that which is more univerfal, and not to that which is partial and particular? And how can we prove that the effence of intelligibles is more allied to demonftration than the effence of fenfibles? For thus they fpeak *: his knowledge is not legitimate, who demonftrates that the ifofceles, the equilateral, or the fcalene triangle, have angles equal to two right; but he poffeffes fcience, properly fo called, who demonftrates this of every triangle fimply, or of triangle itfelf. And again, that univerfals, for the purpofe of demonftration, are fuperior to particulars; that demonftrations concern things more univerfal; but that the principles from which demonftrations are compofed, have a priority of exiftence, and a precedency in nature to fingulars, and are the caufes of the propofitions they prove. It is very remote, therefore, from the nature of Apodictical fciences, that from converfe with things of pofterior origin, and from the dark perceptions of fenfe, they fhould falfely collect their indubitable propofitions. I add farther, that they who affirm this, make the foul of a bafer nature than the material fpecies themfelves. For if matter derives from nature beings effential, and participating a high degree of entity and evidence; but the foul, by a pofterior energy, receives thefe from fenfible objects, and fafhions in herfelf refemblances and images of pofterior origin, contemplating vile effences, and abftracting from matter, the forms infeparable from its nature; do they not make the foul more obfcure and indigent

* Ariftotle, in his laft Analytics. The reader will pleafe to obferve, that the whole force of this nervous, accurate, and elegant reafoning, is directed againft Ariftotle; who feems unfortunately to have confidered, with the moderns, that mathematical fpecies fubfift in the foul, by an abftraction from fenfibles. See the preceding Differtation.

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than matter itfelf? For matter is the receptacle of forms materialized, as the foul is of fpecies immaterialized. But in this cafe, matter would be the place of primary beings, and the foul of fuch as are fecondary and fubordinate: matter and its forms obtaining the lead in being, and exifting as the fources of the fubfiftence of immaterial forms. Laftly, the material forms would have an effential exiftence, the others only an intentional denomination. How then can the foul, which is the firft participant of intellect, and an intellective effence, and which derives from thence confummate knowledge, and a plenitude of life, become the receptacle of the moft obfcure fpecies, the loweft in the order of things, and participating the moft imperfect exiftence. But this opinion, which has been fufficiently exploded by others, needs no farther confutation.

If then, mathematical fpecies do not fubfift by material abftraction, nor by a collection of thofe common properties inherent in individuals; nor are at all, in their origin, pofterior to fenfibles, nor derived in any manner from them: it is neceffary that the foul fhould either deduce them from herfelf, or from intellect; or laftly, from herfelf and intellect united. But if from herfelf alone, Whence do the images of intellectual fpecies arife; whence do they derive their middle nature, linking, as it were, the divifible and indivifible effence together, if they do not participate the fullnefs of entity from primary effences? Laftly, how, upon this hypothefis, are the firft exemplars, paradigms, or ideas, which fubfift in intellect, the principles of univerfals? But if they are derived from intellect alone into the foul, how can the foul remain felf-operative, and felf-motive, if her inherent reafons flow from an external fource, and are regulated by its operations? And in what refpect does the foul differ from matter, which is all things in mere dormant capacity, but generates nothing appertaining to material fpecies? It remains, therefore, that the foul deduces thefe fpecies from herfelf, and intellect; and that fhe is the abfolute confummation of the forms which originate from intellectual exemplars, but which are allotted from themfelves a tranfition to permanent being. The foul, therefore, is by no means to be compared to a fmooth tablet, void of all reafons; but fhe is an ever-written tablet, herfelf infcribing the characters in herfelf, of which fhe

derives an eternal plenitude from intellect. For foul is a certain fubordinate intellect, revolving round an intellect prior to herfelf, formed to its image, and participating its divine irradiations. If then, this fuperior intellect is all things intellectually, foul will all things animally; if the firft exifts as the exemplar, foul will be as its image; if as contracted and united in itfelf, foul as divifible and expanded. And this is what Plato underftood, when in his Timæus, he compofes the foul of the world from all things, dividing her according to harmonical reafons, and analogies; affigning to her the firft principles effective of figures, I mean the right and circular line, and giving an intellectual motion to her inherent circles. All mathematical fpecies, therefore, have a primary fubfiftence in the foul: fo that, before fenfible numbers, there are to be found in her inmoft receffes, felf-moving numbers; vital figures, prior to the apparent, ideal proportions of harmony previous to concordant founds; and invifible orbs, prior to the bodies which revolve in a circle. So that foul is the prolific abundance of all thefe, and is another ornament producing herfelf, and produced from a proper principle, filling herfelf with life, and at the fame time filled from the demiurgus of the univerfe, in an incorporeal and indiftant manner. When, therefore, fhe produces and unfolds her latent reafons, fhe then detects every fcience and virtue. The effence of foul then confifts in thefe fpecies, nor muft we fuppofe her inherent numbers to be a multitude of units, nor her archytipal ideas of divifible forms to be corporeal: but we muft conceive all thefe as fubfifting ever vitally, and intellectually, as the exemplars of apparent numbers, figures, reafons and motions. And here we muft follow the doctrine of Timæus, who derives the origin, and confummates the fabric of the foul, from mathematical forms, and repofes in her nature the caufes of every thing which exifts. For the feven bounding terms *, comprehending the principles of all numbers, lines, planes and folids, pre-exift in foul according to caufe. And again, the principles of figures are placed in her effence, according to a demiurgical power. And laftly, the firft of all motions, which embraces every other motion in its com-

* Viz. 1, 2, 4, 8, 3, 9, 27. Concerning which, fee lib. iii. of Proclus's excellent Commentary on the Timæus.

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prehenfive ambit, is co-exiftent with foul.   For the principle of every thing which is moved is a circle, and the circular motion.   The mathematical reafons, therefore, which fully confummate the foul, are effential, and felf-moving: and the foul, by her cogitative power, diffufing, propagating, and evolving thefe, from her profound receffes, conftitutes all the fair variety of mathematical fciences.   Nor will fhe ever ceafe to generate, and waken into energy, fucceeding fpecies, while fhe divefts her indivifible reafons of their intellectual fimplicity. For fhe previoufly received all things, after a primary manner; and according to her infinite power, from pre-exiftent principles, deduces a beautiful feries of various fpeculations.

C H A P.   VII.

What the Employments and Powers are of the Mathematical Science, and how far they extend themfelves in their Energies.

BUT, after contemplating the effence of mathematical forms, it is neceffary we fhould recur to that one mafter-fcience of thefe, which we have fhewn is prior to a multitude of others, and that we fhould contemplate what its employment is, what are its powers, and how far it advances in its energies.   The employment, therefore, of the whole mathematical fcience, poffeffing, as we have before faid, the power of cogitation, muft not be placed fo high as that of intelli- gence; which is firmly feated in its own ftable effence, is perfect, is contained by itfelf, and in itfelf continually verges.   Nor muft it be fituated fo low as that of opinion and fenfe, fince thefe cognitions dwell upon external concerns, energize upon them, and do not poffefs the caufes of the objects of their knowledge.   But the mathematical fcience, receives its commencement, indeed, extrinfically from * re- collection, but ends in the moft intimate reafons, refiding in the depths

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* Plato frequently, both in the Meno and elfewhere, fhews that fcience is Reminifcence; and I think not without the ftrongeft reafon.   For fince the foul is immaterial, as we have de- monftrated in the differtation to this work, fhe muft be truly immortal, i. e. both *a parte ante*, & *a parte poft*.   That fhe muft be eternal, indeed, with refpect to futurity, if immaterial, is ad- mitted by all; and we may prove, with Ariftotle, in his firft book de Cælo, that fhe is immor- tal, likewife *a parte ante*, as follows.   Every thing without generation, is incorruptible, and every thing incorruptible, is without generation: for that which is without generation, has a

of the foul; and is excited, indeed, from things pofterior, but arrives by gradual advances at the principal effence of forms. Nor is its energy immoveable, like that of intelligence, nor is it affected with local motion and alteration, like fenfe, but it revolves with a vital energy, and runs through the ornament of incorporeal reafons, fome-times advancing from principles to fuch things as are perfected by principles, but at other times yielding in a retrogade progreffion from conclufions to their forming principles: and fometimes proceeding from things previoufly known, to fuch as are the fubject of invefti-gation: but at other times, from things placed in the queftion, to fuch as precede in cognition. Befides, it does not excel all inquifition, as if it were perfect from itfelf, like intellect, nor is it perfected from

neceffity of exifting infinitely *a parte ante* (from the hypothefis); and therefore, if it poffeffes a capacity of being deftroyed, fince there is no greater reafon why it fhould be corrupted now, rather than in fome former period, it is endued with a capacity of being deftroyed and ceafing to be, in every inftant of infinite time, in which it neceffarily is. In like manner, that which is incorruptible, has a neceffity of exifting infinitely *a parte poft*; therefore, if it poffeffes a ca-pacity of being generated, fince there is no greater reafon why it fhould be generated now rather than afterwards, it poffeffes a capacity of being generated, in every inftant of time, in which it neceffarily is. If then the foul is effentially immortal, with refpect to the paft and future cir-culations of time; and if fhe is replete with forms or ideas of every kind, as we have proved in the differtation, fhe muft, from her circulating nature, have been for ever converfant in alter-nately poffeffing and lofing the knowledge of thefe. Now, the recovery of this knowledge by fcience, is called by Plato, reminifcence; and is nothing more than a renewed contemplation of thofe divine forms, fo familiar to the foul, before fhe became involved in the dark veftment of an earthly body. So that we may fay, with the elegant Maximus Tyrus, (Differ. 28.) "Re-mifcence is fimilar to that which happens to the corporeal eye, which, though always endued with a power of vifion, yet darknefs fometimes obftructs its paffage, and averts it from the per-ception of things. Art therefore, approaches, which though it does not give to the eye the power of vifion, yet removes its impediments, and affords a free egrefs to its rays. Conceive now, that our rational foul is fuch a power of perceiving, which fees and knows the nature of beings. To this the common calamity of bodies happens, that darknefs fpreading round it, hurries away its afpect, blunts its fharpnefs, and extinguifhes its proper light. Afterwards, the art of reafon approaches, which, like a phyfician, does not bring or afford it a new fcience, but roufes that which it poffeffes, though very flender, confufed, and unfteady." Hence, fince the foul, by her immerfion in body, is in a dormant ftate, until fhe is roufed by fcience to an exer-tion of her latent energies; and yet even previous to this awakening, fince fhe contains the vivid fparks, as it were, of all knowledge, which only require to be ventilated by the wings of learning, in order to rekindle the light of ideas, fhe may be faid in this cafe to know all things as in a dream, and to be ignorant of them with refpect to vigilant perceptions. Hence too, we may infer that time does not antecede our effential knowledge of forms, becaufe we poffefs it from eternity: but it precedes our knowledge with refpect to a production of thefe reafons into perfect energy. I only add, that I would recommend the liberal Englifh reader, to Mr. Syden-enham's excellent tranflation of Plato's Meno, where he will find a familiar and elegant demon-ftration of the doctrine of Reminifcence.

others,

others, like fenfe, but it proceeds by enquiry to invention, and afcends from the imperfe& to perfection. But it likewife poffeffes twofold powers, one kind of thefe deducing principles into multitude, and generating the different paths of contemplation : but the other endued with a power of colle&ing many tranfitions into proper fup-pofitions. For fince it propofes to itfelf as principles, as well unity, and multitude, as bound and infinite, and fuch things as are fubje& to its comprehenfion, are allotted a middle order, between forms indivi-fible and every way divifible; with great propriety (I think) the gnoftic powers of the whole fcience of thefe are effentially two-fold. One fpecies indeed, haftens to union, and contra&s the expanfion of multitude: but the other poffeffes a power of diftinguifhing things fimple into fuch as are various, more univerfals into more particulars, and reafons digefted in their principle, into things fecondary and multifarioufly multiplied from their principles. For rifing higher from its commencement it penetrates even to fuch things as are the perfe&ions of fenfible concerns, is joined with nature, and demon-ftrates many things together with natural fcience. Since afcending from inferiors, it accedes in a certain refpe& proximate to intelle&ual knowledge, and touches the contemplation of things primary and divine. And hence, in the limits which flow from its effence, it pro-duces the whole mechanic, optic, and catoptric fpeculation, together with many other fciences which are inwoven and entangled with fenfible concerns, and which operate through their affiftance. Befides, in its afcenfions from corporeal natures, it derives intelligences indi-vifible and deftitute of matter: and with thefe it perfe&s its divifible apprehenfions, thofe cognitions which fubfift in progreffions, and its own genera and forms: it likewife indicates the truth refpe&ing the gods themfelves, and in its peculiar treatifes exhibits a contemplation of the things which *are*. And thus much concerning the employ-ment and powers of the Mathematical Science.

CHAP.

C H A P. VIII.

Concerning the Utility of the Mathematical Science.

BUT let us now confider the utility of this Science, which extends itfelf from the moft principal to the laft cognitions. Timæus, therefore, calls the knowledge of the mathematical difciplines the path of erudition, becaufe, indeed, it has the fame proportion to univerfal fcience, and the firft philofophy, which learning has to virtue. For this laft frames our foul to a perfect life, by the poffeffion of worthy manners; but the former prepares our cogitation, and the divine eye of our foul to an elevation from the obfcurity of fenfible information. Hence, Socrates in the Republic, fays, " That the eye of the foul, which is darkened and buried by other ftudies, can by the mathematical difciplines alone be invigorated, and again excited to the contemplation of that which *is*, and transferred from refemblances to real beings, from an obfcure light to that light which has the power of intelligence, and from a cave, and thofe bonds which exift in it as the authors of generation, and from material impediments be able to rife to an incorporeal and indivifible effence. For the beauty and order of mathematical reafons, and the firmnefs and ftability of the contemplations they afford, conjoins us with intelligible objects, and perfectly determines us in their effences; which perpetually remain the fame, ever fhining with divine beauty, and preferving a mutual order without end. But Socrates, in the Phædrus, delivers to us three characters who are elevated from fenfe, becaufe they fill up and accomplifh the primary life of the foul, i. e. the philofopher, the lover, and the mufician. But the beginning and path of elevation to the lover, is a progreffion from apparent beauty, ufing as excitations the middle forms of beautiful objects. But to the mufician, who is allotted the third feat, the way confifts in a tranfition from fenfible to invifible harmonies, and to the reafons exifting in thefe. So that to the one, fight is the inftrument of reminifcence, and to the other, hearing. But to him who is by nature a philofopher, from whence and by what means is reminifcence the prelude of intellectual knowledge, and an excitation to that which truly is, and to truth itfelf? For this

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character alfo, on account of its imperfection, requires a proper principle: for it is allotted a natural virtue, an imperfect eye, and a degraded manner.   It muſt therefore be excited from itſelf; and he who is of ſuch a nature, rejoices in that which *is*.   But to the philoſopher, ſays Plotinus, the mathematical diſciplines muſt be exhibited, that they may accuſtom him to an incorporeal nature, and that afterwards uſing theſe as figures, he may be led to dialectic reaſons, and to the contemplation of all the things which *are*.   And thus it is manifeſt, from hence, that the mathematics are of the greateſt utility to philoſophy.   But it is requiſite that we ſhould be more explicit, and mention the ſeveral particulars to which they conduce, and evince that they prepare the intellectual apprehenſions of theology.   For whatever to imperfect natures appears difficult and arduous in obtaining the true knowledge of the gods, the mathematical reaſons render, by their images, credible, manifeſt, and certain.   Thus, in numbers, they indicate the ſignifications of ſuper-eſſential properties, but they evince the powers of intellectual figures, in thoſe figures which fall under cogitation.   Hence it is, that Plato, by mathematical forms teaches us many and admirable ſentences concerning the gods, and the philoſophy of the Pythagoreans, uſing theſe as veils, conceals from vulgar inſpection the diſcipline of divine ſentences.   For ſuch is the whole of the *Sacred and Divine Diſcourſe* *, that of Philolaus in his Bacchics, and the univerſal method of the Pythagoric narration concerning the Gods.   But it eſpecially refers to the contemplation of nature, ſince it diſcloſes the order of thoſe reaſons by which the univerſe is fabricated, and that proportion which binds, as Timæus ſays,

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* Concerning this valuable work, entitled ΙΕΡΟΣ ΛΟΓΟΣ, ſee the Bibliotheca Græca of Fabricius, vol. i. p. 118 and 462, and in the commentary of Syrianus on Ariſtotle's metaphyſics, p. 7, 71, 83, and 108, the reader will find ſome curious extracts from this celebrated diſcourſe; particularly in p. 83. Syrianus informs us, " that he who conſults this work will find all the orders both of Monads and Numbers, without neglecting one, fully celebrated (ὑμνημέναις.)" There is no doubt, but that Pythagoras and his diſciples concealed the ſublimeſt truths, under the ſymbols of numbers; of which he who reads and underſtands the writings of the Platoniſts will be fully convinced. Hence Proclus, in the third book of his excellent commentary on the Timæus, obſerves, " that Plato employed mathematical terms for the ſake of myſtery and concealment, as certain veils, by which the penetralia of truth might be ſecluded from vulgar inſpection, juſt as the theologiſts made fables, but the Pythagoreans ſymbols, ſubſervient to the ſame purpoſe: for in images we may ſpeculate their exemplars, and the former afford us the means of acceſs to the latter."

whatever

whatever the world contains, in union and confent; befides, it conciliates in amity things mutually oppofing each other, and gives convenience and confent to things mutually difagreeing, and exhibits to our view fimple and primary elements, from which the univerfe is compofed, on every fide comprehended by commenfurability and equality, becaufe it receives convenient figures in its proportions, and numbers proper to every production, and finds out their revolutions and renovations, by which we are enabled to reafon concerning the beft origin, and the contrary diffolution of particulars. In confequence of this, as it appears to me, Timæus difclofes the contemplation concerning the nature of the univerfe, by mathematical names, adorns the origin of the elements with numbers and figures, referring to thefe their powers, paffions, and energies; and efteeming as well the acutenefs as the obtrufenefs of angles, the levity of fides, or contrary powers, and their multitude and paucity to be the caufe of the all-various mutation of the elements. But why may we not fay, that it profits much, and in an admirable manner, to that philofophy which is called Politic, as well by meafuring the times of actions as affording the various revolutions of the univerfe, and numbers convenient to things rifing into being; I mean the affimilating, and authors of diffimilitude, the prolific too and the perfect, and the contraries to thefe; together with orderly and elegant minifters of life, and inelegance; and finally, fuch numbers as procure fertility and fterility. Which, indeed, the fpeech of the Mufes in the Republic * evinces, placing the univerfal Geometric Number as the author of better and more debafed generations, and as the caufe of the indiffoluble perfeverance of good manners, and of the mutation of the beft Republics into fuch as are remote from reafon, and are given to affections. For it is fufficiently evident, that it belongs to the whole mathematical difcipline to deliver the fcience of this number which is called geometrical, and not to one particular fcience, fuch as arithmetic, or geometry: fince the reafons or proportions of abundance and fterility, permeate through all the mathematical difciplines. Again, it is the means of our inftitution in moral philofophy which it brings to its ultimate

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* Concerning this Geometric Number, in the 8th book of Plato's Republic, than which Cicero affirms there is nothing more obfcure, fee the notes of Bullialdus to Theo. p. 292.

perfection,

perfection, and gives order and an elegant life to our manners. Befides this, it delivers to us figures, and modulations and motions convenient to virtue, by which the Athenian gueft wifhes thofe to be inftituted and perfected, who are deftined to purfue moral virtue from their early youth. Add too, that it places before our view the reafons of virtues, in one manner, indeed, in numbers, in another in figures, but differently in mufical fymphonies; and laftly, it indicates the excefs and defect of vices, by which we are enabled to moderate and adorn our manners. Hence it is, that Socrates, in the Gorgias, accufing Calicles of an inordinate and intemperate life, fays to him, " You neglect geometry and geometric equality :" but, in the Republic, he finds out the proportion of tyrannic pleafure to a royal interval, according to a plane and folid generation. But we fhall learn what great utility is derived to other fciences and arts from the mathematical fcience, when we confider that it adds order and perfection to contemplative arts; I mean rhetoric, and all fuch as confift in difcourfe. But it propofes to the poetic arts, the reafons of poems in the place of an example, becaufe it prefides over the meafures exifting in thefe. But to the active arts it determines action and motion, by its own abiding and immoveable forms. For all arts, as Socrates fays, in the Philebus, require arithmetic, menfuration, and ftatics, either in all, or in fome of their operations. But all thefe are contained in the difcourfes of the mathematical fcience, and are terminated according to their diverfity. For from this fcience the divifions of numbers, and the variety of dimenfions, and the difference of weights are known. The utility, therefore, of the whole mathematical fcience to philofophy itfelf, and to other fciences and arts, may be from hence known to intelligent hearers.

C H A P. IX.

A Solution of an Objection raised by some against the Utility of the Mathematical Sciences.

BUT some, who are prone to contradiction through those who wish to subvert geometry, endeavour to destroy the dignity of this science. One part, indeed, depriving it of ornament and good, because it does not discourse on these. But another part * affirming that sensible experiments are more useful than the universal objects of its speculation; I mean, that Geodesia (for instance,) or the mensuration of the earth, is preferable to geometry, and vulgar arithmetic to that arithmetic which is conversant with theorems alone : and that nautical astrology is more useful than that which teaches universally, abstracted from any application to sensible concerns. For we are not, say they, made rich by our knowledge of riches, but by using them; nor are we happy by the merely understanding felicity, but by living happily. Hence we must confess that those mathematical sciences, which are conversant with cognition, do not profit human life, and confer to action, but those only which are engaged in exercise. For those who are ignorant of the reasons of things, but are exercised in particular and sensible experiments, are in every respect more excellent, for the purposes of human life, than those who are employed in contemplation alone. Against objections then, of this kind, we shall reply, by shewing the beauty of the mathematical disciplines from those arguments by which Aristotle endeavours to persuade us. We must therefore confess that there are three things which especially cause beauty, both in bodies and souls; I mean, order, convenience, and determination. Since corporeal basenefs, indeed, arises from material inordination, deformity, and inconvenience, and from the do-

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* I am sorry to say, that this part of the enemies to pure geometry and arithmetic, are at the present time very numerous; conceptions of utility in these sciences, extending no farther than the sordid purposes of a mere animal life. But surely, if intellect is a part of our composition, and the noblest part too, there must be an object of its contemplation; and this, which is no other than truth in the most exalted sense, must be the most noble and useful subject of speculation to every rational being.

minion of the indefinite in the compofite body. But the bafenefs of the foul originates from its irrationality, and inordinate motion, and from its being in a ftate of difcord with reafon, and not receiving from thence its proper limitation. Hence, beauty exifts even in contraries, by means of order, convenience and determination. But we may behold thefe in a more eminent degree in the mathematical fcience; order, indeed, in the perpetual exhibition of things pofterior and more various, from fuch as are primary and more fimple; for things fubfequent are always annexed to their precedents, the latter ranking as principles, and the former as the firft fuppofitions of things confequent: but convenience is evinced in the mutual confonance of things demonftrated, and in the relation of all of them to intellect, fince intellect is the common meafure of all fcience, from which it receives its principles, and to which it converts the learner: but determination is perceived in its perpetually abiding and immoveable reafons, for the objects of its knowledge are not, at times, fubject to variation, like thofe of opinion and fenfe, but prefent themfelves for ever the fame, and are bounded by intellectual forms. If fuch then, are the principal requifites of beauty, it is evident, that in thefe fciences that illuftrious ornament and gracefulnefs is found. For how is it poffible this fhould not be the cafe with a fcience receiving a fupernal illumination from intellect, to which it continually advances, haftening to transfer us from the obfcure light of fenfible information? With refpect to the fecond objection, we think it proper to judge of its utility, without regarding the conveniencies and neceffities of human life. For otherwife, we muft confefs that contemplative virtue is alfo ufelefs, which feparates itfelf from human concerns, which it is very little defirous to look down upon and underftand. Indeed Socrates, in the Theætetus, affirming this concerning noblemen endued with the prophetic power, fays, " that it withdraws them from all regard to human life, and raifes their thoughts, properly liberated, from all neceffity and ufe, to the very fummit of all true being. The mathematical fcience, therefore, muft be confidered as defirable for its own fake, and for the contemplation it affords, and not on account of the utility it adminifters to human concerns. But if it is neceffary to refer the utility it produces to fomething different

from itfelf, it muft be referred to intellectual knowledge. For it leads us to this, and prepares the eye of the foul for the knowledge of univerfals, removing and obliterating the impediments arifing from the fenfes, and from corporeal involution. As therefore we call the whole of purgative virtue ufeful, or the contrary, not regarding the ufe of the fenfible life, but of that which is contemplative, fo indeed it is requifite to refer the end of mathematics to intellect, and univerfal wifdom. Hence its energy is worthy our ftudy, both on its own account, and on account of an intellectual life. But it appears, as Ariftotle * fays, that this fcience is defirable of itfelf to its votaries, becaufe though no reward is propofed to its enquirers, yet the mathematical contemplation receives, in a fmall time, an abundant increafe. Befides, this is farther evident from hence, that all men are willingly employed in its purfuit, and wifh to dwell on its fpeculations, omitting every other concern ; even thofe who have, with their lips, as it were, but juft touched its utility. And hence it follows, that they who defpife the knowledge of the mathematical difciplines, have very little tafted of the pleafures they contain. The mathematics, therefore, are not to be defpifed becaufe their fpeculative parts do not immediately confer to human utility, (for the ultimate limits of its progreffions, and whatever operates with matter, confider a ufe of this kind ;) but on the contrary we fhould admire its immateriality, and the good which it contains, confidered by itfelf alone. For when mankind were entirely difengaged from the care of neceffary concerns, they converted themfelves to the inveftigation of the mathematical difciplines ; and this, indeed, with the greateft propriety. Since affairs familiar to human life in its moft imperfect ftate, and which are immediately connected with its origin, firft of all employed the ftudies of mankind : but, in the fecond place, thofe concerns fucceeded which feparate the foul from generation, and reftore its memory of that which *IS*. After this manner, then, we are engaged in neceffaries, before things honourable for their own fakes, on account of their intrinfic dignity and worth; and in things related to fenfe before fuch as are apprehended by the nobler energies of mind.

* In the 13th book of his Metaphyfics, cap. iii.

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For every origin and life of the foul which is converted into herfelf,. is naturally adapted to proceed from the imperfect to the perfect. And thus much againft thofe who defpife the mathematical fcience.

CHAP. X.

A Solution of another Objection of certain Platonifts, againft the Utility of the Mathematical Sciences.

BUT, perhaps, fome of our own family will here rife up againft us, and, propofing Plato as a witnefs, will endeavour to provoke ruder underftandings into a contemptuous difregard of the mathematical difciplines. For they will fay, that this philofopher entirely excludes (in his Republic) the mathematical knowledge from the choir of the fciences, and that he accufes it as being ignorant of its own principles, that its very principle is to itfelf unknown, and its ends and mediums compofed from things of which it is ignorant. To thefe objections they may likewife add whatever other reproaches are there urged by Socrates againft this contemplation. In anfwer then, to the objections of our friends, we fhall recall into their memory, that Plato himfelf perfpicuoufly afferts the mathematical fcience to be the purgation of the foul, and that it is endued with a power of leading it on high; becaufe, like the Homeric Minerva, it removes the darknefs of a fenfible nature from the intellectual light of thought, which is better worth faving than ten thoufand corporeal eyes, and which not only participates of a mercurial gift, (preferving us from the incantations and delufions of this material abode, which is fimilar to the facinating realms of Circe,) but alfo of the more divine arts of Minerva. He likewife every where calls it by the name of fcience, and afferts that it is the caufe of the greateft felicity to thofe who are exercifed in its contemplation. But I will briefly explain why, in the Republic he takes from it the furname of fcience: for my prefent difcourfe is addreffed to the learned. Plato, indeed, in moft places, calls all the knowledge (as I may fay) of univerfals by the name of fcience, oppofing it in a divifion to fenfe which apprehends only particulars, whether fuch a mode of cognition is accomplished by art

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or experience. And in this fenfe, as it appears to me in the *Civil*
Dialogue, and in the Sophifta, he feems to ufe the name of fcience;
placing likewife the illuftrious Sophiftic fcience, which Socrates in
the Gorgias, fays, is a certain experience: alfo, the adulatory, and
many others, which are experiences, but not true fciences. But
again, dividing this knowledge of univerfals into that which knows
caufes, and into that which underftands without a caufe, he thinks
that the one fhould be called fcience, but the other experience. And
hence, to arts he fometimes attributes the name of fcience, but to ex-
perience never. For how (fays he in the Banquet) can a thing which
poffeffes no reafon be fcience? All knowledge, therefore, which con-
tains the reafon and caufe of the things known, is a certain fcience.
Again, therefore, he divides this fcience which is endued with a
power from the caufe of knowing, by the peculiarity of its fubjects,
and he places one, conjectural of things divifible; but the other of
fuch as fubfift by themfelves, and are ever knowable after the fame
manner. And according to this divifion he feparates from fcience,
medicine, and every faculty which is converfant with material con-
cerns. But he calls mathematical knowledge, and whatever poffeffes
a power of contemplating eternal objects, by the name of fcience.
Laftly, dividing this fcience, which we diftinguifhed from arts, he
confiders one part as void of fuppofition; but the other as flowing
from fuppofition. And that the one which is void of fuppofition,
has a power of knowing univerfals: that it rifes to good, and the
fupreme caufe of all; and that it confiders good as the end of its
elevation: but that the other, which previoufly fabricates for itfelf
definite and determinate principles, from which it evinces things con-
fequent to fuch principles, does not tend to the principle, but to the
conclufion. And hence he afferts, that mathematical knowledge,
becaufe it makes ufe of fuppofition, falls fhort of that fcience which
is without fuppofition, and is perfect. For there is one true fcience,
by means of which we are difpofed to know all the things *which are*,
and from which alfo principles emerge to all fciences; to fome, indeed,
conftituted more proximately, but to others more remotely. We
muft not fay, therefore, that Plato expels mathematical knowledge
from the number of the fciences, but that he afferts it to be the fe-
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cond from that one fcience, which poffeffes the fupreme feat of all: nor muft we affirm, that he accufes it as ignorant of its own principles, but that receiving thefe from the mafter fcience dialectic, and poffeffing them without any demonftration, it demonftrates from thefe its confequent propofitions. For, indeed, he fometimes allows the foul, which is conftituted from mathematical reafons, to be the principle of motion: and fometimes he affirms, that it receives its motion from genera which are fubject to intelligence. And thefe variations accord among themfelves. For to fuch things as are moved by another, the foul is a certain caufe of motion, but it is not the caufe of every motion. After the fame manner, the mathematical fcience is indeed the fecond from the firft of all fciences, and, with reference to it, imperfect: but it is, neverthelefs, a fcience, not as being free from fuppofition, but as knowing the peculiar reafons refident in the foul, and as bringing the caufes of conclufions, and containing the reafon of fuch things as are fubject to its knowledge. And thus much for the opinion of Plato refpecting mathematics.

C H A P. XI.

BUT let us now confider what are the things which may be required of a mathematician, and how any one may rightly judge concerning his diftinguifhing peculiarities. For * Ariftotle indeed, fays, that he who is fimply learned in all difciplines, is adapted to judge of all: but that he who is alone fkilled in the mathematical fciences, can alone determine concerning the magnitude of reafons inherent in thefe. It is requifite, therefore, that we fhould previoufly affume the terms of judging, and that we fhould know, in the firft place, in what things it is proper to demonftrate generally, and in what to regard the peculiarities of fingulars. For many of the fame properties refide in things differing in fpecies, as two right angles in all triangles: but many have indeed the fame predicament, yet differ in their individuals in a common fpecies, as fimilitude in figures and numbers. But one demonftration is not to be fought for by the

* In. I. De Partib. Animalium, et in primo Ethic. cap. ii.

mathe-

mathematician in thefe, for the principles of figures and numbers are not the fame, but differ in their fubject genus. And if the effential accident is óne, the demonftration will alfo be one *: for the pof-feffion of two right angles is the fame in all triangles, and that general fomething to which this pertains is the fame in all, I mean triangle, and a triangular reafon. In the fame manner, likewife, the poffeffion of external angles to four right ones, not only pertains to triangles, but alfo to all right-lined figures; and the demonftration, fo far as they are right-lined, agrees in all. For every reafon brings with it, at the fame time, a certain property and paffion, of which all participate through that reafon, whether triangular, or rectilinear, or univerfally figure. But the fecond limit by which a mathematician is to be judged, is, if he demonftrates according to his fubject-matter, and renders neceffary reafons, and fuch as cannot be confuted, but are at the fame time neither probable, nor replenifhed with a fimilitude of truth. For,, fays Ariftotle, it is juft the fame to require demonftrations from a rhetorician, and to affent to a mathematician difputing probably; fince every one, endued with fcience and art, ought to render reafons adapted to the fubjects of his inveftigation. In like manner alfo, Plato in the Timæus, requires credible reafons of the natural philofopher, as one who is employed in the refem-blances of truth: but of him who difcourfes concerning intelligibles, and a ftable effence, he demands reafons which can neither be con-futed nor moved. For fubjects every where caufe a difference in fciences and arts, fince, if fome of them are immoveable, others are converfant with motion; and fome are more fimple, but others more compofite; and fome are intelligibles, but others fenfibles. Hence we muft not require the fame certainty from every part of the mathematical fcience. For if one part, after a manner, borders upon fenfibles, but another part is the knowledge of intelligible fubjects, they cannot both be equally certain, but one muft inherit a higher degree of evidence than the other. And hence it is, that we call arithmetic more certain than the fcience of harmony. Nor muft we think it juft that mathematics and other fciences fhould ufe the fame demonftrations; for their fubjects afford them no fmall variety. In

* See more concerning this in the Differtation.

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the third place, we muſt affirm, that he who rightly judges mathe-
matical reaſons, muſt conſider ſameneſs and difference, what ſubſiſts
by itſelf, and what is accidental, what proportion is, and every con-
ſideration of a ſimilar kind.   For almoſt all errors of this ſort happen
to thoſe who think they demonſtrate mathematically, when at the
ſame time they by no means demonſtrate, ſince they either demon-
ſtrate the ſame thing as if different in each ſpecies, or that which is
different as if it were the ſame : or when they regard that which is
accidental, as if it were an eſſential property ; or that which ſubſiſts
by itſelf, as if it were accidental.   For inſtance, when they endeavour
to demonſtrate that the circumference of a circle is more beautiful than
a right line, or an equilateral than an iſoſceles triangle.   For the de-
termination of theſe does not belong to the mathematician, but to the
firſt philoſopher alone.   Laſtly, in the fourth place, we muſt affirm,
that ſince the mathematical ſcience obtains a middle ſituation between
intelligibles and ſenſibles, and exhibits in itſelf many images of
divine concerns, and many exemplars of natural reaſons, we may
behold in it three kinds of demonſtration *, one approaching nearer
to intellect, the ſecond more accommodated to cogitation, and the
third bordering on opinion.   For it is requiſite that demonſtrations
ſhould differ according to the varieties of problems, and receive a
diviſion correſpondent to the genera of beings, ſince the mathematical
ſcience is connected with all theſe, and adapts its reaſons to the
univerſality of things.   And thus much for a diſcuſſion of the ſubject
propoſed.

* Since number is prior to magnitude, the demonſtrations of arithmetic muſt be more intel-
lectual, but thoſe of geometry more accommodated to the rational power.   And when either
arithmetic or geometry is applied to ſenſible concerns, the demonſtrations, from the nature of
the ſubjects, muſt participate of the obſcurity of opinion.   If this is the caſe, a true mathe-
matician will value thoſe parts of his ſcience moſt, which participate moſt of evidence; and
will conſider them as degraded, when applied to the common purpoſes of life.

C H A P. XII.

What and how many the Species of the whole Mathematical Science are,
according to the Opinion of the Pythagoreans.

BUT after thefe confiderations, it is requifite to determine con-
cerning the parts of the mathematical fcience, what, and how
many they are. For it is juft, after fpeculating its whole and entire
genus, to confider the differences of its more particular fciences,.
according to their fpecies. The Pythagoreans *, therefore, thought
that the whole mathematical fcience fhould receive a fourfold diftri-
bution, attributing one of its parts to the how-many, but the other
to the how-much; and they affigned to each of thefe parts a twofold
divifion. For they faid, that difcrete quantity, or the how-many,
either fubfifts by itfelf, or muft be confidered with relation to fome
other; but that continued quantity, or the how-much, is either
ftable or in motion. Hence they affirmed, that arithmetic contem-
plates that difcrete quantity which fubfifts by itfelf, but mufic that
which is related to another; and that geometry confiders continued
quantity fo far as it is immoveable; but fpherics contemplates conti-
nued quantity as moving from itfelf, in confequence of its union
with a felf-motive nature. They affirmed befides, that thefe two
fciences, difcrete and continued quantity, did not confider either
magnitude or multitude abfolutely, but that alone which in each of
thefe is definite from the participation of bound. For fciences alone
fpeculate the definite, rejecting as vain the comprehenfion of infinite
quantity. But when thefe wife men affigned this diftribution, we muft

* This divifion of the mathematical fcience, according to the Pythagoreans, which is nearly
coincident with that of Plato, is blamed by Dr. Barrow in his Mathematical Lectures, p. 15.
as being confined within too narrow limits: and the reafon he affigns for fo partial a divifion,
is, " becaufe, in Plato's time, others were either not yet invented, or not fufficiently cultivated,
or at leaft were not yet received into the number of the mathematical fciences." But I muft
beg leave to differ from this moft illuftrious mathematician in this affair; and to affert that the
reafon of fo confined a diftribution (as it is conceived by the moderns) arofe from the exalted
conceptions thefe wife men entertained of the mathematical fciences, which they confidered as
fo many preludes to the knowledge of divinity, when properly purfued; but they reckoned
them degraded and perverted, when they became mixed with fenfible objects, and were applied
to the common purpofes of life.

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not fuppofe they underftood that difcrete quantity which is found in fenfible natures, nor that continued quantity which fubfifts about the fluctuating order of bodies. For, I think, the contemplation of thefe pertains to the natural and not to the mathematical fcience. But becaufe the demiurgus of the univerfe, employed the union, divifion, and identity of general natures, together with difference, ftation, and motion, for the purpofe of completing the effence of the foul, and compofed it from thefe genera, as Timæus informs us, we muft affirm, that cogitation, abiding according to its diverfity, its divifion of reafons, and its multitude, and underftanding itfelf to be both one and many, propófes indeed to itfelf, and produces numbers, together with an arithmetical knowledge of thefe: but it provides for itfelf mufic according to an union of its multitude, and a communication and junction with itfelf; and hence it is that arithmetic excels mufic in antiquity; fince, according to the narration of Plato, the demiurgus firft divided the foul, and afterwards collected it in harmonical proportions. Again, thought eftablifhing its energy according to the ftability which it contains, draws from its inmoft retreats geometry, together with one effential figure, and the demiurgical principles of all figures *: but, according to its inherent motion, it produces the fpherical fcience. For it is moved alfo by circles, but abides perpetually the fame from the caufes of circles. Hence, likewife, geometry precedes fpherics, in the fame manner as ftation is prior to motion. But becaufe cogitation itfelf produces thefe fciences, not by looking back upon its convolution of forms, endued with an infinite power, but upon the inclofure of *bound* according to its definite genera; hence they fay, that the mathematical fciences take away infinite from multitude and magnitude, and are only converfant about finite quantity. Indeed, intellect has placed in cogitation all the principles both of multitude and magnitude. For fince it wholly confifts, with reference to itfelf, of fimilar parts, and is one and indivifible, and again divifible, educing the ornament of forms, it participates of *bound* and *infinite*, from intelligible effences themfelves. But it underftands, indeed, from its participation of bound, and gene-

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* That is, a right and circular line.

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rates vital energies, and various reasons from the nature of infinite. The intellections, therefore, of thought, constitute these sciences according to the *bound* which they contain, and not according to an infinity of life; since they bring with them an image of intellect, but not of life. Such then is the opinion of the Pythagoreans, and the division of the four mathematical sciences.

C H A P. XIII.

Another Division of the Mathematical Science, according to Geminus.

AGAIN, some think (among whom is Geminus) that the mathematical science is to be divided in a different manner from the preceding. And they consider that one of its parts is conversant with intelligibles only, but the other with sensibles, upon which it borders; denominating as intelligibles whatever inspections the soul rouses into energy by herself, when separating herself from material forms. And of that which is conversant with intelligibles they establish two, by far the first and most principal parts, arithmetic and geometry: but of that which unfolds its office and employment in sensibles, they appoint six parts, mechanics, astrology, optics, geodæsia, canonics, and logistics, or the art of reckoning. But they do not think that the military art, or tactics, should be called any one part of mathematics, according to the opinion of some *; but they consider it as using at one time the art of reckoning, as in the numbering of legions; but at another time geodæsia, as in dividing and measuring the spaces filled by a field of camps. As, say they, neither the art of writing, nor the art of healing, are any part of mathematics, though frequently both the historian and physician use mathematical theorems. This is the case with historians indeed, when relating the situation of climates, or collecting the magnitudes and dimensions of

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* I am afraid there are few in the present day, who do not consider tactics as one of the most principal parts of mathematics; and who would not fail to cite, in defence of their opinion, that great reformer of philosophy, *as he is called,* Lord Bacon, commending pursuits which come home to men's businesses and bosoms. Indeed, if what is lowest in the true order of things, and best administers to the vilest part of human nature, is to have the preference, their opinion is right, and Lord Bacon is a *philosopher !*

cities,

cities, or their compass and circuit: but with phyficians, when elucidating by ways of this kind, many things in their art. For Hippocrates himfelf fhews the utility derived to medicine from aftrology, and almoft all who fpeak of opportune times and places. By the fame reafon he alfo, who accommodates his work to tactics, ufes indeed mathematical theorems, yet is not on this account a mathematician, although he is fometimes willing that a numerous camp fhould exhibit a very fmall multitude, and forms his army according to a circular figure; but fometimes in a quadrangular, quinquangular, or fome other multangular figure, when he defires it to appear numerous. But fince thefe are the fpecies of the whole mathematical fcience, geometry is again divided into the contemplation of planes, and the dimenfion of folids, which is called ftereometry. For there is not any peculiar treatife about points and lines, becaufe no figure can be produced from thefe without planes or folids. For geometry treats of nothing elfe in every one of its parts, than that it may conftitute either planes or folids: or that when conftituted, it may compare and divide them among themfelves. In like manner, arithmetic is diftributed into the contemplation of linear, plane, and folid numbers. For it confiders the fpecies of numbers feparate from fenfible connections, proceeding from unity, and the origin of plane numbers; I mean of the fimilar, diffimilar, and folid, even to the third increafe. But geodæfia, and the art of reckoning, are divided fimilarly to arithmetic and geometry, as they do not difcourfe concerning intelligible numbers or figures, but of fuch as are fenfible alone. For neither is it the office of geodefia to meafure the cylinder or the cone, but material maffes as if they were cones, and wells as if they were cylinders. Neither does it accomplifh this purpofe by intelligible right lines, but by fuch as are fenfible, fometimes indeed by a more certain means, as by the folar rays; but at other times by groffer ones, as by a line and perpendicular. In like manner, the reckoner does not furvey the paffions of numbers by themfelves, but as they are refident in fenfible objects. From whence he alfo impofes a name upon thefe derived from the things which he reckons, calling them μηλίαι, & φιαλίται. Befides this, he does not admit of any leaft, like the arithmetician, who receives that minimum, as a genus of
relation.

·relation. For fome one man is considered by him as the meafure of the whole multitude of men, as unity alfo is the common meafure of all numbers. Again, optics and canonics are produced from geome-try and arithmetic. And optics ufes the vifual rays which are conflituted by the rays of the eyes, as lines and angles. But it is divided into that which is properly called optics (becaufe it renders the caufe of thofe appearances, which are accuftomed to prefent them-felves to us different from their reality, on account of the different fituations and diftances of vifible objects, as the coincidence of parallel lines, or the appearance of quadrangles as if they were circles); and into univerfal catoptrics, which is converfant about various and ma-nifold refractions, and is connected with imaginative or conjectural knowledge: as alfo into that which is called fciography *, or the delineation of fhadows, which fhews how appearances in images may feem neither inelegant nor deformed, on account of the diftances and altitudes of the things defigned. But canonics (mufic) or the regular art, confiders the apparent reafons of harmonies, finding out the fections of rules, every where ufing the affiftance of fenfe, and, as Plato fays, feeming to prefer the teftimony of the ears to intellect itfelf. But to the parts we have hitherto enumerated, mechanics muft be added, as it is a certain part of the whole fcience, and of the knowledge of fenfible objects, and of things united with matter. But under this exifts the art effective of inftruments, which is called (ὀργανοποιητικη) I mean of thofe inftruments proper for the purpofes of war: fuch, indeed, as Archimedes is reported to have conftructed, refifting the befiegers of fea and land; and that which is effective of miracles, and which is called (θαυματοποιητικη.) One part of this con-ftructs with the greateft artifice pneumatic engines, fuch as Ctefibius and Heron fabricated: but another operates with weights, the motion of which is reckoned to be the caufe of inequilibrity; but their ftation of equilibrity, as Timæus alfo has determined: and again, another part imi-tates animate foldings and motions by ftrings and ropes. Again, under mechanics is placed the knowledge of equilibriums, and of fuch inftruments as are called centroponderants: alfo (σφαιροποιια) or the

* By this is to be underftood the art new called Perfpective: from whence it is evident that this art was not unknown to the ancients, though it is queftioned by the moderns.

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art effective of fpheres, imitating the celeftial revolutions, fuch as
Archimedes fabricated ; and laftly, every thing endued with a power
of moving matter.   But the laft of all is aftrology, which treats of
the mundane motions, of the magnitudes of the celeftial bodies, their
figures and illuminations, their diftances from the earth, and every
thing of this kind ; affuming many things indeed to itfelf from fenfe,
but communicating much with the natural fpeculation.   One part of
this is gnomonics, which is exercifed in fettling the dimenfion of
hororary gnomons: but the other is metheorofcopics, which finds
out the differences of elevations, and the diftances of the ftars, and alfo
teaches many other and various aftrological theorems.   The third part is
dioptrics, which afcertains by dioptric inftruments of this kind the di-
ftances of the fun and moon, and of the five other ftars.   And fuch
is the account of the parts of the mathematical fcience, delivered by
the ancients, and tranfmitted to our memory by the informing hand
of time.

C H A P.   XIV.

*How Dialectic is the Top of the Mathematical Sciences, and what
their Conjunction is, according to Plato.*

LET us again confider after what manner Plato, in his Republic,
calls dialectic the top of the mathematical difciplines; and what
their conjunction is, according to the tradition of the author of the
Epinomis *.   And in order to this we muft affert, that as intellect
is fuperior to cogitation, fupplying it with fupernal principles,
and from itfelf giving perfection to cogitation; in the fame manner
dialectic alfo, being the pureft part of philofophy, excels in fimplicity
the mathematical difciplines, to which it is proximate, and with
which it is conjoined.   Indeed it embraces the complete circle of

* From hence it appears, that it is doubtful whether Plato is the author of the dialogue
called Epinomis; and I think it may with great propriety be queftioned.   For though it bears
evident marks of high antiquity, and is replete with genuine wifdom, it does not feem to be
perfectly after Plato's manner ; nor to contain that great depth of thought with which the
writings of this philofopher abound.   Fabricius (in his Bibliotheca Græca, lib. i. p. 27.)
wonders that Suidas fhould afcribe this work to a philofopher who diftributed Plato's laws into
twelve books, becaufe it was an ufual opinion; from whence it feems, that accurate critic had
not attended to the prefent paffage.

thefe

these sciences, to which it elevates from itself various energies, endued with a power of causing perfection, judgment, and intelligence. And these energies consist in resolving, dividing, defining, and demonstrating; by which mathematics itself, receiving assistance and perfection, invents some things by resolution, but others by composition: and some things it explains by division, others by definition: but collects other subjects of its investigation by demonstration; accommodating, indeed, these ways to its subjects, but using each of them for the purpose of beholding its middle enquiries. From whence indeed, both the resolutions, definitions, divisions, and demonstrations which it contains, are peculiar, and adapted to its nature, and revolve according to the mode of mathematical cognition. Not undeservedly, therefore, is dialectic the vertex as it were, and summit of mathematics. Since it perfects all which mathematics contains of intelligence; renders its certainty free from reprehension, preserves the stability of its immoveable essence, and refers what it contains destitute of matter and pure to the simplicity of intellect, and a nature separated from material connections. Besides, it distinguishes the first principles of these sciences, by definitions: exhibits the separations of genera and forms contained under the genera themselves: and besides this, teaches the compositions, which, from principles, produce things consequent to principles: and the resolutions which rise and mount up to things first, and to principles themselves. But with respect to what remains, proportion itself is not to be considered (as Eratosthenes thought it was) as the conjunction of the mathematical disciplines. Since proportion is said to be, and indeed is one of those things common to the mathematics. But in short, many other things besides proportion regard all the mathematical disciplines, which are essentially inherent in the common nature of the mathematics. But as it appears to me, we should say, that there is one proximate conjunction of these, and of the whole mathematical science, which especially embraces in itself, in a more simple manner, the principles of all sciences; which considers their community and difference; teaches whatever is found in these the same; together with what things are inherent in a many, and what in a few. So

that

that to thofe who aptly learn there is a reverfion from many other fciences to this alone.*. But dialectic is a conjunction of the mathematical difeiplines fuperior to the preceding; which Plato, as I have already obferved, calls in his Republic their vertex: for, indeed, it perfects the whole of mathematics, brings it back to intellect by its powers, fhews it to be a true fcience, and caufes it to be certain and obnoxious to no reproof. But, intellect obtains the third order between thefe conjunctions, which comprehends in itfelf uniformly all the dialectic powers, contracts their variety by its fimplicity, their partition by its indivifible knowledge, and their multitude by its occult union. Hence, intellect itfelf congregates indeed the involutions and deviations of the dialectic paths, into an intelligible effence, but it collects fupernally all the progreffion of mathematical difcourfes: and it is the beft end both of the elevating power of the foul, and of the energy confifting in cognition. And fuch are the fentiments declared by me on the prefent enquiry.

C H A P. XV.

From whence the Name Mathematics originated.

AGAIN, from whence fhall we fay this name of mathematics, and mathematical difciplines, was affigned by the ancients, and what apt reafon can we render of its pofition? Indeed, it appears to me, that fuch an appellation of a fcience which refpects cogitative reafons, was not, like moft names, invented by indifferent perfons, but (as the truth of the cafe is, and according to report) by the Pythagoreans alone. And this, when they perceived, that whatever is called mathefis or difcipline, is nothing more than reminifcence; which does not approach the foul extrinfically, like the images which rifing from fenfible objects are formed in the phantafy: nor is it adventitious and foreign, like the knowledge confifting in opinion, but it

* This proximate conjunction of the mathematical fciences, which Proclus confiders as fubordinate to dialectic, feems to differ from that vertex of fcience in this, that the former merely embraces the principles of all fcience, but the latter comprehends the univerfal genera of being, and fpeculates the principle of all.

is excited, indeed, from apparent objects, and is perfected within, by thought intimately converted to itself. And when they likewife perceived that though reminifcence might be fhewn from many particulars, yet it was evinced in a more eminent manner (as Plato alfo fays *) from the mathematical difciplines. For if any one, fays he, is led into the defcriptions, he will there eafily prove that difcipline is reminifcence. From whence Socrates alfo, in the Meno, fhews by this method of arguing, that learning is nothing elfe than the foul's recollection of her inherent reafons. And this, becaufe that which recollects, is alone the cogitative part of the foul; but this perfects her effence in the reafons of the mathematical difciplines, the fciences of which fhe previoufly received into herfelf, though fhe does not always energize on their fair variety. Indeed, fhe contains them all effentially and occultly; but fhe produces each of them when fhe is freed from the impediments originating from fenfe. For fenfe unites her with divifible objects: the phantafy fills her with forming motions, and appetite bends her to an indulgent and luxurious life. But every thing divifible is an obftacle to our felf-converfion. And whatever invefts with form, difturbs and offends that knowledge which is deftitute of form. And whatever is obnoxious to perturbations is an impediment to that energy which is unimpaired by affections. When, therefore, we have removed all thefe from the cogitative power, then fhall we be able to underftand by thought itfelf, the reafons which thought contains: then fhall we become fcientific in energy; and unfold our effential knowledge. But whilft we are captive and bound, and winking with the eye of the foul, we cannot by any means attain to a perfection convenient to our nature. Such then is mathefis or difcipline: a reminifcence of the eternal reafons contained in the foul. And the mathematical or difciplinative fcience, is on this account particularly denominated that knowledge which efpecially confers to our reminifcence of thefe effential reafons. Hence, the bufinefs and office of this fcience †, is apparent from its
name.

* In the Meno.

† This is certainly the true or philofophical employment of the mathematical fcience; for by this means we fhall be enabled to afcend from fenfe to intellect, and rekindle in the foul that divine light of truth, which, previous to fuch an energy, was buried in the obfcurity of a
corporeal

name. For its duty is to move the inherent knowledge of the foul; to awaken its intelligence; to purify its cogitation; to call forth its effential forms from their dormant retreats; to remove that oblivion and ignorance, which are congenial with our birth; and to diffolve the bonds arifing from our union with an irrational nature. It plainly leads us to a fimilitude of that divinity who prefides over this fcience, who manifefts intellectual gifts, and fills the univerfe with divine reafons; who elevates fouls to intellect, wakens them as from a profound fleep, converts them by enquiry to themfelves; and by a certain obftetric art, and invention of pure intellect, brings them to a bleffed life. To whom indeed, dedicating the prefent work, we here conclude our contemplation of the mathematical fcience.

corporeal nature. But by a contrary procefs, I mean, by applying mathematical fpeculations, to experimental purpofes, we fhall blind the liberal eye of the foul, and leave nothing in its ftead but the darknefs of corporeal vifion, and the phantoms of a degraded imagination.

BOOK.

B O O K II.

C H A P. I.

What Part Geometry is of Mathematics, and what the Matter is of which it confifts.

IN the preceding difcourfes we have confidered thofe common pro-
perties which refpect the whole of the mathematical fcience; and
this we have done agreeable to the doctrine of Plato; at the fame
time collecting fuch particulars as pertain to our prefent defign. But
confequent to this it is requifite that we fhould difcourfe on geometry
itfelf, and on the propofed inftitution of the elements, for the fake
of which we have undertaken the whole of the prefent work. That
geometry then, is a part of the whole of mathematics, and that it
obtains the fecond place after arithmetic, fince it is perfected and
bounded by this, (for whatever in geometry may be expreffed and
known, is determined by arithemetical reafons) has been afferted by
the ancients, and requires no long difcuffion in the prefent enquiry.
But we alfo may be able to relate our opinion on this particular, if
we confider what place, and what effence its fubject matter * is

allotted

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* The defign of the prefent chapter is to prove that the figures which are the fubjects of
geometric fpeculation, do not fubfift in external and fenfible matter, but in the receptacle of
imagination, or the matter of the phantafy. And this our philofopher proves with his ufual
elegance, fubtilty, and depth.   Indeed, it muft be evident to every attentive obferver, that fen-
fible figures fall far fhort of that accuracy and perfection which are required in geometrical
definitions: for there is no fenfible circle perfectly round, fince the point from which it is
defcribed is not without parts; and, as Voffius well obferves, (de Mathem. p. 4.) there is not
any fphere in the nature of things, that only touches in a point, for with fome part of its
fuperficies it always touches the fubjected plane in a line, as Ariftotle fhews Protagoras to have
objected againft the geometricians.   Nor muft we fay, with that great mathematician Dr. Barrow,
in his Mathematical Lectures, page 76, " that all imaginable geometrical figures, are really in-
herent in every particle of matter, in the utmoft perfection, though not apparent to fenfe; juft
as the effigies of Cæfar lies hid in the unhewn marble, and is no new thing made by the ftatuary,
but only is difcovered and brought to fight by his workmanfhip, i. e. by removing the parts of
matter by which it is overfhadowed and involved.   Which made Michael Angelus, the moft
famous carver, fay, that fculpture was nothing but a purgation from things fuperfluous.   For
take all that is fuperfluous, (fays he) from the wood or ftone, and the reft will be the figure you
intend.   So, if the hand of an angel (at leaft the power of God) fhould think fit to polifh any

particle

allotted among the univerfality of things. For from a proper furvey of this, the power of the fcience which knows this fubject matter, the utility arifing 'from it, and the good acquired by its learners, will immediately appear. Indeed, fome one may doubt in what genus of things he ought to place geometrical matter, fo as not to deviate from the truth it contains. For if the figures concerning which geometry difcourfes, exift in fenfible natures, and cannot be feparated from the dark receptacle of matter; how can we affert that geometry frees us from fenfible objects, that it brings us to an incorporeal effence, that it accuftoms us to an infpection of intelligibles, and prepares us for intellectual energy? Where fhall we ever furvey among fenfible objects a point without parts, or a line deftitute of breadth, or a fuperficies without profundity, or the equality of lines from the centre to the circumference; or the multangles, and all the figures of many bafes, concerning which geometry informs us? Laftly, after what manner can the reafons of fuch a fcience remain free from all poffible confutation; fince, indeed, fenfible forms and figures are fufceptive of the more and the lefs, are all moveable and mutable, and are full of material variety; among which equality fubfifts mixt and confufed with its contrary inequality, and into which things without parts have proceeded into partition, and interval, darkened with the fhades of matter, and loft in its infinite folds? But if the fubjects of geometry are removed from matter, are pure forms, and are feparated from

particle of matter, without vacuity, a fpherical fuperficies would appear to the eyes, of a figure exactly round; not as created anew, but as unveiled and laid open from the difguifes and covers of its circumjacent matter." For this would be giving a perfection to fenfible matter, which it is naturally incapable of receiving: fince external body is effentially full of pores and irregularities, which muft eternally prevent its receiving the accuracy of geometrical body, though polifhed by the hand of an angel. Befides, what polifhing would ever produce a point without parts, and a line without breadth? For though body may be reduced to the greateft exility, it will not by this means ever pafs into an incorporeal nature, and defert its triple dimenfion. Since external matter, therefore, is by no means the receptacle of geometrical figures, they muft neceffarily refide in the catoptric matter of the phantafy, where they fubfift with an accuracy fufficient for the energies of this fcience. It is true, indeed, that even in the purer matter of imagination, the point does not appear perfectly impartible, nor the line without latitude: but then the magnitude of the point, and the breadth of the line is indefinite, and they are, at the fame time, unattended with the qualities of body, and exhibit to the eye of thought, magnitude alone. Hence, the figures in the phantafy, are the proper recipients of that univerfal, which is the object of geometrical fpeculation, and reprefent, as in a mirror, the participated fubfiftence of thofe vital and immaterial forms which effentially refide in the foul.

fenfible.

3

senfible objects: they will be all of them, without doubt, void of parts, incorporeal, and deftitute of magnitude. For extenfion, tumor, and interval, approach to forms, on account of the material receptacle in which they are involved, and which receives things deftitute of parts, diftributed into parts ; things void of dimenfion, extended into dimenfion; and immoveable natures accompanied with motion. How then, if this is the cafe, fhall we cut a right line, triangle, and circle? How can we fpeak of the diverfities of angles, and the increments and decrements of triangular and quadrangular figures? Or how exhibit the contacts of circles or right lines? For all thefe evince that the geometric matter confifts of parts, and does not refide among indivifible reafons. Such then are the doubts concerning the matter of geometry, to which we may add, that Plato confiders the forms of geometry as placed in cogitation ; and grants, that we advance from fenfibles to forms of this kind, and that we rife from fenfibles to intellect, though (as we have previoufly obferved) the reafons fubfifting in cogitation are indivifible, are feparated by no interval, and fubfift according to the peculiarity of the foul. But if reafons are to be rendered agreeable to things themfelves, and to the doctrine of Plato, the following divifion muft be adopted.   * Every univerfal,

and

*· This divifion is elegantly explained by Ammonius, (in Porphyr. p. 12.) as follows, " Conceive a feal-ring, which has the image of fome particular perfon, for inftance, of Achilles, engraved in its feal, and let there be many portions of wax, which are impreffed by the ring. Afterwards conceive that fome one approaches, and perceives all the portions of wax, ftamped with the impreffion of this one ring, and keeps the impreffion of the ring in his mind : the feal engraved in the ring, reprefents the univerfal, prior to the many : the impreffion in the portions of wax, the univerfal in the many : but that which remains in the intelligence of the beholder, may be called the univerfal, after and pofterior to the many.. The fame muft we conceive in genera and fpecies. For that beft and moft excellent artificer of the world, poffeffes within himfelf the forms and exemplars of all things : fo that in the fabrication of man, he looks back upon the form of man refident in his effence, and fafhions all the reft according to its exemplar. But if any one fhould oppofe this doctrine, and affert that the forms of things do not refide with their artificer, let him attend to the following arguments. The artificer cither knows, or is ignorant of that which he produces: but he who is ignorant will never produce any thing. For who will attempt to do that, which he is ignorant how to perform ? fince he cannot act from an irrational power like nature, whofe operations are not attended with animad-verfion. But if he produces any thing by a certain reafon, he muft poffefs a knowledge of every thing which he produces. If, therefore, it is not impious to affert, that the operations of the Deity, like thofe of men, are attended with knowledge, it is evident that the forms of things muft refide in his effence: but forms are in the demiurgus, like the feal in the ring; and thefe forms are faid to be prior to the many, and feparated from matter. But the fpecies man, is

contained

and one thing containing many, is either naturally difpofed to be
thought of in particulars, or to appear fuch, becaufe it poffeffes its
exiftence in thefe; is infeparable from them; is difpofed and diftri-
buted in them; and together with thefe is either moved, or firmly
and immoveably abides. Or it is adapted to fubfift prior to many,
and to poffefs a power of generating multitude, affording to many
things images from itfelf, being furnifhed with a nature deftitute of
parts, from the effences which it participates, and raifing various
participations to fecondary natures: or it is difpofed to be formed by
thought, from the many, to poffefs a generating exiftence, and to refide
in the laft place in the many. For, according to thefe three modes of
fubfiftence, we fhall find, I think, that fome fubfift before the many,
others in the many, and others from the relation and predication
which they poffefs to thefe. But, that I may abfolve all in one word,
univerfal forms being threefold, we fhall confider the differences of
that form which many participate, which exifts in many, and fills
particular natures according to its fubject matter. Befides this, efta-
blifhing a twofold order of participants, one fubfifting in fenfible
objects, but the other in the phantafy, (fince matter is twofold; one
indeed, of things united with fenfe, but the other of fuch as fall under
the infpection of phantafy, as Ariftotle afferts, in a certain place *)
we muft allow that the univerfal, which is diftributed in the many,
is likewife twofold. The one, indeed, fenfible, as being that which
fenfible objects participate; but the other imaginative, as that which
fubfifts in the *many* of the phantafy. For the phantafy, on account
of its forming motion, and becaufe it fubfifts with, and in body,
always receives impreffions which are both divided and figured. So

contained in each particular man, like the impreffion of the feal in the wax, and is faid to fubfift
in the many, without a feparation from matter. And when we behold particular men, and
perceive the fame form and effigy in each, that form feated in our foul, is faid to be after the
many, and to have a pofterior generation : juft as we obferved in him, who beheld many feals
impreffed in the wax from one and the fame ring. And this one, pofterior to the many, may
be feparated from body, when it is conceived as not inherent in body, but in the foul: but is
incapable of a real feparation from its fubject." We muft here, however, obferve, that when
Ammonius fpeaks of the knowledge of the Deity, it muft be conceived as far fuperior to ours.
For he poffeffes a nature more true than all effence, and a perception clearer than all knowledge.
And as he produced all things by his unity, fo by an ineffable unity of apprehenfion, he knows
the univerfality of things.

* In lib. vii. Metaphyf. 35 & 39.

that

that whatever is known by it, is allotted a correfpondent exiftence: on which account, Ariftotle * does not hefitate to call it paffive intellect. But if it is intellect, why is it not impaffive, and deftitute of matter? And if it operates with paffion, how can it with propriety be called intellect? For impaffivity, indeed, properly belongs to intellect and an intelligent nature: but paffivity is very remote from fuch an effence. But (unlefs I am deceived) Ariftotle being willing to explain its middle nature between cognitions the moft primary, and fuch as are the laft, calls it at the fame time intellect, becaufe fimilar to primary cognitions, and paffive from that alliance which it poffeffes with fuch as are pofterior. For firft cognitions are indeed deftitute of figures and forms; comprehending in themfelves, intelligible natures, energizing about themfelves, united with the objects of knowledge, and free from all extrinfical impreffion and paffion. But laft cognitions exercife themfelves through the medium of inftruments, are rather paffions than energies, admit extrinfical knowledge, and move themfelves together with their various fubjects. For fuch (fays Plato) are the fenfations which arife from violent paffions. But the phantafy, obtaining a middle centre in the order of cognitions, is excited, indeed, by itfelf, and produces that which falls under cogitation: but becaufe it is not feparate from body, it deduces into partition, interval, and figure, the objects of its knowledge, from the indivifibility of an intellectual life. Hence, whatever it knows, is a certain impreffion and form of intelligence. For it underftands the circle, together with its interval, void, indeed, of external matter, but poffeffing intelligible matter. On this account, like fenfible matter, it does not contain one circle only: for we behold in its receptacle, diftance, together with the more and the lefs, and a multitude of circles and triangles. If then an univerfal nature is diftributed in fenfible circles, fince each of thefe completes a circular figure, and they are all mutually fimilar, fubfifting in one reafon, but differing in magnitudes or fubjects: in like manner, there is a common fomething in the circles, which fubfift in the receptacle of the phantafy, of which all its circles participate, and according to which they all

* In lib. iii. de Anima, tex. 20.

poffefs.

poſſeſs the ſame form; but in the phantaſy they poſſeſs but one dif-
ference only, that of magnitude.   For when you imagine many
circles about the ſame centre, they all of them exiſt in one immaterial
ſubject and life, which is inſeparable from a ſimple body, which, by
the poſſeſſion of interval, exceeds an eſſence deſtitute of parts; but
they differ in magnitude and parvitude, and becauſe they are con-
tained and contain.   Hence, that univerſal is two-fold, which is
underſtood as ſubſiſting in the many : one, indeed, in ſenſible forms;
but the other in ſuch as are imaginative.   And the reaſon of a cir-
cular and triangular figure, and of figure univerſal, is twofold.   The
one ſubſiſting in intelligible, but the other in ſenſible matter.   But
prior to theſe is the reaſon which reſides in cogitation, and that
which is ſeated in nature herſelf.   The former being the author of
imaginative circles, and of the one form which they contain; but
the other, of ſuch as are ſenſible.   For there are circles exiſting in the
heavens, and univerſally thoſe produced by nature, the reaſon of which
does not fall under a cogitative diſtribution.   For in incorporeal
cauſes, things poſſeſſing interval, are diſtinguiſhed by no intervals :
ſuch as are endued with parts, ſubſiſt without parts : and magnitudes
without the diffuſion of magnitude, as on the contrary in corporeal
cauſes, things without parts ſubſiſt diviſibly, and ſuch as are void of
magnitude with the extenſion of magnitude.   Hence, the circle re-
ſident in cogitation, is one, ſimple and free from interval : and mag-
nitude itſelf is there deſtitute of magnitude; and figure expreſſed by
no figure: for ſuch are reaſons ſeparate from matter.   But the circle
ſubſiſting in the phantaſy, is diviſible, figured, endued with interval,
not one only, but one and many, nor form alone, but diſtributed
with form.   And the circle, in ſenſible objects, is compoſite, diſtant
with magnitude, diminiſhed by a certain reaſon, full of ineptitude,
and very remote from the purity of immaterial natures.   We muſt
therefore ſay, that geometry, when it aſſerts any thing of circle
and diameter, and of the paſſions and affections which regard the
circle; as of contacts, diviſions, and the like: neither teaches nor
diſcourſes concerning ſenſible forms, (ſince it endeavours to ſepara-e
us from theſe), nor yet concerning the form reſident in cogitation,
(for here the circle is one, but geometry diſcourſes of many, propoſing

fomething of each, and contemplating the fame of all: and here it is indivifible, but the geometric circle is divifible); but we muft confefs, that it confiders univerfal itfelf; yet as diftributed in imaginative circles. And that it beholds, indeed, one circle *: and by the medium of another, contemplates the circle refident in the depths of cogitation: but by another, different from the preceding, fabricates the fair variety of its demonftrations. For fince cogitation is endued with reafons, but cannot behold them contractedly, feparated from material figure; it diftributes and removes them, and draws them forth feated in the fhadowy bofom of the phantafy, and placed in the veftibules of primary forms; revolving in it, or together with it, the knowledge of thefe: loving, indeed, a feparation from fenfibles, but finding imaginative matter proper for the reception of its univerfal forms. Hence, its intellection does not fubfift without the phantafy. And the compofitions and divifions of figures are imaginative; and their knowledge is the way which leads us to that effence purfued by cogitation: but cogitation itfelf, does not yet arrive at this ftable effence, while it looks abroad to externals, contemplates its internal forms according to thefe, ufes the impreffions of reafons, and is moved from itfelf to external and material forms. But if it fhould ever be able to return to itfelf, when it has contracted intervals and impreffions, and beholds multitude without impreffion, and fubfifting uniformly; then it will excellently perceive geometrical reafons, void of divifion and interval, effential and vital, of which there is a copious variety. And this energy will be the beft end of the geometric ftudy; and truly the employment of a Mercurial gift, bringing it back as from a certain Calypfo, and her detaining charms, to a more intellectual knowledge; and freeing it from thofe *forming* apprehenfions with which the mirror of the phantafy is replete. Indeed, it is requifite that a true geometrician fhould be employed in this meditation, and fhould eftablifh, as his proper end, the excitation and tranfition from the phantafy to cogitation alone; and that he fhould

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* That is, geometry firft fpeculates the circle delineated on paper, or in the duft: but by the medium of the circular figure in the phantafy, contemplates the circle refident in cogitation; and by that univerfal, or circular reafon, participated in the circle of the phantafy, frames its demonftrations.

accom-

accomplifh this by feparating himfelf from intervals, and the paffive intellect to that energy which cogitation contains. For by this means he will perceive all things without an interval, the circle and diameter without a part, the polygons in the circle, all in all, and yet every one feparate and a-part. Since, on this account, we exhibit alfo in the phantafy, both circles infcribed in polygons, and polygons in circles ; imitating the alternate exhibition of reafons deftitute of parts. Hence, therefore, we defcribe the conftitutions, the origin, divifions, pofitions, and applications of figures : becaufe we ufe the phantafy, and diftances of this kind proceeding from its material nature ; fince form itfelf is immoveable, without generation, indivifible, and free from every fubject. But whatever form contains occultly, and in an indiftant manner, is produced into the phantafy fubfifting with intervals, divifibly and expanded. And that which, indeed, produces the forms of geometric fpeculation, is cogitation: but that from which they are produced, is the form refident in cogitation : and that in which the produced figure refides is what is called the paffive intellect. Which folds itfelf about the impartibility of true intellect, feparates from itfelf the power of pure intelligence free from interval, conforms itfelf according to all formlefs fpecies, and becomes perfectly every thing from which cogitation itfelf, and our indivifible reafon confifts. And thus much concerning the geometric matter, as we are not ignorant of whatever Porphyry the Philofopher has obferved in his mifcellanies, and whatever many of the Platonifts defcribe. But we think that the prefent difcuffions are more agreeable to geometric differtations, and to Plato himfelf, who fubjects to geometry the objects of cogitation. For thefe mutually agree among themfelves; becaufe the caufes, indeed, of geometrical forms, by which cogitation produces demonftrations, pre-exift in demonftration itfelf: but the particular figures which are divided and compounded, are fituated in the receptacle of the phantafy.

C H A P.

CHAP. II.

What kind of Science Geometry is.

BUT let us now speak of that science which poffeffes a power of contemplating the univerfal forms participated by imaginative matter. Geometry, therefore, is endued with the knowledge of magnitudes and figures, and of the terms and reafons fubfifting in thefe; together with the paffions, various pofitions and motions which are contingent about thefe. For it proceeds, indeed, from an impartible point, but defcends even to folids, and finds out their multiform diverfities. And again, runs back from things more compofite, to things more fimple, and to the principles of thefe: fince it ufes compofitions and refolutions, always beginning from fuppofitions, and affuming its principles from a previous fcience; but employing all the dialectic ways. In principles, by the divifions of forms from their genera, and by defining its orations. But in things pofterior to principles, by demonftrations and refolutions. As likewife, it exhibits things more various, proceeding from fuch as are more fimple, and returning to them again. Befides this, it feparately difcourfes of its fubjects; feparately of its axioms; from which it rifes to demonftrations; and feparately of effential accidents, which it fhews likewife are refident in its fubjects. For every fcience has, indeed, a genus, about which it is converfant, and whofe paffions it propofes to confider: and befides this, principles, which it ufes in demonftrations; and effential accidents. Axioms, indeed, are common to all fciences (though each employs them in its peculiar fubject matter), but genus and effential accident vary according to the fciential variety. The fubjects of geometry are therefore, indeed, triangles, quadrangles, circles, and univerfally figures and magnitudes, and the boundaries of thefe. But its effential accidents are divifions, ratios, contacts, equalities, applications, exceffes, defects, and the like. But its petitions and axioms, by which it demonftrates every particular are, this, to draw a right line from any point to any point; and that, if from equals you take away equals, the remainders will

be

be equal; together with the petitions and axioms confequent to thefe. Hence, not every problem nor thing fought is geometrical, but fuch only as flow from geometric principles. And he who is reproved and convicted from thefe, is convinced as a geometrician. But whoever is convinced from principles different from thefe, is not a geometrician, but is foreign from the geometric contemplation. But the objects of the non-geometric inveftigation, are of two kinds. For the thing fought for, is either from entirely different principles, as we fay that a mufical enquiry is foreign from geometry, becaufe it emanates from other fuppofitions, and not from the principles of geometry: or it is fuch as ufes, indeed, geometrical principles, but at the fame time perverfely, as if any one fhould fay, that parallels coincide. And on this account, geometry alfo exhibits to us inftruments of judging, by which we may know what things are confequent to its principles, and what thofe are which fall from the truth of its principles: for fome things attend geometrical, but others arithmetical principles. And why fhould we fpeak of others, fince they are far diftant from thefe? For one fcience is more certain than another (as Ariftotle fays *) that, indeed, which emanates from more fimple fuppofitions, than that which ufes more various principles; and that which tells the *why*, than that which knows only the fimple exiftence of a thing; and that which is converfant about intelligibles, than that which touches and is employed about fenfibles. And according to thefe definitions of certainty, arithmetic is, indeed, more certain than geometry, fince its principles excel by their fimplicity. For unity is void of pofition, with which a point is endued. And a point, indeed, when it receives pofition, is the principle of geometry: but unity, of arithmetic. But geometry is more certain than fpherics; and arithmetic, than mufic. For thefe render univerfally the caufes of thofe theorems, which are contained under them. Again, geometry is more certain than mechanics, optics, and catoptrics. Becaufe thefe difcourfe only on fenfible objects. The principles, therefore, of geometry and arithmetic, differ, indeed, from the principles of other fciences; but the hypothefes of thefe two,

* In his firft Analytics, t. 42. See the Differtation to this work.

alternately

alternately differ and agree according to the difference we have already defcribed. Hence, alfo, with refpect to the theorems which are demonftrated in thefe fciences, fome are, indeed, common to them, but others peculiar. For the theorem which fays, *every proportion may be exprefled,* alone belongs to arithmetic; but by no means to geometry: fince this laft fcience contains things which cannot be exprefled *. That theorem alfo, which affirms, that *the gnomons of quadrangles are terminated according to the leaft* †, is the property of arithmetic: for in geometry, a minimum cannot be given. But thofe things are peculiar to geometry, which are converfant about pofitions; for numbers have no pofition: which refpect contacts; for contact is found in continued quantities: and which are converfant about ineffable proportions; for where divifion proceeds to infinity, there, alfo that which is ineffable is found ‡. But things common to both thefe fciences, are fuch as refpect divifions, which Euclid treats of in the fecond book; except that propofition which divides a right line into extreme and mean proportion §. Again, of thefe common theorems, fome, indeed, are transferred from geometry into arithmetic; but others, on the contrary, from arithmetic into geometry: and others fimilarly accord with both, which are derived into them from the whole mathematical fcience. For the permutation, indeed,

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* Such as the proportion of the diagonal of a fquare to its fide ; and that of the diameter of a circle, to the periphery.

† The gnomons, from which fquare numbers are produced, are odd numbers in a natural feries from unity, i. e. 1, 3, 5, 7, 9, 11, &c. for thefe, added to each other continually, produce fquare numbers *ad infinitum.* But thefe gnomons continually decreafe from the higheft, and are at length terminated by indivifible unity.

‡ This doctrine of ineffable quantities, or fuch whofe proportion cannot be exprefled, is largely and accurately difcuffed by Euclid, in the tenth book of his Elements : but its ftudy is neglected by modern mathematicians, *becaufe it is of no ufe,* that is, becaufe it contributes to nothing mechanical.

§ This propofition is the 11th of the fecond book: at leaft, the method of dividing a line into extreme and mean proportion, is immediately deduced from it ; which is done by Euclid, in the 30th, of the fixth book. Thus, Euclid fhews (11. 2.) how to divide the line (A_____G___B) A B, fo that the rectangle under the whole A B, and the fegment G B, may be equal to the fquare made from A G: for when this is done, it follows, that as A B is to A G, fo is A G to G B; as is well known, But this propofition, as Dr. Barrow obferves, cannot be explained by numbers; becaufe there is not any number which can be fo divided, that the product from the whole into one part, may be equal to the fquare from the other part.

converfions,

converfions, compofitions, and divifions of ratios are, after this man-
ner, common to both.   But fuch things as are commenfurable,
arithmetic firft beholds; but afterwards geometry, imitating arith-
metic.   From whence, alfo, it determines fuch things to be commen-
furables of this kind, which have the fame mutual ratio to one another,
as number to number; becaufe commenfurability principally fubfifts
in numbers.   For where number is, there alfo that which is com-
menfurable is found; and where commenfurable is, there alfo number.
Laftly, geometry firft infpects triangles and quadrangles: but, arith-
metic, receiving thefe from geometry, confiders them according to
proportion.   For in numbers, figures refide in a caufal manner.
Being excited, therefore, from effects, we pafs to their caufes, which
are contained in numbers.   And at one time, we indifferently behold
the fame accidents, as when every polygon is refolved by us into
triangles *: but, at another time, we are content with what is neareft
to the truth, as when we find in geometry one quadrangle the double
of another, but not finding this in numbers, we fay that one fquare
is double of another, except by a deficience of unity.   As for in-

* All polygonous figures, may, it is well known, be refolved into triangles; and this is no lefs
true of polygonous numbers, as the following obfervations evince.   All number originates from
indivifible unity, which correfponds to a point: and it is either linear, correfponding to a line;
or fuperficial, which correfponds to a fuperficies; or folid, which imitates a geometrical folid.
After unity, therefore, the firft of linear numbers is the duad; juft as every finite line is allotted
two extremities.   The triad is the firft of fuperficial numbers; as the triangle of geometrical
figures.   And the tetrad, is the firft of folids; becaufe a triangular pyramid, is the firft among
folid numbers, as well as among folid figures.   As, therefore, the monad is affimilated to the
point, fo the duad to the line, the triad to the fuperficies, and the tetrad to the folid.   Now,
of fuperficial numbers, fome are triangles, others fquares, others pentagons, hexagons, hep-
tagons, &c.   Triangular numbers are generated from the continual addition of numbers in a
natural feries, beginning from unity.   Thus, if the numbers 1, 2, 3, 4, 5, &c. be added to
each other continually, they will produce the triangular numbers 1, 3, 6, 10, 15, &c. and if
every triangular number be added to its preceding number, it will produce a fquare number.
Thus 3 added to 1 makes 4; 6 added to 3 is equal to 9; 10 added to 6 is equal to 16; and fo
of the reft.   Pentagons, are produced from the junction of triangular and fquare numbers, as
follows.   Let there be a feries of triangular numbers 1, 3, 6, 10, 15, &c.
                    And of fquares   1, 4, 9, 16, 25, &c.
Then the fecond fquare number, added to the firft triangle, will produce the firft pentagon
from unity, i. e. 5.   The third fquare added to the fecond triangle, will produce the fecond
pentagon, i. e. 12; and fo of the reft, by a fimilar addition.   In like manner, the fecond
pentagon, added to the firft triangle, will form the firft hexagon from unity; the third pentagon
and the fecond triangle, will form the fecond hexagon, &c.   And, by a fimilar proceeding, all
the other polygons may be obtained.

                                                                                ftance,

stance, the square from 7, is double the square from 5, wanting one: But we have produced our discussion to this length, for the purpose of evincing the communion and difference in the principles of these two sciences. Since it belongs to a geometrician to survey from what common principles common theorems are divided; and from what principles such as are peculiar proceed; and thus to distinguish between the geometrical, and non-geometrical, referring each of them to different sciences.

C H A P.   III.

From whence the whole of Geometry originated, how far it proceeds, and in what its Utility consists.

BUT, beginning still higher, let us contemplate the whole of geometry, from whence it originated, and how far it proceeds in its energies: for thus we shall properly perceive the ornament which it contains. Indeed, it is necessary to understand that it is extended through the universality of things: that it accommodates its animadversions * to all beings; and contains in itself the forms of all things: that, according to its supreme part, and which is endued with the highest power of intelligence, it surveys true beings; and teaches by images the properties of divine ornaments, and the powers of intellectual forms: for it contains the reasons of these also in its peculiar contemplations. And it exhibits what figures are convenient to the god, to primary essences, and to the natures of souls. But, according to its middle cognitions, it evolves cogitative reasons; explains and beholds the variety which they contain; exhibits their existence, and inherent passions; as also, their communities and di-

* Intellections are universally correspondent to their objects, and participate of evidence or the contrary, in proportion as their subjects are lucid or obscure. Hence, Porphyry, in his sentences, justly observes, that " we do not understand in a similar manner with all the powers of the soul, but according to the particular essence of each. For with the intellect we understand intellectually; and with the soul, rationally: our knowledge of plants is according to a seminal conception; our understanding of bodies is imaginative; and our intellection of the divinely solitary principle of the universe, who is above all things, is in a manner superior to intellectual perception, and by a super-essential energy." Ἀφορμαὶ πρὸς τὰ Νοητὰ, (10.) So that, in consequence of this reasoning, the speculations of geometry are then most true, when most abstracted from sensible and material natures.

versities.

verfities. From which, indeed, it comprehends, in terminated bounds, the imaginative formations of figures, and reduces them to the effential fubftance of reafons. But, according to the third propagations of cogitative intelligence, it confiders nature, and delivers the manner in which the forms of fenfible elements, and the powers which they contain, are previoufly received according to caufe, in the reafons themfelves. For it poffeffes, indeed, the images of univerfal intelligible genera; but the exemplars of fuch as are fenfible: and completes its own effence, according to fuch things as are fubject to cogitation. And through thefe, as through proper mediums, it afcends and defcends to thofe univerfals which truly are, and to fenfible forms which are in a ftate of perpetual formation. But always geometrically philofophifing concerning the things which are, it comprehends in all the proportions of virtues, the images of intellectual, animal, and natural concerns. And it delivers, in an orderly manner, all the ornaments of republics: and exhibits in itfelf their various mutations. Such then are its energies arifing from a certain immaterial power of cognition: but when it touches upon matter, it produces from itfelf a multitude of fciences; fuch as geodæfia, mechanics, and perfpective: by which it procures the greateft benefit to the life of mortals. For it conftructs by thefe fciences, war-inftruments, and the bulwarks of cities; and makes known the circuits of mountains, and the fituations of places. Laftly, it inftructs us in meafures: at one time of the diverfified ways of the earth; and at another, of the reftlefs paths of the deep. Add too, that it conftructs balances and fcales, by which it renders to cities a fure equality according to the invariable ftandard of number. Likewife, it clearly expreffes, by images, the order of the whole orb of the earth; and by thefe, manifefts many things incredible to mankind, and renders them credible to all. Such, indeed, as Hiero of Syracufe is reported to have faid of Archimedes *, when he had fabricated a fhip furnifhed with three fails, which he had prepared to fend to Ptolemy king of Egypt. For when all the Syracufians together, were unable to draw this fhip, Archimedes enabled Hiero

* See Plutarch, in the life of Marcellus.

to draw it himſelf, without any aſſiſtance from others. But he, being aſtoniſhed, ſaid, From this day, Archimedes ſhall be believed in whatever he ſhall affirm. They alſo report, that Gelo ſaid the ſame, when Archimedes diſcovered the weight of the ſeveral materials from which his crown was compoſed, without diſſolving their union. And ſuch are the narrations which many of the ancients have delivered to our memory, who were willing to ſpeak in praiſe of the mathematics : and, on this account, we have placed before the reader, for the preſent, a few out of the many, as not foreign from our deſign of exhibiting the knowledge and utility of geometry.

C H A P.   IV.

On the Origin of Geometry, and its Inventors.

BUT let us now explain the origin of geometry, as exiſting in the preſent age of the world. For the demoniaçal Ariſtotle * obſerves, that the ſame opinions often ſubſiſt among men, according to certain orderly revolutions of the world : and that ſciences did not receive their firſt conſtitution in our times, nor in thoſe periods which are known to us from hiſtorical tradition, but have appeared and vaniſhed again in other revolutions of the univerſe ; nor is it poſſible to ſay how often this has happened in paſt ages, and will again take place in the future circulations of time. But, becauſe the origin of arts and ſciences is to be conſidered according to the preſent revolution of the univerſe, we muſt affirm, in conformity with the moſt general tradition, that geometry was firſt invented by the Egyptians, deriving its origin from the menſuration of their fields : ſince this, indeed, was neceſſary to them, on account of the inundation of the Nile waſhing away the boundaries of land belonging to each. Nor ought it to ſeem wonderful, that the invention of this as well as of other ſciences, ſhould receive its commencement from convenience and opportunity. Since whatever is carried in the circle of genera-

* In lib. i. de Cœlo, tex. 22. et lib. i. Meteo. cap. 3. Ariſtotle was called demoniacal by the Platonic philoſophers, in conſequence of the encomium beſtowed on him by his maſter, Plato, " That he was tne dæmon of nature." Indeed, his great knowledge in things ſubject to the dominion of nature, well deſerved this encomium ; and the epithet *divine*, has been univerſally aſcribed to Plato, from his profound knowledge of the intelligible world.

tion,

tion, proceeds from the imperfect to the perfect. A tranſition, therefore, is not undeſervedly made from ſenſe to conſideration, and from this to the nobler energies of intellect *. Hence, as the certain knowledge of numbers received its origin among the Phœnicians, on account of merchandiſe and commerce, ſo geometry was found out among the Egyptians from the diſtribution of land. When Thales, therefore, firſt went into Egypt, he transferred this knowledge from thence into Greece: and he invented many things himſelf, and communicated to his ſucceſſors the principles of many. Some of which were, indeed, more univerſal, but others extended to ſenſibles. After him Ameriſtus, the brother of Steſichorus the poet, is celebrated as one who touched upon, and taſted the ſtudy of geometry, and who is mentioned by Hippias the Elean, as reſtoring the glory of geometry. But after theſe, Pythagoras changed that philoſophy, which is converſant about geometry itſelf, into the form of a liberal doctrine, conſidering its principles in a more exalted manner; and inveſtigating its theorems immaterially and intellectually; who likewiſe invented a treatiſe of ſuch things as cannot be explained † in geometry, and diſcovered the conſtitution of the mundane figures. After him, Anaxagoras the Clazomenian ſucceeded, who undertook many things pertaining to geometry. And Oenopides the Chian, was ſomewhat junior to Anaxagoras, and whom Plato mentions in his Rivals, as one who obtained mathematical glory. To theſe, ſucceeded Hippocrates, the Chian, who invented the quadrature of the lunula ‡,

and

* Εἰς τῶν, is wanting in the original, but is ſupplied by the excellent tranſlation of Barocius.

† 'Αλόγων, in the printed Greek, which Fabricius, in his Bibliotheca Græca, vol. i. page 385, is of opinion, ſhould be read ἀναλόγων; but I have rendered the word according to the tranſlation of Barocius, who is likely to have obtained the true reading, from the variety of manuſcripts which he conſulted.

‡ The quadrature of the Lunula is as follows.
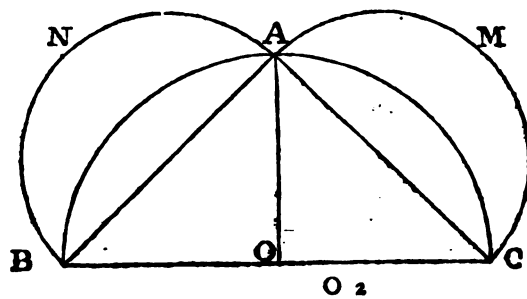
Let

and Theodorus the Cyrenean, both of them eminent in geometrical knowledge. For the firft of thefe, Hippocrates compofed geometrical elements: but Plato, who was pofterior to thefe, caufed as well geometry itfelf, as the other mathematical difciplines, to receive a remarkable addition, on account of the great ftudy he beftowed in their inveftigation. This he himfelf manifefts, and his books, replete with mathematical difcourfes, evince: to which we may add, that he every where excites whatever in them is wonderful, and extends to philofophy. But in his time alfo lived Leodamas the Thafian, Architas the Tarentine, and Theætetus the Athenian; by whom theorems were increafed, and advanced to a more fkilful conftitution. But Neoclides was junior to Leodamas, and his difciple was Leon; who added many things to thofe thought of by former geometricians. So that Leon alfo conftructed elements more accurate, both on account of their multitude, and on account of the ufe which they exhibit: and befides this, he difcovered a method of determining when a problem, whofe inveftigation is fought for, is poffible, and when it is impoffible. But Eudoxus the Cnidian, who was fomewhat junior to Leon, and the companion of Plato, firft of all rendered the multitude of thofe theorems which are called univerfals more abundant; and to three proportions added three others; and things relative to a fection, which received their commencement from Plato, he diffufed into a richer multitude, employing alfo refolutions in the profecution of thefe. Again, Amyclas the Heracleotean, one of Plato's familiars, and Menæchmus, the difciple, indeed, of Eudoxus, but converfant with Plato, and his brother Dinoftratus, rendered the whole of geometry as yet more perfect. But Theudius, the Magnian, appears

Let A B C be a right-angled triangle, and B A C a femi-circle on the diameter B C: B N A a femi-circle defcribed on the diameter A B; A M C a femi-circle defcribed on the diameter A C. Then the femi-circle B A C is equal to the femi-circles B N A, and A M C together: (becaufe circles are to each other as the fquares of their diameters, 31, 6.) If, therefore, you take away the two fpaces B A, A C common on both fides, there will remain the two lunulas B N A, A M C, bounded on both fides with circular lines, equal to the right-angled triangle B A C. And if the line B A, be equal to the line A C, and you let fall a perpendicular to the hypothenufe B C, the triangle B A O will be equal to the lunular fpace B N A, and the triangle C O A will be equal to the lunula C M A. Thofe who are curious, may fee a long account of an attempt of Hippocrates to fquare the circle, by the invention of the lunulas, in Simplicius on Ariftotle's Phyfics, lib. i.

to have excelled, as well in mathematical difciplines, as in the reft of philofophy. For he conftruded elements egregioufly, and rendered many particulars more univerfal. Befides, Cyzicinus the Athenian, flourifhed at the fame period, and became illuftrious in other mathematical difciplines, but efpecially in geometry. Thefe, therefore, reforted by turns to the Academy, and employed themfelves in propofing common queftions. But Hermotimus, the Colophonian, rendered more abundant what was formerly publifhed by Eudoxus and Theætetus, and invented a multitude of elements, and wrote concerning fome geometrical places. But Philippus the Mendæan *, a difciple of Plato, and by him inflamed in the mathematical difciplines, both compofed queftions, according to the inftitutions of Plato, and propofed as the objed of his enquiry whatever he thought conduced to the Platonic philofophy. And thus far hiftorians produce the perfedion of this fcience. But Euclid was not much junior to thefe, who colleded elements, and conftruded many of thofe things which were invented by Eudoxus; and perfeded many which were difcovered by Theætetus. Befides, he reduced to invincible demonftrations, fuch things as were exhibited by others with a weaker arm. But he lived in the times of the firft Ptolemy: for Archimedes mentions Euclid, in his firft book, and alfo in others. Befides, they relate that Euclid was afked by Ptolomy, whether there was any fhorter way to the attainment of geometry than by his elementary inftitution, and that he anfwered, there was no other royal path which led to geometry. Euclid, therefore, was junior to the familiars of Plato, but more ancient than Eratofthenes and Archimedes (for thefe lived at one and the fame time, according to the tradition of Eratofthenes) but he was of the Platonic fed, and familiar with its philofophy: and from hence he appointed the conftitution of thofe figures which are called Platonic †, as the end of his elementary inftitutions.

* So Barocius reads, but Fabricius Μελμαιος.

† i. e. The five regular bodies, the pyramid, cube, odaedron, dodecaedron and icofaedron; concerning which, and their application to the theory of the univerfe, fee Kepler's admirable work, De Harmonia Mundi.

C H A P.

C H A P. V.

What Mathematical Volumes Euclid compofed.

THERE are, therefore, many other mathematical volumes of this man, full of admirable diligence, and fkilful confideration: for fuch are his Optics *, and Catoptrics: and fuch alfo, are his elementary inftitutions, which conduce to the attainment of mufic †; and his book concerning divifions ‡. But his geometrical inftitution of the Elements is efpecially admirable, on account of the order and election of thofe theorems and problems, which are diftributed through the Elements. For he does not affume all which might be faid, but that only which could be delivered in an elementary order. Befides this, he exhibits modes of fyllogifms of every kind; fome, indeed, receiving credibility from caufes, but others proceeding from certain figns; but all of them invincible and fure, and accommodated to fcience. But, befides thefe, he employs all the dialectic ways, dividing, indeed, in the inventions of forms; but defining in effential reafons: and again, demonftrating in the progreffions from principles to things fought, but refolving in the reverfions from things fought to principles. Befides this, we may view in his geometrical elements, the various fpecies of converfions, as well of fuch as are fimple as of fuch as are more compofite. And again, what wholes may be converted with wholes: what wholes with parts; and on the other hand, what as parts with parts §. Befides this, we muft fay, that in the continuation of inventions, the difpofitions and order of things preceding and following, and in the power with which he treats

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* It may be doubted whether the optics and catoptrics, afcribed to Euclid in the editions of his works are genuine: for Savil, and Dr. Gregory, think them fcarcely worthy fo great a man.

† There are two excellent editions of this work, one by Meibomius, in his collection of ancient authors on harmony; and the other by Dr. Gregory, in his collection of Euclid's works.

‡ This work is moft probably loft. See Dr. Gregory's Euclid.

§ All this is fhewn by Proclus in the following Commentaries; and is furely moft admirable and worthy the inveftigation of every liberal mind; but I am afraid modern mathematicians very little regard fuch knowledge, becaufe it cannot be applied to practical and mechanical purpofes.

every

every particular, he is not deceived, as if falling from fcience, and carried to its contrary, falfehood and ignorance. But becaufe we may imagine many things as adhering to truth, and which are confequent to principles producing fcience, which neverthelefs tend to that error which flows from the principles, and which deceives ruder minds, he has alfo delivered methods of the perfpicacious prudence belonging to thefe. From the poffeffion of which, we may exercife thofe in the invention of fallacies, who undertake this infpection, and may preferve ourfelves from all deception. And this book, by which he procures us this preparation, is infcribed ψευδαρίος, or, concerning fallacies *. Becaufe he enumerates in order their various modes, and in each exercifes our cogitation with various theorems. And he compares truth with falfehood, and adapts the confutation of deception to experience itfelf. This book, therefore, contains a purgative and exercifing power. But the inftitution of his elementary, fkilful contemplation of geometrical concerns, poffeffes an invincible and perfect narration.

C H A P. VI.

Concerning the Purport of Geometry.

BUT, perhaps, fome one may enquire in what the defign of this treatife confifts? To this I anfwer, that its defign is to be diftinguifhed as well according to the objects of enquiry, as according to the learner. And, indeed, regarding the fubject, we muft affirm, that all the difcourfe of geometry is concerning the mundane figures. Becaufe it begins from fuch things as are fimple, but ends in the variety of their conftitution. And, indeed, it conftitutes each of them feparately, but at the fame time delivers their infcriptions in a fphere, and the proportions which they contain. On which account fome have thought, that the defign of each of the books is to be referred to the world; and they have delivered to our memory, the utility which they afford us in the contemplation of the univerfe.

* This work is unfortunately loft.

But

But diftinguifhing the defign with refpect to the learner, we muft affirm, that its purpofe is the inftitution of elements; and the perfection of the learners cogitative powers in univerfal geometry. For beginning from thefe, we are enabled to underftand the other parts of this fcience, and to comprehend the variety which they contain. And, indeed, without thefe, the difcipline of the reft, is to us impoffible and incomprehenfible. For fuch theorems as are moft principal and fimple, and are moft allied to firft fuppofitions, are here collected in a becoming order. And the demonftrations of other mathematicians, ufe thefe as moft known, and advance from thefe in their moft complicated progreffions. For thus Archimedes, in what he has writ concerning the fphere and cylinder, and Apollonius, and the reft of mathematicians, ufe, as evident principles, the things exhibited in this treatife. Its purpofe, therefore, is the inftitution of learners in the whole geometric fcience, and to deliver the determinate conftitutions of the mundane figures.

C H A P. VII.

From whence the Name of Elementary Inftitution originated, and why Euclid is called the Inftitutor of Elements.

BUT what gave rife to the name of elementary inftitution, and of element itfelf, from which elementary inftitution was derived? To this we fhall reply, by obferving, that of theorems fome are ufually called elements, but others elementary, and others again are determined beyond the power of thefe. Hence, an element is that whofe confideration paffes to the fcience of other things, and from which we derive a folution of the doubts incident to the particular fcience we inveftigate. For as there are certain firft principles of fpeech, moft fimple and indivifible, which we denominate elements, and from which all difcourfe is compofed; fo there are certain principal theorems of the whole of geometry, denominated elements, which have the refpect of principles to the following theorems; which regard all the fubfequent propofitions, and afford the demonftrations of many accidents effential to the fubjects of geometric fpeculation.

But

But things elementary are fuch as extend themfelves to a multitude of propofitions, and poffefs a certain fimplicity and fweetnefs, yet are not of the fame dignity with elements; becaufe their contemplation is not common to all the fcience to which they belong, as is the cafe in the following theorem, that in triangles, perpendiculars, drawn from their angles to their fides, coincide in one point *. Laftly, whatever neither poffeffes a knowledge extended into multitude, nor exhibits any thing fkilful and elegant, falls beyond the elementary power. Again, an element, as Menæchmus fays, may have a twofold definition. For that which confirms, is an element of that which is confirmed; as the firft propofition of Euclid with refpect to the fecond, and the fourth with regard to the fifth. And thus, indeed, many things may be mutually called elements one of another; for they are mutually confirmed. Thus, becaufe the external angles of right-lined figures, are equal to four right angles, the multitude of internalones equal to right angles; and, on the contrary, that from this is exhibited †. Befides, an element is otherwife called that into which, becaufe it is more fimple, a compofite is diffolved. But it muft be obferved, that every element cannot be called the element of every thing : but fuch as are more principal are the elements of fuch as are conftituted in the reafon of the thing effected; as petitions are the elements of theorems. And, according to this fignification of an element, Euclid's elements are conftructed. Some, indeed, of that geometry which is converfant about planes ; but others of ftereometry. In the fame manner, likewife, in arithmetic and aftronomy, many have compofed elementary inftitutions. But it is difficult, in each fcience, to chufe and conveniently ordain elements, from which all the peculiarities of that fcience originate, and into which they may be refolved. And among thofe who have undertaken this employment, fome have been able to collect more, but others fewer elements. And fome, indeed, have ufed fhorter demonftrations ; but others have extended their treatife to an infinite length. And fome have omitted the method by an impoffibility; but others

* Becaufe this is true only in ifofceles and equilateral triangles.

† This follows from the 32d propofition of the firft book of Euclid; and is demonftrated by Dr. Barrow, in his fchelium to that propofition.

that by proportion; and others, again, have attempted preparations againft arguments deftroying principles. So that many methods of elementary inftitution have been invented by particular writers on this fubject. But it is requifite that this treatife fhould entirely remove every thing fuperfluous, becaufe it is an impediment to fcience. But every thing fhould be chofen, which contains and concludes the thing propofed; for this is moft convenient and ufeful in fcience. The greateft care, likewife, fhould. be paid to clearnefs and brevity; for the contraries to thefe, difturb our cogitation. Laftly, it fhould vin-dicate to itfelf, the univerfal comprehenfion of theorems, in their proper bounds: for fuch things as divide learning into particular, fragments, produce an incomprehenfible knowledge. But in all thefe modes, any one may eafily find, that the elementary inftitution of Euclid excels the inftitutions of others. For its utility, indeed, efpecially confers to the contemplation of primary figures: but the tranfition from things more fimple to fuch as are more various, and alfo that perception, which from axioms poffeffes the beginning of knowledge, produces clearnefs, and an orderly tradition: and the migration from firft and principal theorems to the objects of enquiry, effects the univerfality of demonftration. For whatever he feems to omit, may either be known by the fame ways, as the conftruction of a fcalene and ifofceles triangle * : or, becaufe they are difficult, and capable of infinite variety, they are far remote from the election of elements, fuch as the doctrine of perturbate proportions, which Apollonius has copioufly handled : or, laftly, becaufe they may be eafily conftructed from the things delivered, as from caufes, fuch as many fpecies of angles and lines. For thefe, indeed, were omitted by Euclid, and are largely difcourfed of by others, and are known from fimple propofitions. And thus much concerning the univerfal elementary inftitution of geometry.

* The method of conftructing thefe is fhewn by our philofopher, in his comment on the firft propofition, as will appear in the fecond volume of this work.

C H A P. VIII.

Concerning the Order of Geometrical Difcourfes.

BUT let us now explain the univerfal order of the difcourfes contained in geometry. Becaufe then, we affert that this fcience confifls from hypothefis *, and demonftrates its confequent propofitions from definite principles (for one fcience only, I mean the firft philofophy, is without fuppofition, but all the reft affume their principles from this) it is neceffary that he who conftructs the geometrical inftitution of elements, fhould feparately deliver the principles of the fcience, and feparately the conclufions which flow from thofe principles; and that he fhould render no reafon concerning the nature or truth of the principles, but fhould confirm by reafons, the things confequent to thefe geometric principles. For no fcience demonftrates its own principles, nor difcourfes concerning them; but procures to itfelf a belief of their reality, and they become more evident to the particular fcience to which they belong than the things derived from them as their fource. And thefe, indeed, fcience knows by themfelves; but their confequents, through the medium of thefe. For thus, alfo, the natural philofopher propagates his reafons from a definite principle, fuppofing the exiftence of motion. Thus too, the phyfician, and he who is fkilled in any of the other fciences and arts. For if any one mingles principles, and things flowing from principles into one and the fame, he difturbs the whole order of knowledge, and conglutinates things which can never mutually agree; fince a principle, and its emanating confequent, are naturally diftinct from each other. In the firft place, therefore (as I have faid), principles in the geometric inftitution are to be diftinguifhed from their confequents, which is performed by Euclid in each of his books; who, before every treatife, exhibits the common principles of this fcience; and afterwards divides thefe common principles into hypothefes, petitions, and axioms. For all thefe mutually differ; nor is an axiom, petition,

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* The reader will pleafe to obferve, that the definitions are, indeed, hypothefes, according to the doctrine of Plato, as may be feen in the note to chap. i. book I. of this work.

and

and hypothesis the fame, according to the demoniacal Ariftotle; but when that which is affumed in the order of a principle, is indeed known to the learner, and credible by itfelf, it is an axiom: fuch as, that things equal to the fame, are mutually equal to each other. But when any one, hearing another fpeak concerning that of which he has no felf-evident knowledge, gives his affent to its affumption, this is hypothefis. For that a circle is a figure of fuch a particular kind, we prefume (not according to any common conception) without any preceding doctrine. But when, again, that which is afferted was neither known, nor admitted by the learner, yet is affumed, then (fays he) we call it petition; as the affumption that all right angles are equal. But the truth of this is evinced by thofe who ftudy to treat of fome petition, as of that which cannot by itfelf be admitted by any one. And thus, according to the doctrine of Ariflotle *, are axiom, petition, and fuppofition diftinguifhed. But oftentimes, fome denominate all thefe hypothefes, in the fame manner as the Stoics call every fimple enunciation an axiom. So that, according to their opinion, hypothefes alfo will be axioms; but, according to the opinion of others, axioms will be called fuppofitions. Again, fuch things as flow from principles are divided into problems and theorems. The firft, indeed, containing the origin, fections, ablations, or additions of figures, and all the affections with which they are converfant; but the other exhibiting the accidents effential to each figure. For, as things effective of fcience, participate of contemplation, in the fame manner things contemplative previoufly affume problems in the place of operations. But formerly fome of the ancient mathematicians thought that all geometrical propofitions fhould be called theorems, as the followers of Speufippus and Amphinomus, believing, that to contemplative fciences, the appellation of theorems is more proper than that of problems; efpecially fince they difcourfe concerning eternal and immutable objects. For origin does not fubfift among things eternal: on which account, problems cannot have any place in thefe fciences; fince they enunciate origin, and the production of that which formerly had no exiftence, as the conftruction of an equi-

* In his laft Analytics. See the preceding Differtation.

lateral.

lateral triangle, or the defcription of a fquare on a given right line, or the pofition of a right line at a given point. It is better, therefore (fay they), to affert that all propofitions are of the fpeculative kind: but that we perceive their origin, not by production, but by knowledge, receiving things eternal as if they were generated; and on this account we ought to conceive all thofe theorematically, but not problematically. But others, on the contrary, think that all fhould be called problems: as thofe mathematicians who have followsd Menæchmus. But that the office of problems is two-fold, fometimes, indeed, to procure the thing fought; but at other times when they have received the determinate object of enquiry, to fee, either what it is, or of what kind it is, or what affection it poffeffes, or what its relation is to another. And, indeed, the affertions of each are right; for the followers of Speufippus well perceive. Since the problems of geometry are not of the fame kind, with fuch as are mechanical. For thefe are fenfibles, and are endued with origin, and mutation of every kind. And, on the other hand, thofe who follow Menæchmus do not diffent from truth: fince the inventions of theorems cannot by any means take place without an approach into matter; I mean intelligible matter. Reafons, therefore, proceeding into this, and giving form to its formlefs nature, are not undefervedly faid to be affimilated to generations. For we fay that the motion of our cogitation, and the production of its inherent reafons, is the origin of the figures fituated in the phantafy, and of the affections with which they are converfant: for there conftructions and fections, pofitions and applications, additions and ablations, exift: but every thing refident in cogitation, fubfifts without origin and mutation. There are, therefore, both geometrical problems and theorems. But, becaufe contemplation abounds in geometry, as production in mechanics, all problems participate of contemplation; but every thing contemplative is not problematical. For demonftrations are entirely the work of contemplation; but every thing in geometry pofterior to the principles, is affumed by demonftration. Hence, a theorem is more common: but all theorems do not require problems; for there are fome which poffefs from themfelves the demonftration of the thing fought. But others, diftinguifhing a theorem from a problem, fay, that

that

... On a given right line to describe a quadrangle; for a figure that is not quadrangular may be described. And, in short, all of this kind belong to the problematical order. But the followers of Zenodotus, who was familiar with the doctrine of Oenopides, but the disciple of Andron, distinguish a theorem from a problem, so far as a theorem enquires what the symptom is which is predicated of the matter it contains; but a problem enquires what that is, the existence of which is granted. From whence the followers of Possidonius define a theorem a proposition, by which it is enquired whether a thing exists or not; but a problem, a proposition, in which it is enquired what a thing is, or the manner of its existence. And they say that we ought to form the contemplating proposition by enunciating, as that every triangle has two sides greater than the remaining one, and that

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the angles at the bafe of every ifofceles triangle are equal: but we muft form the problematical propofition, as if enquiring whether a triangle is to be conftructed upon this right line. For there is a difference, fay they, abfolutely and indefinitely, to enquire whether the thing propofed is from a given point to erect a right line at right angles to a given line, and to behold what the perpendicular is. And thus, from what has been faid, it is manifeft there is fome difference between a problem and a theorem. But that the elementary inftitution of Euclid, alfo, confifts partly of problems, and partly of theorems, will be manifeft from confidering the feveral propofitions. Since, in the conclufion of his demonftrations, he fometimes adds (which was to be fhewn) fometimes (which was to be done) the latter fentence being the mark or fymbol of problems, and the former of theorems. For although, as we have faid, demonftration takes place in problems, yet it is often for the fake of generation; for we affume demonftration in order to fhew, that what was commanded is accomplifhed: but fometimes it is worthy by itfelf, fince the nature of the thing fought after may be brought into the midft. But you will find Euclid fometimes combining theorems with problems, and ufing them alternately, as in the firft book; but fometimes abounding with the one and not the other. For the fourth book is wholly problematical; but the fifth is entirely compofed from theorems. And thus much concerning the order of geometrical propofitions.

C H A P. IX.

Concerning the Defign of the firft Book,—its Divifion,—and a previous Admonition to the Reader.

BUT, after thefe confiderations, when we have determined the defign of the firft book, and have exhibited its divifion, we fhall enter upon the treatife of the definitions. The defign, then, of this book, is to deliver the principles of the contemplation of right lines. For though a circle, and its confideration, is more excellent than the effence and knowledge of right lines, yet the doctrine concerning thefe is more adapted to us, who are haftening to transfer our cogitation

that indeed every problem receives whatever is predicated of its matter, together with its own oppofite : but that every theorem receives, indeed, its fymptom predicate, but not its oppofite. But I call the matter of thefe, that genus which is the fubject of enquiry; as for inftance, a triangle, quadrangle, or a circle : but the fymptom predicate, that which is denominated an effential accident, as equality, or fection, or pofition, or fome other affection of this kind. When, therefore, any one propofes to infcribe an equilateral triangle in a circle, he propofes a problem : for it is poffible to infcribe one that is not equilateral. But when any one afferts that the angles at the bafe of an ifofceles triangle are equal, we muft affirm that he propofes a theorem; for it is not poffible that the angles at the bafe of an ifofceles triangle fhould be unequal to each other. On which account, if any one forming problematically, fhould fay that he wifhes to infcribe a right angle in a femi-circle, he muft be confidered as ignorant of geometry; fince every angle in a femi-circle is neceffarily a right one. Hence, propofitions which have an univerfal fymptom, attending the whole matter, muft be called theorems ; but thofe in which the fymptom is not univerfal, and does not attend its fubject, muft be confidered as problems. As to bifect a given terminated right line, or to cut it into equal parts : for it is poffible to cut it into unequal parts. To bifect every rectilinear angle, or divide it into equal parts ; for a divifion may be given into unequal parts. On a given right line to defcribe a quadrangle; for a figure that is not quadrangular may be defcribed. And, in fhort, all of this kind belong to the problematical order. But the followers of Zenodotus, who was familiar with the doctrine of Oenopides, but the difciple of Andron, diftinguifh a theorem from a problem, fo far as a theorem enquires what the fymptom is which is predicated of the matter it contains ; but a problem enquires what that is, the exiftence of which is granted. From whence the followers of Poffidonius define a theorem a propofition, by which it is enquired whether a thing exifts or not; but a problem, a propofition, in which it is enquired what a thing is, or the manner of its exiftence. And they fay that we ought to form the contemplating propofition by enunciating, as that every triangle has two fides greater than the remaining one, and that

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the angles at the end of every isosceles triangle are equal. But we must form the problematical proposition, as if enquiring whether a triangle is to be constituted upon this right line. For there is a difference, say they, whether to enquire whether the thing proposed is from a given point to draw a right line at right angles to a given line, and to behold what the perpendicular is. And thus, from what has been said, it is manifest there is some difference between a problem and a theorem. But that the elementary institution of Euclid, consists partly of problems, and partly of theorems, will be manifest from considering the several propositions. Since, in the execution of his demonstrations, he sometimes adds which was to be shewn, sometimes which was to be done; the latter sentence being the mark or symbol of problems, and the former of theorems. For although, as we have said, demonstration takes place in problems, yet it is often for the like of generation, for we assume demonstration in order to shew, that what was commanded is accomplished; so sometimes it is worthy by mind, from the nature of the thing sought what may be brought into act. But you will find Euclid sometimes combining theorems with problems, and using them alternately, as in the first book, but sometimes abounding with the one and not the other. For the fourth book is wholly problematical, and the fifth is entirely composed from theorems. And thus much concerning the order of geometrical propositions.

CHAP. X

Concerning the Design of the first Book. — Its Division. — That a treatise is addressed to the Studious.

BUT, after these considerations, when we have determined the design of the first book, and have exhibited its division, we shall enter upon the tumult of its inspection. The design, then, of this book is to deliver the principles of the constitution of right lines. For though a circle, and in particular, is more excellent than the circle, and knowledge of right lines, yet the science concerning these is more adapted to us, who are labouring to consider the cogi-

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tation from more imperfect and fenfible natures, to fuch as are intelligible. For, indeed, right lined figures are proper to fenfibles, but a circle to intelligibles. Becaufe that which is fimple, uniform, and definite, is proper to the nature of the things which are: but that which is various, and which increafes indefinitely from the number of its containing fides, regards the fluctuating effence of fenfible particulars. Hence, in this book, the firft and moft principal of right lined figures are delivered; I mean the triangle and parallelogram. For in thefe, as under their proper genus, the caufes of the elements are contained: viz. the ifofceles and fcalene, and thofe which are formed from thefe, the equilateral triangle, and the quadrangle, from which the four figures of the elements are compofed. We fhall find, therefore, as well the origin of the equilateral triangle as of the quadrangle; of the laft, indeed, upon, but of the firft from a given right line. [* An equilateral triangle, therefore, is the proximate caufe of the three elements, fire, air, and water: but a quadrangle is annexed to earth. And laftly, the defign of the firft book is adapted to the whole treatife, and confers to the univerfal knowledge of the mundane elements. Befides, it inftructs learners in the fcience concerning right-lined figures; fince it rightly invents, and accurately collects, the firft principles of thefe.

But this book is divided into three greateft parts, of which the firft declares the origin and properties of triangles, as well according to angles, as alfo according to fides. Befides, it makes mutual comparifons of thefe, and beholds every one by itfelf. For receiving one triangle, fometimes it confiders the angles from the fides; but fometimes the fides from the angles: and this according to equality and inequality. And fuppofing two triangles, it difcovers the fame property again, by various methods. But the fecond part combines the contemplation of parallelograms, defcribing their properties and generations. And the third part fhews the communication of triangles

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* That part of this work enclofed within the brackets, is wanting in the original; which I have reflored from the excellent verfion of Barocius. The philofophical reader, therefore, of the original, who may not have Barocius in his poffeffion, will, I hope, be pleafed, to fee fo great a vacancy fupplied; efpecially, as it contains the beginning of the commentary on the definition of a point.

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and parallelograms, both in fymptoms and mutual comparifons. For it fhews that triangles and parallelograms conftituted on the fame and on equal bafes, are affected with the fame paffions; and by complication, when both ftand upon one bafe : and again, after what manner a parallelogram may be made equal to a triangle; and laftly, concerning the proportion which in right angled triangles, the fquare made from the fide fubtending, has to the fquares containing the right angle. And fuch is the divifion of the firft Book.

But, previous to our enquiry into each of thefe parts, we think it requifite to admonifh the reader, that he muft not require of us, thofe fmall affumptions, and cafes, and whatever elfe there may be of that kind, which has been divulged by our predeceffors. For we are fatiated with thefe, and fhall, therefore, but rarely adopt them in our difcourfe. But whatever has a more difficult contemplation, and regards univerfal philofophy, of this we fhall make a particular relation : imitating the Pythagoreans, with whom this ænigma was common, " a * figure and a ftep : but not a figure and three oboli." fhewing by this, that it is requifite to purfue that philofophy which afcends every theorem by a ftep, and raifes the foul on high; but does not fuffer it to remain among fenfibles, to fill up the ufe attendant on mortals, and, confulting for this, to neglect the elevation which rifes from hence to an intelligible effence.

* I do not find this ænigma among the Pythagoric fymbols which are extant; fo that it is probably no where mentioned but in the prefent work. And I am forry to add, that a *figure and three oboli*, is too much the general cry of the prefent times.

D E F I N I T I O N S.

DEFINITION I.

A POINT is that which has no PARTS.

THAT geometry, according to the tranfition which takes place
from things more compofite to fuch as are more fimple, runs
from body, which is diffufed into diftance by three dimenfions, to a
fuperficies by which it is bounded; but from fuperficies to a line, the
boundary of fuperficies; and from a line to a point deftitute of all
dimenfion, has been often faid, and is perfectly manifeft. But becaufe
thefe terms, in many places, on account of their fimplicity, appear
to be more excellent than the nature of compofites; but in many, as
when they fubfift in things which they terminate, they are fimilar to
accidents, it is neceffary to determine in what genera of beings each
of thefe may be beheld *. I fay then, that fuch things as are deftitute
of matter, which fubfift in feparate reafons, and in thofe forms which
are placed under themfelves, are always allotted a fubfiftence of more
fimple effences, fuperior to the fubfiftence of fuch as are more com-
pofite. On this account, both in intellect, and in the ornaments, as
well of the middle kind as among thofe peculiar to the foul, and in
natures themfelves, the terms which proximately vivify bodies, excel
according to effence the things which are terminated; and are more
impartible, more uniform, and more primary than thefe. For in
immaterial forms, unity is more perfect than multitude; that which

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* The prefent Comment, and indeed moft of the following, eminently evinces the truth of
Kepler's obfervation, in his excellent work, *De Harmonia Mundi*, p. 118. For, fpeaking of our
author's compofition in the prefent work, which he every where admires and defends, he
remarks as follows, "oratio fluit ipfi torrentis inftar, ripas inundans, et cæca dubitationum vada
gurgitefque occultans, dum mens plena majeftatis tantarum rerum, luctatur in anguftiis linguæ,
et conclufio nunquam fibi ipfi verborum copia fatisfaciens, propofitionum fimplicitatem excedit."
But Kepler was fkilled in the Platonic philofophy, and appears to have been no lefs acquainted
with the great depth of our author's mind than with the magnificence and fublimity of his
language. Perhaps Kepler is the only inftance among the moderns, of the philofophical and
mathematical genius being united in the fame perfon.

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is impartible, than that which is endued with unbounded progreffion; and that which terminates, than that which receives bound from another. But fuch things as are indigent of matter, and abide in others, and degenerate from the perfection of their effence, which are fcattered about fubjects, and have an unnatural union, are allotted more compofite reafons, prior to fuch as are more fimple. Hence, things which appear in the phantafy invefted with form, and the matter of the figures which the phantafy contains, and whatever in fenfibles is generated by nature, have, in a preceding order, the reafons of the things terminated; but the reafons which terminate, in a following and adventitious rank *.] For left that which is diftributed into three dimenfions, fhould be extended into infinite magnitude, either according to intelligence or fenfe, it was every way terminated by fuperficies. And left a plane fuperficies fhould conceal itfelf in an infinite progreffion, a line approaching oppofed its diffufion, and gave bound to its indefinite extenfion. And, in like manner, a point limited the progreffions of a line; compofite natures deriving their fubfiftence from fuch as are fimple. For this alfo is again manifeft, that in feparate forms the reafons of terms fubfift in themfelves, but not in thofe which are terminated; and abiding fuch as they are in reality, poffefs a power of conftituting fecondary natures. But, in infeparable forms they give themfelves up to things which are terminated, refide in them, become, as it were, their parts, and are replenifhed with bafer natures. On which account, that which is impartible is there endued with a partible effence, and that which is void of latitude is diffufed into breadth. And terms are no longer able to preferve their fimplicity and purity. For fince they abide in another, they neceffarily change their own nature into the matter of their containing fubject. Matter, indeed, difturbs the perfection of thefe, and carries the reafon of a plane to become a profound plane; but obfcuring the one dimenfion of a line, carries it to be every way partible; and gives corporeity to the indivifibility of a point, and feparates it together with the natures which it terminates. For all thefe reafons falling into matter, the one kind from cogitation into intelli-

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* That is, the reafon of a triangular figure, for inftance, in the phantafy, or image itfelf, is fuperior as the triangular nature participated in that figure.

gible

gible matter, but the other from nature into that which is fenfible, are replenifhed with their containing fubjeds; and depart from their own fimplicity, into foreign compofitions and intervals. But here a doubt arifes how all thefe, exifting in intelled and foul in an impartible manner, and without any dimenfion, are diftributed into matter, fome indeed, principally, but others on account of its nature? Shall we fay that there is a certain order in immaterial forms, fo that fome are allotted the firft, fome the middle, and others the laft place; and that of forms fome are more uniform, but that others are more multiplied ; and that fome have their powers colleded together, but others tending into interval; and that fome, again, border upon bound, but that others are proximate to infinity? For though all participate of thefe two principles, yet fome originate from bound, but others from infinity, of which they more largely participate. Hence, a point is entirely impartible, fince it fubfifts according to bound, yet it occultly contains an infinite power, by which it produces every interval, and the progreffion of all intervals, unfolds its infinite power. But body, and the reafon of body, participates more of an infinite nature; on which account it is among the number of things terminated by another, and divifible in infinitum, according to all dimenfions. But the mediums between thefe, according to the diftance of the extremes, are either among the number of things which have an abundance of bound; or among fuch as have an affluence of infinity: on which account they both terminate and are terminated. For, indeed, fo far as they confift from bound, they are able to terminate others; but fo far as they participate of infinity, they are indigent of termination from others. Hence, fince a point is alfo a bound, it preferves its proper power in participation: but fince it likewife contains infinity occultly, and is compelled to be every where prefent with the natures which it terminates, it refides with them infinitely. And, becaufe among immaterial forms there was a certain infinite power capable of producing things diftant from each other by intervals, a point is prefent with its participants in capacity. For infinity in intelligibles is the primary caufe and prolific power of the univerfe; but in material natures it is imperfed, and is alone all things in dormant capacity. And in fhort, thofe forms which, on account of their

fimplicity

fimplicity and impartibility, hold a fuperior rank among principles, preferve, indeed, (in conformity to their nature,) their own property in their participations, but become worfe than more compofite reafons. For matter is able to participate thefe more clearly, and to be prepared for their reception, rather than that of the moft fimple caufes of beings. On which account, the veftigies of feparate principles defcend into matter; but the participations of thofe in a fecond and third order, become more confpicuous. Hence, matter participates more of the caufe of body, than of a plane; and of this more than the form of a line; and of this ftill more than that of a point, which contains all thefe, and is the boundary of them all. For the reafon of a point prefides over this whole feries, unites and contains all partible natures, terminates their progreffions, produces them all by its infinite power, and comprehends them in its indivifible bound. On which account alfo, in the images of immaterial forms, fome are the boundaries of others; but a point is the limit of them all. But that we muft not think with the Stoics, that thefe boundaries of bodies alone fubfift from cogitation; but that there are certain natures of this kind among beings, which previoufly contain the demiurgical reafons of things, we fhall be enabled to remember, if we regard the whole world, the convolutions of its parts, the centres of thofe convolutions, and the axes which penetrate through the whole of thefe revolving circles. For the centres fubfift in energy, fince they contain the fpheres, preferve them in their proper ftate, unite their intervals, and bind and eftablifh to themfelves the powers which they poffefs. But the axes themfelves being in an immoveable pofition, evolve the fpheres, give them a circular motion, and a revolution round their own abiding nature. And the poles of the fpheres, which both terminate the axes, and bind in themfelves the other convolutions, do they not perfpicuoufly evince, that points are endued with demiurgical and capacious powers, that they are perfective of every thing diftant by intervals, and are the fources of union, and an unceafing motion? From whence, indeed, Plato * alfo fays, that they have an adamantine fubfiftence; fhewing by this, the immutable, eternal, and ftable power

* In the tenth book of his Republic.

of

of their effence, ever preferving itfelf in the fame uniform mode of exiftence. He adds too, that the whole fpindle of the Fates, is turned about thefe, and leaps round their coercive union. But other more recondite and abftrufe difcourfes affirm, that the demiurgus prefides over the world, feated in the poles, and, by his divine love, converting the univerfe to himfelf. But the Pythagoreans thought that the pole fhould be called the Seal of Rhea *; becaufe the zoogonic, or vivific goddefs, pours through thefe into the univerfe, an inexplicable and efficacious power. And the centre they called the prifon of Jupiter; becaufe, fince Jupiter has placed a demiurgical guard in the bofom of the world, he has firmly eftablifhed it in the midft. For, indeed, the centre abiding, the univerfe poffeffes its immoveable ornament, and unceafing convolution: and the gods who prefide over the poles, obtain a power collective of divifible natures, and unific of fuch as are multiplied: and thofe who are allotted the government of the axes, reftrain and eternally evolve their perpetual convolutions. And, if it is lawful to offer our own opinion on this fubject, the centres and poles of all the fpheres are the fymbols of the conciliating gods, fhadowing forth their imperceptible and unifying compofition. But the axes exprefs the coherencies of the univerfal ornaments; and are endued with a power of comprehending the mundane integrities and periods, in the fame manner as their prefiding deities, of fuch as are intellectual. But the fpheres themfelves are images of the gods, called perfectors of works, copulating the principle with the end, and excelling all figures in fimplicity, fimilitude, and perfection. But we have been thus prolix, that we might evince the power of impartibles, and of the terms which the world contains, and that fo far as they bear an image of primary and moft principal caufes, they are allotted the moft excellent order in the univerfe. For centres and poles are not of the fame kind with things which are terminated; but they fubfift in energy, and poffefs an effence, and perfect power, which pervades through all partible natures. But many beholding thofe terms which imperfectly fubfift in terminated effences, confider them as endued with a flender fubfiftence; and fome indeed fay, that

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* See the Hymn to the Mother of the Gods, in my tranflation of the Orphic Initiations.

they

they are alone feparated from fenfibles by thought; but others, that they have an effence no where but in our thoughts. However, fince the forms of all thefe are found both in the nature of intellect, in the ornaments of foul, in the nature of things, and in inferior bodies, let us confider how, according to the order they contain, they fubfift in the genera of beings. And indeed, all of them pre-exift in intellect, but in an impartible and uniform manner: fo that they all fubfift according to one form, the reafon of a point, which exifts occultly and impartibly. But they all fubfift in foul according to the form of a line : on which account Timæus alfo compofes the foul from right and circular lines: for every circle is a line alone *. But they all fubfift in natures, according to the reafon of a plane; and on this account, Plato commands us to manifeft thofe natural reafons, which are endued with a power of conftituting bodies by a plane. And the refolution of bodies into planes leads us to the proximate caufe of appearances. Laftly, they all fubfift in bodies, but in a corporeal manner; fince all forms have their being in thefe, according to the partible nature of bodies. Hence, all of them appear every where, and each according to its proper order ; and diverfity arifes from predominating power. The point, indeed, is every where impartible, and when that which is divifible into parts, excels according to the diminution of beings, it vindicates to itfelf, an illuftrious fubfiftence of partible natures. And fometimes the point is entirely fuperior, according to the excellence of caufe; but fometimes it is connected with divifibles, and fometimes it is allotted in them an adventitious exiftence; and, as if fwallowed up by the partition of the loweft natures, lofes its own proper impartibility. As, therefore, with refpect to the monad, one † is the mother of number, but the other is as

* The philofopher here feems to contradict what he afferts in the end of his comment on the 13th Definition: for there he afferts, that the circle is a certain plane fpace. Perhaps he may be reconciled, by confidering, that as the circle fubfifts moft according to bound, when we fpeculate its effence in this refpect we may define it according to the circumference, which is the caufe of its bound. But when we confider it as participating of infinity alfo, though not in fo eminent a degree, and view it from its emanations from the centre as well as in its regreffions, we may define it a plane fpace.

† That is, the effential one of the foul is the mother of number; but that which fubfifts in opinion is nothing more than the receptacle of the former; juft as matter is the feat of all forms. For a farther account of the fubfiftence of numbers, fee the firft fection of the preceding Differtation.

matter

matter fpread under, and the receptacle of numbers; and each of them
is a principle, (yet neither of them is number), but in a different re-
fpect: in the fame manner a point alfo, is partly the parent and author
of magnitudes; but is partly a principle in another refpect, and not
according to a generative caufe. But is a point, then, the only im-
partible? Or may we affirm this of the now in time, and of ùnity in
numbers? Shall we not fay, that to the philofopher, indeed, dif-
courfing concerning the univerfality of things, it is proper to behold
every thing, however falling under diftribution; but that to him who
is endued with the fcience of particulars, who produces his contem-
plation from certain definite principles, and runs back even to thefe,
but very little fcrutinizes the progreffions of beings. it is requifite to
attempt, confider, and treat concerning that impartible nature alone,
which regards his firft principles; and to behold that fimplicity which
prefides over all the particular fubjects of his knowledge? In confe-
quence of this reafoning, therefore, a point alone, according to the
geometric matter, is deftitute of partition; but unity according to that
which is arithmetical. And the reafon of a point, however in fome
other refpects it may be imperfect, yet is perfect in the prefent fcience.
For, indeed, the phyfician alfo fays, that the elements of bodies are
fire and water, and things fimilar to thefe; and as far as to thefe the
refolution of bodies proceeds. But the natural philofopher paffes on
to more fimple elements; and the one defines an element fimple as to
fenfe, but the other fimple as to reafon; and both of them properly as
to their peculiar fcience. We muft not, therefore, think that the
definition of a point is faulty, nor determine it as imperfect; for fo
far as pertains to the geometric matter, and its principles, it is fuf-
ficiently delivered. This alone, indeed, is wanting to its completion,
that the definition does not clearly fay, *that which is impartible with
me is a point; and my principle, and that which I contain is moft fimple,
is nothing elfe than this.* And after this manner it is proper to hear
the geometrician addreffing us. Euclid, therefore, from a negation
of parts, declares to us a principle, leading to the theory of its whole
fubject nature. For negative difcourfes are proper to principles, as
Parmenides teaches us, who delivers the doctrine concerning the firft
and laft caufe, by negations alone. Since every principle confifts of

an

an effence different from its flowing confequents; and the negations of thefe exhibit to us the property of their fource. For that it is, indeed, the caufe of thefe, yet at the fame time has nothing in common with thefe, becomes perfpicuous from a doctrine of this kind. But here a doubt may arife, how, fince the phantafy receives all things invefted with forms, and in a partible manner, the geometrician beholds in it the point deftitute of parts ? For it is not becaufe they are reafons exifting in cogitation, but the phantafy receives the refemblances of intellectual and divine forms according to its own proper nature, exhibiting in its fhadowy bofom the forms of formlefs natures, and clothing with figure things entirely free from the affections of figure. To this ambiguity we muft fay, that the fpecies of imaginative motion is neither alone partible, nor impartible; but that it proceeds from the impartible to the partible, and from the formlefs nature to that which is expreffed by form. For if it was partible alone, it could not preferve in itfelf many impreffions of forms, fince the fubfequent would obfcure the pre-exiftent figures; for no body can contain at once, and according to the fame fituation, a multitude of figures; but the former will be blotted out by the fucceffion of the latter. But if it was alone impartible, it would not be inferior to cogitation, and to foul, which furveys all things in an impartible manner. Hence, it is neceffary that it fhould indeed begin from an impartible according to its motion, and from thence draw forth the folded and fcattered form of every thing falling under cogitation, and penetrating to its fhadowy receptacle: but, that it fhould at length end in form, figure, and interval. And if it be allotted a nature of this kind, it will, after a certain manner, contain an impartible effence: and a point, according to this, muft be faid to have its principal fubfiftence: for the form of a line is contracted in the phantafy according to this. Hence, becaufe it poffeffes a twofold power, impartible and partible, it will indeed contain a point in an impartible, and intervals in a partible manner. But as the Pythagoreans define a point to be unity having pofition, let us confider what they mean. That numbers, indeed, are more immaterial and more pure than magnitudes, and that the principle of numbers is more fimple than the principle of magnitudes, is manifeft to every one: but when they fay that a

point

point is unity endued with pofition, they appear to me to evince that unity and number fubfift in opinion: I mean monadic number *. On which account, every number, as the pentad and the heptad, is one in every foul, and not many; and they are deftitute of figure and adventitious form. But a point openly prefents itfelf in the phantafy, fubfifts, as it were, in place, and is material according to intelligible matter. Unity, therefore, has no pofition, fo far as it is immaterial, and free from all interval and place: but a point has pofition, fo far as it appears feated in the bofom of the phantafy, and has a material fubfiftence. But unity is ftill more fimple than a point, on account of the community of principles. Since a point exceeds unity according to pofition; but appofitions in incorporeals produce diminutions of thofe natures, by which the appofitions are received.

DEFINITION II.

A Line is a Length without Breadth.

A Line obtains the fecond place in the Definitions, as it is by far the firft and moft fimple interval, which the geometrician calls a length, adding alfo without breadth; fince a line, in refpect of a fuperficies, ranks as a principle. For he defines a point, as it is the principle of all magnitudes, by negation alone; but a line, as well by affirmation as by negation. Hence it is a length, and by this exceeds the impartibility of a point; but it is without breadth, becaufe it is feparated from other dimenfions. For, indeed, every thing which is void of breadth, is alfo deftitute of bulk, but the contrary is not true, that every thing void of bulk is alfo deftitute of breadth. Since, therefore, he has removed breadth from a line, he has alfo removed at the fame time bulk. On which account he does not add, that a line alfo has no thicknefs, becaufe this property is confequent to the notion of being without breadth. But it is defined by others in various ways: for fome call it the flux of a point, but others a magnitude contained by one interval. And this definition,

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* That is, number compofed from units.

indeed,

Indeed, is perfect, and fufficiently explains the effence of a line; but that which calls it the flux of a point, appears to manifeft its nature from its producing caufe; and does not exprefs every line, but alone that which is immaterial. For this is produced by a point, which though impartible itfelf, is the caufe of being to partible natures. But the flux of a point, fhews its progreffion and prolific power, approaching to every interval, receiving no detriment, perpetually abiding the fame, and affording effence to all partible magnitudes. However, thefe obfervations are known, and manifeft to every one. But we fhall recall into our memory, difcourfes more Pythagorical, which determine a point as analogous to unity, a line to the duad, a fuperficies to the triad, and body to the tetrad. [* Yet when we compare thofe which receive interval together, we fhall find a line monadic; but a fuperficies dyadic, and a folid body triadic.] From whence alfo, Ariftotle † fays, that body is perfected by the ternary number. And, indeed, this is not wonderful, that a point, on account of its impartibility, fhould be affimilated to unity; but that things fubfequent to a point, fhould fubfift according to numbers proceeding from unity, and fhould preferve the fame proportion to a point, as numbers to unity; and that every one fhould participate of its proximate fuperior, and have the fame proportion to its kindred, and following degree, as the fuperior to this, which is the immediate confequent. [‡ For example, that a line has the order of the duad with refpect to the point, but of unity to a fuperficies; and that this laft has the relation of a triad to the point, but of the duad to a folid.] And on this account, body is tetradic, with refpect to a point, but triadic as to a line. Each order, therefore, has its proportion; but the order of the Pythagoreans is the more principal, which receives its commencement from an exalted fource, and follows the nature of beings. For a point is indeed twofold; fince it either fubfifts by itfelf, or in a line; in which laft refpect alfo, fince as a boundary it is alone and one, neither having a whole nor parts, it imitates the fupreme

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* This fentence within the brackets, is wholly omitted in the printed Greek.
† In i. De Cælo.
‡ This fentence within the brackets, which is very imperfect in the Greek, I have fupplied from the excellent tranflation of Barocius. In the Greek there is nothing more than λίγω δὲ ὅτι τὴν γραμμὴν δυαδός πρὸς τὸ ςιγιόν.

nature

nature of beings. On which account too, it was placed in a corre-
fpondent proportion to unity. * For as the oracle fays, *Unity is there
firft, where the paternal unity abides.* But a line is the firft endued
with parts and a whole, and it is monadic becaufe it is diftant by
one interval only; and dyadic on account of its progreffion: for if it
be infinite, it participates of the indefinite duad; but if finite, it re-
quires two terms, from whence and to what place; fince, on account
of thefe it imitates totality, and is allotted an order among totals. For
unity, according to the oracle, is extended †, and generates two; and
this produces a progreffion into longitude, together with that which is
diftant extendedly, and with one interval, and the matter of the duad.
But fupérficies, fince it is both a triad and duad, as alfo the recep-
tacle of the primary figures, and that which receives the firft form
and fpecies, is in a certain refpect fimilar to the triadic nature, which
firft terminates beings; and to the duad, by which they are divided
and difperfed. But a folid, fince it has a triple diftance, and is dif-
tinguifhed by the tetrad, which is endued with a power of compre-
hending all reafons, is reduced to that order in which the diftinction
of corporeal ornaments appears; as alfo the divifion of the univerfe
into three parts, together with the tetradic property, which is gene-
rative and female. And thefe obfervations, indeed, might be more
largely difcuffed, but for the prefent, muft be omitted. Again, the
difcourfe of the Pythagoreans, not undefervedly, calls a line, which
is the fecond in order, and is conftituted according to the firft motion
from an impartible nature, dyadic. And that a point is pofterior to
unity, a line to the duad, and a fuperficies to the triad, Parmenides
himfelf fhews, by firft of all taking away multitude from one by
negation, and afterwards the whole. Becaufe, if multitude is before
that which is a whole, number alfo will be prior to that which is
continuous, and the duad to the line, and unity to the point: fince
the epithet *not many*, belongs to unity which generates multitude, but

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* In the Greek, γαρ' η μονα; ικιι πρωτον, οπυ παζικη μοιας ιςι φτοι το λογιον. The latter part
only of this oracle, is to be found in all the printed editions of the Zoroaftrian oracles; though
it is wonderful how this omiffion could efcape the notice of fo many able critics, and learned
men. It feems probable, from hence, that it is only to be found perfect in the prefent work.

† The word ταναν, is omitted in the Greek.

to the point, the term *not a whole*, is proper, becaufe it produces a whole; for this is faid to have no part. And thefe things are affirmed of a line, while we more accurately contemplate its nature. But we fhould alfo admit the followers of Apollonius, who fay, that we obtain a notion of a line, when we are ordered to meafure the lengths alone, either of ways or walls; for we do not then fubjoin either breadth or bulk, but only make one diftance the objeĉt of our confideration. In the fame manner we perceive fuperficies, when we meafure fields; and a folid, when we take the dimenfions of wells. For then, colleĉting all the diftances together, we fay, that the fpace of the well is fo much, according to length, breadth, and depth. But a line may become the objeĉt of our fenfation, if we behold the divifions of lucid places from thofe which are dark, and furvey the moon when dichotomized: for this medium has no diftance with refpeĉt to latitude; but is endued with longitude, which is extended together with the light and fhadow.

D E F I N I T I O N III.

But the Extremities of a Line are Points.

EVERY compofite receives its bound from that which is fimple, and every thing partible from that which is impartible; and the images of thefe openly prefent themfelves in mathematical principles. For when it is faid that a line is terminated by points, it feems manifeftly to make it of itfelf infinite, becaufe, on account of its proper progreffion, it has no extremity. As, therefore, the duad is terminated by unity, and reduces its own intolerable boldnefs under bound, when it is reftrained in its comprehenfive embrace : fo a line alfo is limited by the points which it contains. For, fince it is fimilar to the duad, it participates of a point having the relation of unity, according to the nature of the duad. Indeed, in imaginative, as well as in fenfible forms, the points themfelves terminate the lines in which they refide. But in immaterial forms, the reafon of the impartible point pre-exifts feparate and apart ; but when proceeding from thence by far the firft of all, by determining itfelf with interval, moving itfelf, and flowing

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in infinite progreffion, and imitating the indefinite duad, it is reftrained indeed, by its proper principle, is united by its power, and on every fide feized by its coercive bound. Hence it is, at the fame time, both infinite and finite: infinite, indeed, according to its progreffion; but finite according to its participation of a terminating caufe. So that, when it approaches to this caufe, it is detained in its comprehenfion, and is terminated according to its union. Hence too, in the images of incorporeal forms, a point is faid to terminate a line, by occupying its beginning and end. Bound, therefore, in immaterials, is feparated from that which is bounded: but here it is twofold; for it fubfifts in that which is terminated. And this affords a wonderful fymptom, that forms, indeed, abiding in themfelves, precede their participants according to caufe; but when giving themfelves up to their fubordinate natures, fubfift according to their diverfified properties: fince they are multiplied and diftributed together with thefe, and receive the divifion of their fubjects. Befides, this alfo muft be previoufly received concerning a line, that our geometrician ufes it in a threefold acceptation. As terminated on both fides, and finite; as in the problem * which fays, Upon a given terminated right line to conftruct an equilateral triangle. And as partly infinite and partly finite; as in the problem which commands us from three right lines, which are equal to three given right lines, to conftruct a triangle; for in the conftruction of the problem, he fays, Let there be placed a certain right line, on one part finite, but on the other part infinite. And again, a line is received by Euclid as on both fides infinite; as in the problem which fays, Upon a given infinite right line, from a given point, which is not in that line, to let fall a perpendicular. But, befides this, the following doubts, fince they are worthy of folution, muft

* This and the following problems, are the 1ft, 22d, and 12th propofitions of the firft book. But in the two laft, inftead of the word απειρος or infinite, which is the term employed by Euclid, Mr. Simfon, in his edition of the Elements, ufes the word unlimited. But it is no unufual thing with this great geometrician, to alter the words of Euclid, when they convey a philofophical meaning; as we fhall plainly evince in the courfe of thefe Commentaries. He certainly deferves the greateft praife for his zealous attachment to the ancient geometry: but he would (in my opinion) have deferved ftill more, had he been acquainted with the Greek philofophy; and fathomed the depth of Proclus; for then he would never have attempted to reftore Euclid's Elements, by depriving them of fome very confiderable beauties.

not

not be omitted. How are points called the extremities of a line? and of what line, fince they can neither be the bounds of one that is infinite, nor of every finite? For there is a certain line, which is both finite, and has not points for its extremities. And fuch is a circular line, which returns into itfelf, and is not bounded by points, like a right line. And fuch alfo is the ellipfis, or line like a fhield. Is it therefore requifite to behold a line, confidered as a line? for we muft receive a certain circumference, which is terminated by points, and a part of the elliptic line; having, in like manner, its extremities bounded by points. But every circular and eliptic line, affumes to itfelf another certain property, by which it is not line alone, but is alfo endued with a power of perfecting figure *. Lines, themfelves, therefore, have their extremities terminated by points; but thofe which are effective of fuch like figures, return into themfelves. And, indeed, if you conceive them to be defcribed, you will alfo find how they are bounded by points; but if you receive them already defcribed, and connect the end with the beginning, you can no longer behold their extremes.


DEFINITION IV.

A Right Line, is that which is equally fituated between its *bounding* Points.

PLATO, eftablifhing two moft fimple and principal fpecies of lines, the right and the circular, compofes all the reft from the mixture of thefe; I mean fuch as are called curve lines, fome of which are formed from planes; but others fubfift about folids; and whatever fpecies of curve lines are produced by the fections of folids. And it feems, indeed, that a point (if it be lawful fo to fpeak) bears an image of the one itfelf, according to Plato: for unity has no part, as he alfo fhews in the Parmenides. But, becaufe after unity itfelf there are three hypoftafes, or fubftances, bound, infinite, and that which is

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* This is doubtlefs the reafon why the proportion between a right and circular line, cannot be exactly obtained in numbers: for on this hypothefis, they muft be incommenfurable quantities; becaufe the one contains a property effentially different from the other.

mixed

mixed from thefe, the fpecies of lines, angles, and figures, which
fubfift in the nature of things originate from thence. And, indeed,
a circumference and a circular angle, and a circle among plane figures,
and a fphere among folids, are analogous to *bound*. But a right line
correfponds to *infinity*, according to all thefe; for it properly be-
longs to all, if it is beheld as exifting in each. But that which is
mixed in all thefe, is analogous to the mixt which fubfifts among
intelligibles. For lines are mixed, as thofe which are called fpirals.
And angles, as the femi-circular and cornicular *. And plane figures,
as fegments and apfides; but folids, as cones and cylinders, and others
of that kind. Bound, therefore, infinite, and that which is mixed,

<div align="right">are</div>

* The cornicular angle is that which is made from the periphery of a circle and its tangent;
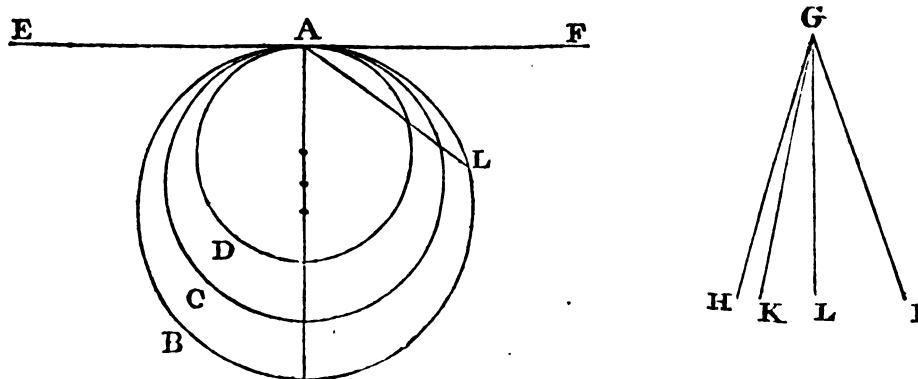
that is, the angle comprehended by the arch L A, and the right line F A, which Euclid in
(16. 3.) proves to be lefs than any right-lined angle. And from this admirable propofition it
follows, by a legitimate confequence, that any quantity may be continually and infinitely in-
creafed, but another infinitely diminifhed; and yet the augment of the firft, how great foever
it may be, fhall always be lefs than the decrement of the fecond: which Cardan demonftrates
as follows. Let there be propofed an angle of contact B A E, and an acute angle H G I.
Now if there be other leffer circles defcribed A C, A D, the angle of contact will be evidently
increafed. And if between the right lines G H, G I, there fall other right lines G K, G L,
the acute angle fhall be continually diminifhed: yet the angle of contact, however increafed, is
always lefs than the acute angle, however diminifhed. Sir Ifaac Newton likewife obferves, in
his Treatife on Fluxions, that there are angles of contact made by other curve lines, and their
tangents infinitely lefs than thofe made by a circle and right line; all which is demonftrably
certain: yet, fuch is the force of prejudice, that Mr. Simfon is of opinion, with Vieta, that
this part of the 16th propofition is adulterated; and that the fpace made by a circular line and
its tangent, is no angle. At leaft his words, in the note upon this propofition, will bear fuch
a conftruction. Peletarius was likewife of the fame opinion; but is elaborately confuted by the
excellent Clavius, as may be feen in his comment on this propofition. But all the difficulties
and paradoxes in this affair, may be eafily folved and admitted, if we confider, with our philo-

<div align="right">fopher</div>

are participated by all thefe. But Ariftotle * likewife affents to Plato; for every fpecies of lines, fays he, is either right or circular, or mixed from thefe two. From whence alfo there are three motions, one according to a right line; the other circular; and the third mixed. But fome oppofe this divifion, and fay that there are not two fimple lines alone, but that there is a certain third line given, i. e. a helix or fpiral, which is defcribed about a cylinder †, when, whilft a right line is moved round the fuperficies of the cylinder, a point in the line is carried along with an equal celerity. For by this means, a helix, or circumvolute line, is produced, which adapts all the parts of itfelf to all, according to a fimilitude of parts, as Apollonius fhews in his book concerning the Cochlea; which paffion, among all fpirals, agrees to this alone. For the parts of a plane helix are diffimilar among themfelves; as alfo of thofe which are defcribed about a cone and fphere. But the cylindric fpiral alone, confifts of fimilar parts in the fame manner as a right and circular line. Are there, then, three fimple lines, and not two only? To which doubt we reply, that a helix of this kind is, indeed, of fimilar parts, as Apollonius teaches, but is by no means fimple; fince among natural productions, gold and filver are compofed of fimilar parts, but are not fimple bodies. But the generation of the cylindric helix evinces that its mixture is from things fimple; for it originates while a right line is circularly moved round the axis of the cylinder, a point at the fame time flowing along in the right line. Two fimple motions, therefore, compofe its nature; and, on this account, it is among the number of mixt lines, and not among fuch as are fimple: for that which is compofed from diffimilars is not fimple, but mixt. Hence, Geminus, with great propriety, when he admits that fome fimple lines may be produced from many motions, does not grant that every fuch line is mixt; but that alone, which arifes from diffimilar motions. For if you conceive

fopher, that the effence of an angle does not fubfift in either quantity, quality, or inclination, taken fingly, but in the aggregate of them all. For if we regard the inclination of a circular line to its tangent, we fhall find it poffefs the property, by which Euclid defines an angle: if we refpect its participation of quantity, we fhall find it capable of being augmented and diminifhed; and if we regard it as poffeffing a peculiar quality, we fhall account for its being incommenfurable with every right-lined angle. See the Comment on the 8th Definition.

* In i. De Cælo.

† It is from this cylindric fpiral that the fcrew is formed.

a fquare, and two motions which are performed with an equal celerity, one according to the length, but the other according to the breadth, a right line or the diameter will be produced; but the right line will not, on this account, be mixed: for no other line precedes it, formed by a fimple motion, as we afferted of the cylindric helix. Nor yet, if you fuppofe a right line, moving in a right angle, and by a bifection to defcribe a circle *, is the circular line, on this account, produced with mixture: for the extremities of that which is moved after this manner, fince they are equally moved, will defcribe a right line; and the bifection, fince it is unequally devolved, will delineate a circle; but the other points will defcribe an ellipfis. On which account, the

* The prefent very obfcure paffage, may be explained by the following figure. Let A B C,
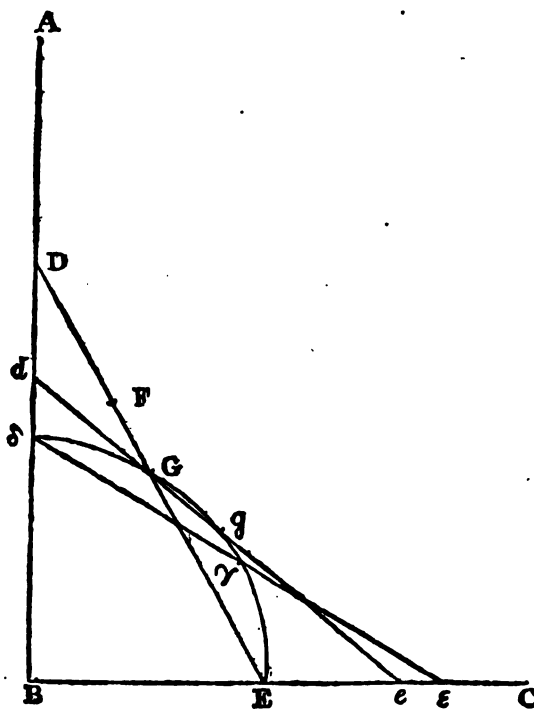
be a right-angle, and D E the line to be moved, which is bifected in G. Now, conceive it to be moved along the lines A B, B C, in fuch a manner, that the point D may always remain in A B, and the point E in B C. Then, when the line D E, is in the fituations *d e*, *δ ι*, the point G, fhall be in *g*, *γ*, and thefe points G, *g*, *γ*, fhall be in a circle. And any other point F in the line D E, will, at the fame time, defcribe an ellipfis; the greater axis being in the line A B, when the point F is between D and G; and in the line B C, when the point F is between G and E.

generation.

generation of a circular line is the confequence of that inequality of lation arifing from the bifection; becaufe a right line was fuppofed to be moved in a right angle, but not in a natural manner. And thus much concerning the generation of lines. But it feems, that of the two fimple lines, the right and the circular, the right line is the more fimple; for in this, diffimilitude cannot be conceived, even in opinion. But in the circular line, the concave and the convex, indicate diffimilitude. And a right line, indeed, does not infer a circumference according to thought; but a circumference brings with it a right line, though not according to its generation, yet with refpect to its centre. But what if it fhould be faid that a circumference requires a right line to its conftruction! For if either extreme of a right line remains fixt, but the other is moved, it will doubtlefs defcribe a circle, whofe centre will be the abiding extreme of the right line. Shall we fay that the generator of the circle is the point which is carried about the abiding point, but not the right line itfelf? For the line only determines the diftance, but the point compofes the circular line, while it is moved in a circular manner: but of this enough. Again, a circumference appears to be proximate to bound, and to have the fame proportion to other lines, as bound to the univerfality of things. For it is finite, and is alone among fimple lines perfective of figure. But a right line is proximate to infinity; for its capacity of infinite extenfion never fails: and as all the reft are produced from bound and infinite, in the fame manner from the circular and right line, every mixt genus of lines is compofed, as well of planes as of thofe which confift in folid bodies. And on this account, the foul alfo * previoufly affumed into herfelf the right and circular according to her effence, that fhe might moderate all the co-ordination of infinite, and all the nature of bound, which the world contains. By a right line, indeed, conftituting the progreffion of thefe principles into the univerfe; but by a circular line, their return to their original fource: and by the one, producing all things into multitude; but by the other, collecting them into one. And not only the foul, but he alfo who produced the foul, and endued her with thefe powers, contains in himfelf both thefe primary caufes,

* That is, the foul of the world.

For when he previoufly affumed the beginning, middle, and end of all things, he terminated right lines (fays Plato *), by a circular progreffion according to nature. And proceeding to all things by provident energies, and returning to himfelf, he eftablifhed himfelf, fays Timæus, after his own peculiar manner. But a right line is the mark or fymbol of a providence, indeclinable, incapable of perverfion, immaculate, never-failing, omnipotent, and prefent to all beings, and to every part of the univerfe. But a circumference, and that which environs, is the fymbol of an energy retiring into union with itfelf, and which rules over all things according to one intellectual bound. When, therefore, the demiurgus of the univerfe had eftablifhed in himfelf thefe two principles, the right and the circular line, and had given them dominion, he produced from himfelf two unities; the one, indeed, energizing according to the circular line, and being effective of intellectual effences; but the other according to the right line, and affording an origin to fenfible natures. But becaufe the foul is allotted a middle fituation between intellectuals and fenfibles, fo far, indeed, as fhe adheres to an intellectual nature, fhe energizes according to the circle; but fo far as fhe prefides over fenfibles, fhe provides for their welfare according to the right line : and thus much concerning the fimilitude of thefe forms to the univerfality of things. But Euclid, indeed, has properly delivered the prefent definition of a line; by which he fhews that a right line alone occupies a fpace equal to that which is fituated between its points : for as much as is the diftance of one point from another, fo great is the magnitude of the lines terminated by the points. And this is the meaning of being equally fituated between its extremes. For if you take two points in a circumference, or in any other certain line, the fpace of line which is included between thefe, exceeds their diftance from each other; and every line, befides a right one, appears to fuffer this property. Hence, according to a common conception, the vulgar alfo fay, that he who walks by a right line, performs only a neceffary journey: but that they neceffarily wander much, who do not proceed in a right line. But Plato thus defines it ; a right line is that whofe middle parts darken its

* In Timæo.

extremes.

extremes. For this paffion neceffarily attends things which have a direct pofition ; but it is not neceffary that things fituated in the circumference of a circle, or in another interval, fhould be endued with this property. Hence, the aftrologers alfo fay, that the fun then fuffers an eclipfe when that luminary, the moon, and our eye are in one right line ; for it is then darkened through the middle pofition of the moon between us and its orb. And perhaps, the paffion of a right line will evince, that in the things which are, according to proceffions emanating from caufes, the mediums are endued with a power of dividing the diftance of the extremes, and their mutual communication with each other. As alfo, according to regreffions, fuch things as are diftant from the extremes, are converted by mediums to their primary caufes. But Archimedes defines a right line the leaft of things having the fame bounds. For fince, according to Euclid, a right line is equally fituated between its points, it is on this account, the leaft of things having the fame bounds : for if a lefs line could be given, it would not lie equally between its extremes: but all the other definitions of a right line, fall into the fame conclufions ; as for inftance, that it is conftituted in its extremities, and that one part of it is not in its fubject plane, but another, in one more fublime : and that all its parts fimilarly agree to all : and that its extremes abiding, it alfo abides. Laftly, that it does not perfect figure, with one line fimilar in fpecies to itfelf : for all thefe definitions exprefs the property of a right line, which it poffeffes from the fimplicity of its effence, and from its having one progreffion the fhorteft of all from one extremity to another. And thus much concerning the definitions of a right line. But again, Geminus divides a line firft into an incompofite and compofite ; calling a compofite, that which is refracted, and forms an angle ; but all the reft of them, he denominates incompofites. Afterwards, he divides a compofite line into that which produces figure, and that which may be infinitely extended. And he calls that which produces figure, a circular line, and the line of a fhield *, and that which is fimilar to an ivy leaf † ; but that which is not effective of figure, the fection of a rectangular and obtufe angular cone, the line fimilar to a

* The ellipfis.
† The ciffoid. For the properties of this curve, fee Dr. Wallis's treatife on the cycloid, p. 81.

fhell

fhell *, the right line, and all of that kind. And again, after another manner, of the incompofite line, one fort is fimple, but the other mixt. And of the fimple, one produces figure, as the circular; but the other is indefinite, as the right line. But of the mixt, one fubfifts in planes, but the other in folids. And of that which is in planes, one coincides in itfelf, as the figure of the ivy leaf, which is called the ciffoid; but the other may be produced in infinitum, as the helix. But of that which is in folids, one may be confidered in the fections of folids; but the other as confifting about the folids themfelves. For the helix, indeed, which is defcribed about a fphere or a cone, confifts about folids; but conic, or fpirical fections are generated from a particular fection of folids. But, with refpect to thefe fections, the conic were invented by Mænechmus, which alfo Eratofthenes relating, fays,

"Nor in a cone Mænechmian ternaries divide."

But the fpiric by Perfeus, who compofed an epigram on their invention, to this purpofe, "When Perfeus had invented three fpiral lines in five fections, he facrificed to the gods on the occafion." And the three fections of a cone, are the parabola, hyperbola, and ellipfis: but of fpiral fections, one kind is twifted and involved, like the fetlock of a horfe; but another is dilated in the middle, and deficient in each extremity: and another which is oblong, has lefs fpace in the middle, but is dilated on each fide. But the multitude of the other mixt lines is infinite. For there is an innumerable multitude of folid figures, from which there are conftituted multiform fections. For a right line, while it is circularly moved †, does not make a certain determinate fuperficies, nor yet conical, nor conchoidal lines, nor circumferences themfelves. Hence, if thefe folids are multifarioufly cut, they will exhibit various fpecies of lines. Laftly, of thofe lines which confift about folids, fome are of fimilar parts, as the helixes

* The conchoid.

† Thus, a right line, when confidered as the fide of a parallelogram, moving circularly, generates a cylindrical fuperficies: when moving circularly, as the fide of a triangle, a conical furface; and fo in other lines, the produced fuperficies varying according to the different pofitions of their generative lines.

about

about a cylinder; but others of diffimilar parts, as all the reft. From thefe divifions, therefore, we may collect, that there are only three lines of fimilar parts, the right, the circular, and the cylindric helix. The two fimple ones, indeed, exifting in a plane, but the one mixt, about a folid. And this Geminus evidently demonftrates, when he fhews, that if two right lines are extended from one point, to a line of fimilar parts, fo as to make equal angles upon that line, they fhall be equal to each other. And the demonftrations of this may be received by the ftudious, from his volumes; fince in thefe he delivers the origin of fpiral, conchoidal, and ciffoidal lines. But we have barely related the names and divifions of thefe lines, for the purpofe of exciting the ingenious to their inveftigation; as we think, that an accurate enquiry after the method of detecting the properties of each, would be fuperfluous in the prefent undertaking: fince the geometrician only unfolds to us in this work, fimple and primary lines, i e. the right line, in the prefent definition; but a circular line, in the tradition of a circle. For he then fays, that the line terminating the circle, is the circumference. But he makes no mention of mixt lines, though he was well acquainted with mixt angles, I mean, the femicircular and cornicular: as alfo with plane mixt figures, i. e. fegments and fectors; and with folids, viz. cones and cylinders. Of each of the reft, therefore, he delivers three fpecies; but of lines only two, i. e. the right and circular: for he thought it requifite in difcourfes concerning things fimple, to affume fimple fpecies; and all the reft are more compofite than lines. Hence, in imitation of the geometrician, we alfo fhall terminate their explanation with fimple lines.

DEFINITION V.

A SUPERFICIES is that which has only Length and Breadth.

AFTER a point and a line, a fuperficies is placed, which is diftant by a twofold interval, length and breadth. But this alfo remaining deftitute of thicknefs or bulk, poffeffes a nature more fimple than body, which is diftant by a triple dimenfion. On which account the geometrician adds to the two intervals the particle *only*, because

becaufe the third interval does not exift in fuperficies. And this is equivalent to a negation of bulk, as here alfo he fhews the excellency of fuperficies compared to a folid with refpect to fimplicity, by negation, or by an addition equivalent to negation : but the diminution which it poffeffes, if compared with the preceding terms, by the affirmations themfelves. But others define a fuperficies to be the boundary of body, which is almoft affirming the fame as the definition of Euclid; fince that which terminates is exceeded in one dimenfion, by that which is terminated. And others, a magnitude different by two intervals. Laftly, others declaring the fame affection, form its affignation in a fomewhat different manner. But they fay we have a knowledge of fuperficies when we meafure fields, and diftinguifh their extremities according to length and breadth; but that we receive a certain fenfation of it, when we behold fhadows. For as they are without bulk, becaufe they cannot penetrate into the interior part of the earth, they have only length and breadth. But the Pythagoreans fay, that it is affimilated to the triad; becaufe the ternary is by far the firft caufe to all the figures; which a fuperficies contains. For a circle, which is the principle of orbicular figures, occultly poffeffes the ternary, by its centre, interval, and circumference. But a triangle, which ranks as the firft among all right-lined figures, on every fide evinces that it is enclofed by the triad, and receives its form from its perfect nature.

DEFINITION VI.

The Extremities of a SUPERFICIES are Lines.

FROM thefe alfo, as images, we may underftand, that things more fimple procure bound and an end to every one of their proximate natures: for foul perfects and determines the operations of nature; and nature the motion of bodies. And prior to thefe, intellect meafures the convolutions of foul; and unity the life of intellect; for that is the meafure of all. Juft as in thefe alfo, a folid is terminated by a fuperficies; but a fuperficies by a line; and a line by a point; for that is the boundary of them all. Hence, the line exift-

ing

ing uniformly in immaterial forms and impartible reafons, terminates and reftrains the various motion of a fuperficies in its progreffion, and proximately unites its infinity. But in the images of thefe, when that which bounds fupervenes that which is bounded, it caufes, by this means, its limitation and bound. But if it fhould be enquired how lines are the extremities of every fuperficies, fince they are not the extremes of every finite figure; for the fuperficies of a fphere is terminated indeed, yet not by lines, but by itfelf? In anfwer to this, we muft fay, that by receiving a fuperficies fo far as it is diftant by a two-fold interval, we fhall find it terminated by lines according to length and breadth. But if we behold a fpherical fuperficies, we muft receive it as that which is endued with figure; which poffeffes another quality, and conjoins the end with the beginning; and lofes its two extremities in the comprehenfive embraces of one: and this one extremity fubfifts in capacity only, and not in energy.

DEFINITION VII.

A Plane SUPERFICIES is that which is equally fituated between its bounding Lines.

IT was not agreeable to the ancient philofophers to eftablifh a plane fpecies of fuperficies; but they confidered fuperficies in general, as the reprefentative of magnitude, which is diftant by a two-fold interval. For thus the divine Plato * fays, that geometry is contemplative of planes, oppofing it in divifion to ftereometry, as if a plane and a fuperficies were the fame. And this was likewife the opinion of the demoniacal Ariftotle †. But Euclid and his followers confider fuperficies as a genus, but a plane as its fpecies, in the fame manner as rectitude of a line. And on this account he defines a plane feparate from a fuperficies, after the fimilitude of a right line. For he defines this laft as equal to the fpace, placed between its points. And in like manner, he fays, that two right lines being given, a plane fuperficies occupies a place equal to the fpace fituated between thofe two lines.

* Inv ii. De Rep.   † In multis locis.

For this is equally fituated between its lines; and others alfo explaining the fame boundary, affert that it is conftituted in its extremities. But others define it as that to all the parts of which a right line may be adapted *. But perhaps others will fay, that it is the fhorteft of fuperficies, having the fame boundaries; and that its middle parts darken its extremities; and that all the definitions of a right line may be transferred into a plane fuperficies, by only changing the genus: fince a right, circular, and mixt line, commencing from lines, arrive even at folids, as we have afferted above; for they are proportionally, both in fuperficies and folids. Hence alfo, Parmenides fays, that every figure is either right, or circular, or mixt. But if you wifh to confider the right in fuperficies, take a plane, to which a right line agrees in various ways; but if a circular receive a fpherical fuperficies; and if a mixt, a conic or cylindric, or fome one of that genus. But it is requifite (fays Geminus) fince a line, and alfo a fuperficies is called mixt, to know the meafure of mixture, becaufe it is various. For mixture in lines, is neither by compofition, nor by temperament only: fince, indeed, a helix is mixed, yet one part of it is not ftraight, and another part circular, like thofe things which are mixed by compofition: nor if a helix is cut after any manner, does it exhibit an image of things fimple, fuch as thofe which are mixed through temperament; but in thefe the extremes are, at the fame time, corrupted and confufed. Hence, Theodorus the mathematician, does not rightly perceive, in thinking that this mixture is in lines. But mixture in fuperficies, is neither by compofition, nor by confufion; but fubfifts rather by a certain temperament. For conceiving a circle in a fubject plane, and a point on high, and producing a right line from the point to the circumference of the circle, the revolution of this line will produce a conical fuperficies which is mixt. And we again refolve it into its fimple elements, by a parallel fection: for by drawing

* This definition is the fame with that which Mr. Simfon has adopted inftead of Euclid's, expreffed in different words: for he fays, "a plane fuperficies is that in which any two points being taken, the ftraight line between them lies wholly in that fuperficies." But he does not mention to whom he was indebted for the definition; and this, doubtlefs, becaufe he confidered it was not worth while to relate the trifles of Proclus at full length: for thefe are his own words, in his note to propofition 7, book i. Nor has he informed us in what refpect Euclid's definition is indiftinct.

a section between the vertex and the base, which shall cut the plane of the generative right line, we effect a circular line. But the idea of lines, shews that the mode of mixture is not by temperament; for neither does it send us back to the simple nature of elements : on the contrary, when superficies are cut, they immediately exhibit to us their producing lines The mode of mixture, therefore, is not the same in lines and superficies. But as among lines there were some simple, that is, the right and circular, of which the vulgar also possess an anticipated knowledge without any previous instruction ; but the species of mixt lines require a more artificial apprehension: so among superficies, we possess an innate notion of those which are especially elementary, the plane and spherical; but science and its reason investigates the variety of those which are composed through mixture. But this is an admirable property of superficies, that their mixture in generation is oftentimes produced from a circular line; and this also happens to a spiral superficies. For this is understood by the revolution of a circle remaining erect, and turning itself about the same point which is not its centre. And on this account, a spiral also is threefold ; for its centre is either in a circumference, or within, or external to a circumference. If the centre is in the circumference, a continued spiral is produced : if within the circumference, an intangled one ; if without, a divided one. And there are three spiral sections corresponding to these three differences. But every spiral line is mixt, although the motion from which it is produced is one and circular. And mixt superficies are produced as well from simple lines, (as we have said,) while they are moved with a motion of this kind, as from mixt lines. Since, therefore, there are three conic lines, they produce four mixt superficies, which they call conoids. For a rectangular conoid, is produced from the revolution of the parabola about its axis : but that which is formed by the ellipsis, is called a spheroid ; and if the revolution is made about the greater axis, it is an oblong ; but if about the lesser a broad spheroid. Lastly, an obtuse-angled conoid is generated from the revolution of the hyperbola But it is requisite to know, that sometimes we arrive at the knowledge of superficies from lines, and sometimes the contrary ; for from conical and spiral superficies, we apprehend conical and spiral lines. Be-

sides,

fides, this alſo muſt be previouſly received concerning the difference of lines and ſuperficies, that there are three lines of ſimilar parts (as we have already obſerved), but only two ſuperficies, the plane and the ſpherical. For this is not true of the cylindric, ſince all parts of the cylindric ſuperficies cannot agree to all. And thus much concerning the differences of ſuperficies, one of which the geometrician having choſen (I mean the plane), this alſo he has defined ; and in this, as a ſubjeǮ, he contemplates figures, and their attendant paſſions : for his diſcourſe is more copious in this than in other ſuperficies : ſince, indeed, we may underſtand right lines, and circles, and helixes in a plane; alſo the ſeǮions of circles and right lines, contaǮs, and appli- eations, and the conſtruǮions of angles of every kind. But in other ſuperficies, all theſe cannot be beheld. For how in one that is ſphe- rical, can we apprehend a right line, or a right-lined angle ? How, laſtly, in a conic or cylindric ſuperficies, can we behold ſeǮions of circles or right lines ? Not undeſervedly, therefore, does he both define this ſuperficies, and diſcuſs his geometrical concerns, by exhi- biting every thing in this as in a ſubjeǮ ; for from hence he calls the preſent treatiſe plane. And, after this manner, it is requiſite to under- ſtand that which is plane, as projeǮed and conſtituted before the eyes : but cogitation as deſcribing all things in this, the phantaſy corre- ſponding to a plane mirror, and the reaſons reſident in cogitation as dropping their images * into its ſhadowy receptacle.

DEFINITION VIII.

† A PLANE ANGLE, is the inclination of two Lines to each other in a Plane, which meet together, but are not in the ſame direction.

SOME of the ancient philoſophers, placing an angle in the pre- dicament of relation, have ſaid, that it is the mutual inclination of lines or planes to each other. But others, including this in quality,

as

* In the Greek εικοιας, but it ſhould doubtleſs be read εικοιας, *images*, as in the tranſlation of Barocius.

† Mr. Simſon, in his note on this definition, ſuppoſes it to be the addition of ſome leſs ſkilful editor ; on which account, and becauſe it is quite uſeleſs (in his opinion) he diſtinguiſhes

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as well as rectitude and obliquity, fay, that it is a certain paffion of a fuperficies or a folid. And others, referring it to quantity, confefs that it is a fuperficies or a folid. For the angle which fubfifts in fuperficies is divided by a line ; but that which is in folids, by a fuperficies. But (fay they) that which is divided by thefe, is no other than magnitude, and this is not linear, fince a line is divided by a point ; and therefore it follows that it muft be either a fuperficies or a folid. But if it is magnitude, and all finite magnitudes of the fame kind have a mutual proportion ; all angles of the fame kind, i. e. which fubfift in fuperficies, will have a mutual proportion. And hence, the cornicular will be proportionable to a right-lined angle. But things which have a mutual proportion, may, by multiplication, exceed each other ; and therefore it may be poffible for the cornicular to exceed a right-lined angle, which, it is well known, is impoffible, fince it is fhewn to be lefs than every right-lined angle. But if it is quality alone, like heat and cold, how is it divifible into equal parts ? For equality, inequality, and divifibility, are not lefs refident in angles than in magnitudes ; but they are, in like manner, effential. But if the things in which thefe are effentially inherent, are quantities, and not qualities, it is manifeft that angles alfo are not qualities. Since the more and the lefs are the proper paffions of quality *, but not equal and unequal. On this hypothefis, therefore, angles ought not to be called unequal, and this greater, but the other lefs ; but they ought to be denominated diffimilars, and one more an angle, but the other lefs. But that thefe appellations are foreign from the effence of mathematical concerns, is obvious to every one : for every angle receives the fame definition, nor is this more an angle, but that lefs. Thirdly, if an angle is inclination, and belongs to the category of

it from the reft by inverted double commas. But it is furely ftrange that the definition of angle in general fhould be accounted ufelefs, and the work of an unfkilful geometrician. Such an affertion may, indeed, be very fuitable to a profeffor of experimental philofophy, who confiders the ufeful as infeparable from practice ; but is by no means becoming a reftorer of the liberal geometry of the ancients. Befides, Mr. Simfon feems continually to forget that Euclid was of the Platonic fect ; and confequently was a philofopher as well as a mathematician. I only add, that the commentary on the prefent definition is, in my opinion, remarkably fubtle and accurate, and well deferves the profound attention of the greateft geometricians.

* For a philofophical difcuffion of the nature of quality and quantity, confult the Commentaries of Ammonius, and Simplicius on Ariftotle's Categories, Plotinus on the genera of beings, and Mr. Harris's Philofophical Arrangements.

relation,

relation, it muft follow, that from the exiftence of one inclination, there will alfo be one angle, and not more than one.   For if it is no-thing clfe than the relation of lines or planes, how is it poffible there can be one relation of lines or planes, but many angles ? If, therefore, we conceive a cone cut by a triangle from the vertex to the bafe, we fhall behold one inclination of the triangular lines in the femicone to the vertex; but two diftinct angles: one of which is plane, I mean that of the triangle ; but the other fubfifts in the mixt fuperficies of the cone, and both are comprehended by the two triangular lines. The relation, therefore, of thefe, do not make the angle.   Again, it is neceffary to call an angle either quality or quantity, or relation; for figures, indeed, are qualities, but their mutual proportions belong to relation.   It is neceffary, therefore, that an angle fhould be reduced under one of thefe three genera.   Such doubts, then, arifing concern-ing an angle, and Euclid calling it inclination, but Apollonius the collection of a fuperficies, or a folid in one point, under a refracted line or fuperficies (for he feems to define every angle univerfally), we fhall affirm, agreeable to the fentiments of our preceptor Syrianus, that an angle is of itfelf none of the aforefaid ; but is conftituted from the concurrence of them all.   And that, on this account, a doubt arifes among thofe who regard one category alone.   But this is not peculiar to an angle, but is likewife the property of a triangle.   For this, too, participates of quantity, and is called equal and unequal; becaufe it has to quantity the proportion of matter.   But quality alfo, is prefent with this, in confequence of its figure (fince triangles are called as well fimilar as equal) ; but it poffeffes this from one category, and that from another.   Hence, an angle is perfectly indigent of quantity, the fubject of magnitude.   But it is alfo indigent of quality, by which it poffeffes, as it were, its proper form and figure.   Laftly, it is indigent of the relation of lines terminating, or of fuperficies comprehending its form.   So that an angle confifts from all thefe, yet is not any one of them in particular.   And it is indeed divifible, and capable of receiving equality and inequality, according to the quantity which it contains.   But it is not compelled to admit the proportion of magnitudes of the fame kind, fince it has alfo a peculiar quantity, by which angles are alfo incapable of a comparifon with each other.   Nor

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can one inclination perfect one angle: since the quantity also, which is placed between the inclined lines, completes its essence. If then we regard these distinctions, we shall dissolve all absurdities, and discover that the property of an angle is not the collection of a superficies or solid, according to Apollonius (since these also complete its essence,) but that it is nothing else than a superficies itself, collected into one point, and comprehended by inclined lines, or by one line inclined to itself: and that a solid angle is the collection of superficies mutually inclined to each other. Hence, we shall find that a formed quantum, constituted in a certain relation, supplies its perfect definition. And thus much we have thought requisite to assert concerning the substance of angles, previously contemplating the common essence of every triangle, before we divide it into species. But since there are three opinions of an angle, Eudemus the Peripatetic, who composed a book concerning an angle, affirms that it is quality. For, considering the origin of an angle, he says that it is nothing else than the fraction of lines: because, if rectitude is quality, fraction also will be quality. And hence, since its generation is in quality, an angle will be entirely quality. But Euclid, and those who call it inclination, place it in the category of relation. But they call it quantity, who say that it is the first interval under a point, that is immediately subsisting after a point. In the number of which is Plutarch, who constrains Apollonius also into the same opinion. For it is requisite (says he) there should be some first interval, under the inclination of containing lines or superficies. But since the interval, which is under a point, is continuous, it is not possible that a first interval can be assumed; since every interval is divisible in infinitum. Besides, if we any how distinguish a first interval, and through it draw a right line, a triangle is produced, and not one angle. But Carpus Antiochenus says, that an angle is quantity, and is the distance of its comprehending lines, or superficies; and that this is distant by one interval, and yet an angle is not on that account a line: since it is not true that every thing which is distant by only one interval, is a line. But this surely is the most absurd of all, that there should be any magnitude except a line, which is distant only by one interval. And thus much concerning the nature of an angle. But with respect to the division of

<div align="right">angles,</div>

angles, fome confift in fuperficies, but others in folids. And of thofe which are in fuperficies, fome are in fimple ones, but others in fuch as are mixt. For an angle may be produced in a cylindric, conic, fpherical, and plane fuperficies. But of thofe which confift in fimple fuperficies, fome are conftituted in the fpherical; but others in the plane. For the zodiac itfelf forms angles, dividing the equinoctial in two parts, at the vertex of the cutting fuperficies. And angles of this kind fubfift in a fpherical fuperficies. But of thofe which are in planes, fome are comprehended by fimple lines, others by mixt ones; and others, again, by both. For in the fhield-like figure *, an angle is comprehended by the axis, and the line of the fhield: but one of thefe lines is mixt, and the other fimple. But if a circle cuts the fhield, the angle will be comprehended by the circumference, and the ellipfis. And when ciffoids, or lines fimilar to an ivy leaf, clofing in one point like the leaves of ivy (from whence they derive their appellation) make an angle, fuch an angle is comprehended by mixt lines. Alfo, when the hippopeda, or line fimilar to the foot of a mare, which is one of the fpirals, inclining to another line, forms an angle, it is comprehended by mixt lines. Laftly, the angles contained by a circumference and a right line, are comprehended by fimple lines. But of thefe again, fome are contained by fuch as are fimilar in fpecies, but others by fuch as are diffimilar. For two circumferences, mutually cutting, or touching each other, produce angles: and thefe triple, for they are either on both fides convex, when the convexities of the circumferences are external: or on both fides concave, when both the concavities are external; which they call fiftroides; or mixt from convex and concave lines, as the lines called lunulas. But befides this, angles are contained in a twofold manner, by a right line and a circumference: for they are either contained by a right line, and a concave circumference, as the femicircular angle; or by a right line and a convex circumference, as the cornicular angle. But all thofe which are comprehended by two right lines, are called rectilinear angles, which have likewife a triple difference †. The geometrician, therefore, in the prefent hypothefis, defines all thofe angles which are conftituted in plane fuperficies, and gives them the common name of

* That is, the ellipfis.]   † That is, they are either right, acute, or obtufe.

a plane angle. And the genus of thefe he denominates inclination: but the place, the plane itfelf, for angles have pofition: but their origin fuch, that it is requifite there fhould be two lines at leaft, and not three as in a folid. And that thefe fhould touch each other, and by touching, muft not lie in a right line, as an angle is the inclination and comprehenfion of lines: but is not diftance only, according to one interval. But if we examine this definition, in the firft place it appears that it does not admit, an angle can be perfected by one line; though a ciffoid, which is but one, perfects an angle. And, in like manner, the hippopeda. For we call the whole a ciffoid, and not its portions (left any one fhould fay, that the conjunction of thefe forms an angle) and the whole a fpiral, but not its parts. Each, therefore, fince it is one, forms an angle to itfelf, and not to another. But after this, he is faulty, in defining an angle to be inclination. For how, on this hypothefis, will there be two angles, from one inclination? How can we call angles equal and unequal? And whatever elfe is ufually objected againft this opinion. Thirdly, and laftly, that part of the definition, which fays, *and not placed in a right line*, is fuperfluous in certain angles, as in thofe which are formed from orbicular lines. For without the affiftance of this part, the definition is perfect; fince the inclination of one of the lines to the other, forms the angle. And it is not poffible that orbicular angles fhould be placed in a right line. And thus much we have thought proper to fay concerning the definition of Euclid; partly, indeed, interpreting, and partly doubting its truth.

DEFINITION IX.

But when the Lines containing the Angle, are right, the Angle is called RECTILINEAR.

AN angle is the fymbol and image of the connection and compreffion, which fubfifts in the divine genera, and of that order which collects divifibles into one, partibles into an impartible nature, and the many into conciliating community. For it is the bond of a multitude of lines and fuperficies, the collector of magnitude into the

impartibilty

impartibility of points, and the comprehender of every figure which is compofed by its confining nature. On which account, the oracles * call the angular junctions of figures, knots, fo far as they bring with them an image of connecting union, and divine conjunctions, by which difcrete natures mutually cohere with each other. The angles, there-fore, fubfifting in fuperficies, exprefs the more immaterial, fimple, and perfect unions which fuperficies contain : but thofe which are in folids, reprefent the unions, which proceed even to inferiors, and fupply a community to things disjunct, and a conftruction of the fame nature, to things which on every fide receive a perfect partition. But of the angles in fuperficies, fome fhadow forth primary and unmixt unions; but others, fuch as comprehend in themfelves, an infinity of progreffions. And fome, indeed, are the fources of union to intel-lectual forms; but others, to fenfible reafons; and others, again, are copulative of thofe forms which obtain between thefe, a middle fitua-tion. Hence, the angles which are made from circumferences, imitate thofe caufes which envelop intellectual variety in coercive union; for circumferences, haftening to coalefce with each other, are images of intellect, and intellectual forms. On the contrary, rectilineal angles, are the fymbols of thofe unions which prefide over fenfibles, and afford a conjunction of the reafons fubfifting in thefe : but mixt angles reprefent the prefervers of the communion, as well of fenfible, as of intellectual forms, according to one immoveable union. It is requifite, therefore, by regarding thefe paradigms, or exemplars, to render the caufes of each. For among the Pythagoreans we fhall find various angles dedicated to various gods. Thus, Philolaus, confecrates to fome a triangular, but to others a quadrangular angle ; and to others, again, different angles. Likewife, he permits the fame to many gods, and many to the fame god, according to the different powers which they contain. And with a view to this, and to the demiurgic triangle, which is the primary caufe of all the ornament of the elements, it appears to me, that Theodorus Afinæus the philofopher, conftitutes fome of the gods, according to fides ; but others, according to angles. The firft, indeed, fupplying progreffion and power; but the fecond, the conjunction of the univerfe, and the collection of progreffive na-

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* This oracle is not mentioned by any of the collectors of the Zoroaftrian oracles.

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tures again into one. But thefe, indeed, direct us to the knowledge of the things which are. And we muft not wonder that lines are here faid to contain an angle. For the one and impartible nature which is found in thefe, is adventitious: but in the gods themfelves, and in true beings, the whole, and impartible good, precedes things many, and divided.

DEFINITION X.

When a Right Line ftanding on a Right Line, makes the fucceffive Angles on each fide equal to one another, each of the equal Angles, is a Right Angle; and the infifting Right Line, is called a PERPENDICULAR to that upon which it ftands.

DEFINITION XI.

An OBTUSE ANGLE is that which is greater than a RIGHT ANGLE.

DEFINITION XII.

But an ACUTE ANGLE, is that which is lefs than a RIGHT ANGLE.

THESE are the triple fpecies of angles, which Socrates fpeaks of in the Republic, and which are received by geometricians from hypothefis; a right-line conftituting thefe angles, according to a divifion into fpecies; I mean, the right, the obtufe, and the acute. The firft of thefe being defined by equality, identity and fimilitude; but the others being compofed through the nature of the greater and the leffer; and laftly, through inequality and diverfity, and through the more and the lefs, indeterminately affumed. But many geome-, tricians, are unable to render a reafon of this divifion, and ufe the, affertion, that there are three angles, as an hypothefis *. So that,

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* This, indeed, muft always be the cafe with thofe geometricians, who are not at the fame time, philofophers; a conjunction no lefs valuable than rare. Hence, from their ignorance of principles and intellectual concerns, when any contemplative enquiry is propofed, they immediately afk, in what its utility confifts; confidering every thing as fuperfluous, which does not contribute to the folution of fome practical problem.

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when we interrogate them concerning its cause, they answer, this is
not to be required of them as geometricians. However, the Pytha-
goreans, referring the solution of this triple distribution to principles,
are not wanting in rendering the causes of this difference of right-
lined angles. For, since one of the principles subsists according to
bound, and is the cause of limitation, identity, and equality, and lastly,
of the whole of a better co-ordination; but the other is of an infinite
nature, and confers on its progeny, a progression to infinity, increase,
and decrease, inequality, and diversity of every kind, and entirely
presides over the worse series; hence, with great propriety, since the
principles of a right-lined angle are constituted by these, the reason
proceeding from bound, produces a right angle, one, with respect to
the equality of every right angle, endued with similitude, always
finite and determinate, ever abiding the same, and neither receiving
increment nor decrease. But the reason proceeding from infinity,
since it is the second in order, and of a dyadic nature, produces two-
fold angles about the right angle, distinguished by inequality, accord-
ing to the nature of the greater and the lesser, and possessing an in-
finite motion, according to the more and the less, since the one becomes
more or less obtuse; but the other more or less acute. Hence, in
consequence of this reason, they ascribe right angles to the pure and
immaculate gods of the divine ornaments, and divine powers which
proceed into the universe, as the authors of the invariable providence
of inferiors; for rectitude, and an inflexibility and immutability to
subordinate natures accords with these gods: but they affirm, that the
obtuse and acute angles should be ascribed to the gods, who afford
progression, and motion, and a variety of powers. Since obtuseness
is the image of an expanded progression of forms; but acuteness pos-
sesses a similitude to the cause dividing and moving the universe. But
likewise, among the things which are, rectitude is, indeed, similar to
essence, preserving the same bound of its being; but the obtuse and
acute, shadow forth the nature of accidents. For these receive the
more and the less, and are indefinitely changed without ceasing.
Hence, with great propriety, they exhort the soul to make her descent
into generation, according to this invariable species of the right angle,
by not verging to this part more than to that; and by not affecting
. some

fome things mote, and others lefs. For the diftribution of a certain
convenience and fympathy of nature, draws it down into material
error, and indefinite variety *. A perpendicular line is, therefore,
the fymbol of inflexibility, purity, immaculate, and invariable power,
and every thing of this kind. But it is likewife the fymbol of divine
and intellectual meafure: fince we meafure the altitudes of figures by
a perpendicular, and define other rectilineal angles by their relation
to a right angle, as by themfelves they are indefinite and indeterminate.
For they are beheld fubfifting in excefs and defect, each of which is,
by itfelf, indefinite. Hence they fay, that virtue alfo ftands accord-
ing to rectitude; but that vice fubfifts according to the infinity of the
obtufe and acute, that it produces excefles and defects, and that the
more and the lefs exhibit its immoderation, and inordinate nature.
Of rectilineal angles, therefore, we muft eftablifh the right angle, as
the image of perfection, and invariable energy, of limitation, intel-
lectual bound, and the like; but the obtufe and acute, as fhadowing
forth infinite motion, unceafing progreffion, divifion, partition and
infinity. And thus much for the theological fpeculation of angles.
But here we muft take notice, that the genus is to be added to the
definitions of an obtufe and acute angle; for each is right-lined, and
the one is greater, but the other lefs than a right-angle. But it is not
abfolutely true, that every angle which is lefs than a right one, is
acute. For the cornicular is lefs than every right-angle, becaufe lefs
than an acute one, yet is not on this account an acute angle. Alfo,
a femi-circular is lefs than any right-angle, yet is not acute. And the
caufe of this property is becaufe they are mixt, and not rectilineal
angles. Befides, many curve-lined angles appear greater than right-
lined angles, yet are not on this account obtufe; becaufe it is requifite
that an obtufe fhould be a right lined angle. Secondly, as it was the
intention of Euclid, to define a right-angle, he confiders a right line
ftanding upon another right-line, and making the angles on each fide
equal. But he defines an obtufe and acute angle, not from the inclin-
ation of a right line to either part, but from their relation to a right-

angle.

angle. For this is the meafure of angles deviating from the right, in the fame manner as equality of things unequal. But lines inclined to either part, are innumerable, and not one alone, like a perpendicular. But after this, when he fays, *(the angles equal to one another)* he exhibits to us a fpecimen of the greateft geometrical diligence; fince it is poffible that angles may be equal to others, without being right. But when they are equal to one another, it is neceffary they fhould be right. Befides, the word *fucceffive* appears to me not to be added fuperfluoufly, as fome have improperly confidered it; fince it exhibits the reafon of rectitude. For it is on this account that each of the angles is right; becaufe, when they are *fucceffive*, they are equal. And, indeed, the infifting right-line, on account of its inflexibility to either part, is the caufe of equality to both, and of rectitude to each. The caufe, therefore, of the rectitude of angles, is not abfolutely mutual equality, but pofition in a confequent order, together with equality. But, befides all this, I think it here neceffary to call to mind, the purpofe of our author; I mean, that he difcourfes in this place, concerning the angles confifting in one plane. And hence, this definition is not of every perpendicular; but of that which is in one and the fame plane. For it is not his prefent defign to define a folid angle. As, therefore, he defines, in this place, a plane angle, fo likewife a perpendicular of this kind. Becaufe a folid perpendicular ought not to make right angles to one right-line only; but to all which touch it, and are contained in its fubject plane : for this is its neceffary peculiarity.

DEFINITION   XIII.

A BOUND is that which is the Extremity of any thing *.

A Bound, in this place, is not to be referred to all magnitudes, for there is a bound and extremity of a line ; but to the fpaces which are contained in fuperficies, and to folid bodies. For he now calls a bound, the ambit which terminates and diftinguifhes every

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* This Definition too, is marked by Mr. Simfon with inverted commas, as a fymbol of its being interpolated. But for what reafon I know not, unlefs becaufe it is ufelefs, that is, becaufe it is *philofophical!*

fpace.

ſpace. And a bound of this kind, he defines to be an extremity: but not after the manner in which a point is called the extremity of a line, but according to its property of including and excluding from circumjacent figures. But this name is proper to geometry in its infant ſtate, by which they meaſured fields, and preſerved their boundaries diſtinct and without confuſion, and from which they arrived at the knowledge of the preſent ſcience. Since, therefore, Euclid calls the external ambit, a bound, it is not without propriety that he, by this means, defines the extremity of ſpaces. For by this, every thing comprehended is circumſcribed. I ſay, for example, in a circle, its bound and extremity is the circumference; but itſelf, a certain plane ſpace: and ſo of the reſt.

DEFINITION XIV.

A FIGURE is that which is comprehended by one or more Boundaries.

BECAUSE figure is predicated in various ways, and is divided into different ſpecies, it is requiſite, in the firſt place, to behold its differences; and afterwards to diſcourſe concerning that figure which is propoſed in this Definition. There is, then, a certain figure which is conſtituted by mutation, and is produced from paſſion, while the recipients of the figure are diſturbed, divided, or taken away; while they receive additions, or are altered, or ſuffer other various affections. There is alſo a figure, which is produced by the potter's, or ſtatuary's art, according to the pre-exiſtent reaſon, which art itſelf contains: art, indeed, producing the form, but matter receiving from thence, form, and beauty, and elegance. But there are ſtill more noble and more illuſtrious figures than theſe, the ſkilful operations of nature. Some, indeed, exiſting in the elements under the moon *, and having a power of comprehending the reaſons thoſe elements contain: but others are ſituated in the celeſtial regions, diſtinguiſhing their powers, and endleſs revolutions. For the heavenly bodies, both when conſidered by themſelves, and with relation to each other, exhibit an abundant and admirable variety of figures; and at different times they preſent to our view different forms, bringing with them a ſplendid image of intellec-

* That is, the various ſpecies of forms, with which the four elements are replete.

tual

tual species; and, by their elegant and harmonious revolutions, de-
scribing the incorporeal and immaterial powers of figures. But there
are, again, befides all thefe, moft pure and perfect beauties, the figures
of fouls, which, becaufe they are full of life, and felf-motive, have
an exiftence prior to things moved by another; and which, becaufe
they fubfift immaterially, and without any dimenfion, excel the forms
which are endued with dimenfion and matter. In the nature of which
we are inftructed by Timæus, who has explained to us the demiurgic,
and effential figure of fouls *. But again, the figures of intellects are
by far more divine than the figures of fouls; for thefe, on every fide,
excel partible effences; are every where refplendent with impartible
and intellectual light; are prolific, effective, and perfective of the
univerfe; are equally prefent, and firmly abide in all things; and
procure union to the figures of fouls; but recall the mutation of fenfible
figures to the limitation of their proper bound. Laftly, there are,
feparate from all thefe, thofe perfect, uniform, unknown, and in-
effable figures of the gods, which are refident, indeed, in the figures
of intellects; but jointly terminate all figures, and comprehend all
things in their unifying boundaries. The properties of which the
theurgic art, alfo expreffing, furrounds various refemblances of the
gods, with various figures. And fome, indeed, it fafhions by charac-
ters, in an ineffable manner; for characters of this kind, manifeft the
unknown powers of the gods: but others it imitates by forms and
images; fafhioning fome of them erect, and others fitting; and fome
fimilar to a heart, but others fpherical, and others expreffed by dif-
ferent figures. And again, fome it fabricates of a fimple form; but
others it compofes from a multitude of forms; and fome are facred and
venerable; but others are domeftic, exhibiting the peculiar gentlenefs
of the gods. And fome it conftructs of a fevere afpect; and laftly,
attributes to others, different fymbols, according to the fimilitude and
fympathy pertaining to the gods †. Since, therefore, figure derives

<div align="right">its</div>

* That is, the circle.

† An admirer of the moderns, and their purfuits, will doubtlefs confider all this as the
relics of heathen fuperftition and ignorance; and will think, perhaps, he makes a great con-
ceffion in admitting the exiftence of one fupreme god, without acknowledging a multitude of
deities fubordinate to the firft. For what the ancients can urge in defence of this obfolete
opinion, I muft beg leave to refer the reader to the differtation prefixed to my tranflation of
Orpheus; in addition to which let him attend to the following confiderations. Is it poffible
<div align="right">that</div>

its origin from the gods themfelves, it arrives, by a gradual pro-
greffion, even to inferiors, in thefe alfo appearing from primary
caufes. Since it is requifite to fuppofe the perfect before the imper-
fect, and things fituated in the ftability of their own effence, prior to
thofe which fubfift in others, and previous to things full of their own
privation, fuch as preferve their proper nature fincere. Such figures,
therefore, as are material, participate of material inelegance, and do
not poffefs a purity convenient to their nature. But the celeftial
figures are divifible, and fubfift in others. And the figures of fouls
are endued with divifion, and variety, and involution of every kind;
but the figures of intellects, together with immaterial union, poffefs
a progreffion into multitude. And laftly, the figures of the gods are
free, uniform, fimple, and generative; they fubfift before all things,
containing all perfection in themfelves, and extending from themfelves
to all things, the completion of forms. We muft not, therefore, liften
to, and endure the opinions of many, who affirm, that certain addi-
tions, ablations, and alterations, produce fenfible figures, (for motions,
fince they are imperfect, cannot poffefs the principle and primary
caufe of effects; nor could the fame figures often be produced from
contrary motions; for the fame form is fometimes generated from
addition and detraction,) but we muft confider operations of this kind
as fubfervient to other purpofes in generation, and derive the perfection

that the machinery of the gods in Homer could be fo beautiful, if fuch beings had no exiftence ?
Or can any thing be beautiful which is deftitute of all reality ? Do not things univerfally pleafe
in proportion as they refemble reality ? Perhaps it will be anfwered, that the reverfe of this
is true, and that fiction more generally pleafes than truth, as is evident from the great avidity
with which romances are perufed. To this I reply, that fiction itfelf ceafes to be pleafing,
when it fuppofes abfolute impoffibilities: for the exiftence of genii and fairies cannot be proved
impoffible; and thefe compofe all the marvellous of romance. This obfervation is verified
in Spencer's Fairy Queen: for his allegories, in which the paffions are perfonified, are tedious
and unpleafant, becaufe they are not difguifed under the appearance of reality: while the
magic of Circe, the bower of Calypfo, the rocks of Scylla and Charybdis, and the melody
of the Syrens, in the Odyffey of Homer, though nothing but allegories, univerfally enchant
and delight, becaufe they are covered with the femblance of truth. It is on this account
that Milton's battles in heaven are barbarous and ridiculous in the extreme; for every one
fees the impoffibility of fuppofing gun-powder and cannons in the celeftial regions: the ma-
chinery is forced and unnatural, contains no elegance of fancy, and is not replete with
any myftical information. On the contrary, Homer's machinery is natural and poffible, is
full of dignity and elegance, and is pregnant with the fublimeft truths; it delights and enobles
the mind of the reader, aftonifhes him with its magnificence and propriety, and animates him
with the fury of poetic infpiration. And this, becaufe it is poffible and true.

of

of figure from other primogenial caufes. Nor muft we fubfcribe to their opinion, who affert that figures deftitute of matter can have no fubfiftence, but thofe only which appear in matter. Nor to theirs, who acknowledge, indeed, that they are external to matter, but confider them as fubfifting alone, according to thought and abftraction. For where fhall we preferve in fafety, the certainty, beauty, and order of figures, among things which fubfift by abftraction? For, fince they are of the fame kind with fenfibles, they are far diftant from indubitable and pure certainty. But from whence do they derive the certainty, order, and perfection which they receive? For they either derive it from fenfibles (but they have no fubfiftence in thefe), or from intelligibles (but in thefe they are more perfect), fince, to fay from that which is not, is the moft abfurd of all. For nature does not produce imperfect figures, and leave the perfect without any fubfiftence. Nor is it lawful, that our foul fhould fabricate more certain, perfect, and orderly figures, than intellect and the gods themfelves. There are, therefore, prior to fenfible figures, felf-moving, intellectual, and divine reafons of figures. And we are excited, indeed, from the obfcurity of fenfible forms, but we produce internal reafons, which are the lucid images of others. And we poffefs a knowledge of fenfible figures, by their exemplars refident in foul (παραδειγματικως), but we comprehend by images (εικονικως) fuch as are intellectual and divine. For the reafons we contain, emerging from the dark night of oblivion, and propagating themfelves in fciential variety, exhibit the forms of the gods, and the uniform bounds of the univerfe, by which they ineffably convert all things into themfelves. In the gods, therefore, there is both an egregious knowledge of univerfal figures, and a power of generating and conftituting all inferiors. But in natures, figures are endued with a power generative of apparent forms; but are deftitute of cognition and intellectual perception. And, in particular fouls, there is, indeed, an immaterial intellection, and a felf-energizing knowledge; but there is wanting a prolific, and efficacious caufe. As, therefore, nature, by her forming power prefides over fenfible figures, in the fame manner, foul, by her gnoftic energy, drops in the phantafy as in a mirror, the reafons of figures. But the phantafy receiving thefe in her fhadowy forms, and poffeffing images of the

inherent.

inherent reafons of the foul, affords by thefe the means of inward converfion to the foul, and of an energy directed to herfelf, from the fpectres of imagination.   Juft as if any one beholding his image in a mirror, and admiring the power of nature, and his own beauty, fhould defire to fee himfelf in perfection, and fhould receive a power of becoming, at the fame time, the perceiver, and the thing perceived. For the foul, after this manner, looking abroad into the bright mirror of the phantafy, and furveying the fhadowy figures it contains, and admiring their beauty and order, purfues, in confequence of her admiration, the reafons from which thefe images proceed; and being wonderfully delighted, difmiffes their beauty, as converfant about fpectres alone; but afterwards feeks her own purer beauty, and defires to pafs into her own profound retreats, and there to perceive the circle and the triangle, and all things fubfifting together, in an impartible manner, and to infert herfelf in the objects, to contract her multitude into one; and laftly, to behold the occult and ineffable figures of the gods, feated in the moft facred and divine receffes of her nature.   She is likewife defirous of bringing into light, from its awful concealment, the folitary beauty of the gods, and of perceiving the circle, fubfifting in its true perfection, more impartible than any centre, and the triangle without interval; and laftly, by afcending into an union with herfelf, of furveying every object which is fubject to the power of cognition.   The figure, therefore, which is felf-motive, precedes that which is moved by another; and the impartible that which is felf-motive: but that which is the fame with *one*, precedes the impartible itfelf.   For all things are bounded, when they return to the unities of their nature; fince all things pafs through thefe as a divine entrance into being.   And thus much for this long digreffion, which we have delivered according to the fentiments of the Pythagoreans.   But the geometrician, contemplating that figure which is feated in the phantafy, and defining this, in the firft place, (fince this definition agrees with fenfibles, in the fecond place) fays, that figure is that which is comprehended by one or more boundaries.   For, fince he receives it together with matter, and conceives of it as diftant with intervals, he does not improperly call it finite and terminated *.   [Since every thing

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* The fentence within the brackets is omitted in the Greek.

which contains either intelligible or fenfible matter, is allotted an adventitious bound; and is not itfelf bound, but that which is bounded.] Nor is it the bound of itfelf; but one of its powers is terminating, and the other terminated. Nor does it fubfift in bound itfelf, but is contained by bound. For figure is joined to quantity, and fubfifts together with it; and, at the fame time, quantity is fubjected to figure; but the reafon and afpect of that quantity is nothing elfe than figure and form. Since, indeed, reafon terminates quantity, and adds to it a particular character and bound, either fimple or compofite. For, fince this alfo exhibits the two-fold progreffion of *bound* and *infinite* in its proper forms, (in the fame manner as the reafon of an angle,) it invefts the objects of its comprehenfion with one boundary and fimple form, according to *bound*, but with many, according to infinity *. Hence, every thing figured, vindicates to itfelf either one boundary, or a many. Euclid, therefore, denominating that which is figured and material, and annexed to quantity figure, does not improperly fay, that it is contained by one or more terms. But Poffidonius defines figure to be concluding bound, feparating the reafon of figure from quantity; and confidering it as the caufe of terminating, defining, and comprehending quantity. For that which enclofes, is different from that which is enclofed; and bound from that which is bounded. And Poffidonius, indeed, feems to regard the external furrounding bound; but Euclid, the whole fubject. Hence, the one calls a circle a figure, with relation to its whole plane, and exterior ambit; but the other with relation to its circumference only. And the one defines that which is figured, and which is beheld together with its fubject: but the other defires to define the reafon of the circle; I mean that which terminates and concludes its quantity. But if any logician, and captious perfon, fhould blame the definition of Euclid, becaufe he defines genus from fpecies (for things contained by one or more terms, are the fpecies of figure,) we fhall affert, in oppofition to fuch an objection, that genera alfo pre-occupy in themfelves the powers of fpecies. And when men of ancient authority, were willing to manifeft genera themfelves, from thofe powers which genera contain, they appeared, indeed, to enter on their defign from fpecies, but, in reality, they.

* That is, the circular form proceeds from *bound*, but right-lined figures from *infinity*.

explained

3

explained genera from themfelves, and from the powers which they contain. The reafon of figure, therefore, fince it is one, comprehends the differences of many figures, according to the bound and infinity refiding in its nature. And he who defined this reafon, was not void of underftanding, whilft he comprehended in a definition, the differences of the powers it contained. But you will afk, From whence does the reafon of figure originate, and by what caufes is it perfected? I anfwer, that it firft arifes from *bound* and *infinite*, and that which is mixed from thefe. Hence it produces fome fpecies from *bound*, others from *infinite*, and others from the *mixt*. And this it accomplifhes by bringing the form of bound to circles; but that of infinite, to right-lines: and that of the mixt to figures compofed from right and circular lines. But, in the fecond place, this reafon is perfected from that totality which is feparated into diffimilar parts. From whence, indeed, it occafions a whole to every form, and each figure is eut into different fpecies. For a circle, and every right-lined figure may be divided, by reafon or proportion, into diffimilar figures; which is the bufinefs of Euclid in his book of divifions, where he divides one figure into figures fimilar to fuch as are given; but another into fuch as are diffimilar. In the third place, it is invigorated from accumulated multitude, and, on account of this, extends forms of every kind, and produces the multiform reafons of figures. Hence, in propagating itfelf, it does not ceafe till it arrives at fomething laft, and has unfolded all the variety of forms. And, as in the intelligible world, *one* is fhewn to abide in that which *is*; and, at the fame time, that which *is* in *one*, fo likewife, reafon exhibits circular in right-lined figures; and, on the contrary, rectilinear comprehended in circular figures. And it peculiarly manifefts its whole nature in each, and all thefe in all. Since the whole fubfifts in all collectively, and in each feparate and apart. From that order, therefore, it is endued with this power. In the fourth place, it receives from the firft of numbers *, the meafures of the progreffion of forms. From whence it conftitutes all figures according to numbers; fome, indeed, according to the more fimple, but others according to the more compofite. For triangles, quadrangles, quinquangles, and all multangles, proceed in

* That is, the number three.

infi-

infinitum, together with the mutations of numbers. But the caufe of this is, indeed, unknown to the vulgar, though, to thofe who under-ftand where number and figure fubfift, the reafon is manifeft. Fifthly, it is replete with that divifion of forms, which divides forms into other fimilar forms, from another fecond totality, which is alfo diftributed into fimilar parts. And by this, a trianglar reafon is divided into tri-angles, and a quadrangular reafon into quadrangles. And hence, exercifing our inward powers, we effect what I have faid in images, fince it pre-exifted by far the firft in its principles. But by regarding thefe diftributions, we may render many caufes of figures, reducing them to their firft principles. And the more common, or geometrical figure, is allotted an order of this kind, and from fo many caufes, re-ceives the perfection of its nature. But, from hence it advances to the genera of the gods, and is varioufly attributed according to its various forms, and energizes differently in different gods. To fome, indeed, affording more fimple figures ; but to others, fuch as are more com-pofite. And to fome, again, affigning primary figures, and thofe which are produced in fuperficies; but to others (entering the tumor of folid bodies) fuch figures, as in folids are convenient to themfelves. For all figures, indeed, fubfift in all, fince the forms of the gods are accumulated, and full of univerfal powers: but, by their peculiarity, they produce one thing according to another. For one poffeffes all things circularly, another in a triangular manner, but another accord-ing to a quadrangular reafon. And in a fimilar manner in folids.

DEFINITION XV.

A CIRCLE is a Plane Figure, comprehended by one Line, which is called the Circumference, to which all Right Lines falling from a certain Point within the Figure, are equal to each other.

DEFINITION XVI.

And that Point is called the CENTRE of the CIRCLE.

A Circle is the firft, moft fimple, and moft perfect of figures. For it excels all folids, becaufe it exifts in a more fimple place; but it is fuperior to the figures fubfifting in planes, on account of its

fimilitude

fimilitude and identity. And it has a correfponding proportion to bound, and unity, and a better co-ordination of being. Hence, in a diftribution of mundane and fuper-mundane figures, you will always find that the circle is of a diviner nature. For if you make a divifion into the heavens, and the univerfal regions of generation, you muft affign to the heavens a circular form; but to generation, that of a right line. For whatever among generable natures is circular, defcends from the heavens; fince generation revolves into itfelf, through their circumvolutions, and reduces its unftable mutation to a regular and orderly continuance. But if you diftribute incorporeal natures into foul and intellect, you will fay, that the circle belongs to intellect, and the right line to the foul. And on this account, the foul, by its converfion to intellect, is faid to be circularly moved; and it poffeffes the fame proportion to intellect, as generation to the heavens. For it is circularly moved, (fays Socrates *,) becaufe it imitates intellect. But the generation and progreffion of foul is made according to a right-line. For it is the property of the foul to apply herfelf at different times to different forms. But if you wifh to divide into body and foul, you muft conftitute every thing corporeal, according to the right-line; but you muft affign to every animal a participation of the identity and fimilitude of the circle. For body is a compofite, and is endued with various powers, fimilar to right-lined figures: but foul is fimple and intelligent; felf-motive, and felf-operative; converted into, and energizing in herfelf. From whence, indeed, Timæus alfo, when he had compofed the elements of the univerfe from right-lined figures, affigned to them a circular motion and formation, from that divine foul which is feated in the bofom of the world. And thus, that the circle every where holds the firft rank, in refpect of other figures, is fufficiently evident from the preceding obfervations. But it is requifite to furvey its whole feries, beginning fupernally, ending in inferiors, and perfecting all things, according to the aptitude of the natures which receive its alliance. To the gods, therefore, it affords a converfion to their caufes, and ineffable union: it occafions their abiding in themfelves, prevents their departing from their own beatitude, ftrengthens their higheft unions, as centres defirable to

* In Timæo.

inferior

inferior natures, and ftably places about thefe the multitude of the powers which the gods poffefs, containing them in the fimplicity of their effences. But the circle affords to intellectual natures, a perpetual energy in themfelves, is the caufe of their being filled with knowledge from themfelves, and of poffeffing in their effences, intelligibles contractedly; and of perfecting intellections in themfelves. For every intellect, propofes to itfelf that which is intelligible; and this is as a centre to intellect, about which it continually revolves : for intellect folds itfelf, and operates about this, and is united within itfelf on all fides, by univerfal intellectual energies. But it extends to fouls by illumination, a felf-vital, and felf-motive power, and an ability of turning, and leaping round intellect, and of returning according to proper convolutions, unfolding the impartibility of intellect. Again, the intellectual orders excel fouls after the manner of centres, but fouls energize circularly about their nature. For every foul, according to its intellectual part, and the fupreme *one*, which is the very flower of its effence, receives a centre: but, according to its multitude, it has a circular revolution, defiring, by this means, to embrace the intellect which it participates. But, to the celeftial bodies, the circle affords an affimilation to intellect, equality, a comprehenfion of the univerfe, in proper limits, revolutions which take place in determinate meafures, a perpetual fubfiftence, a nature without beginning and end, and every thing of this kind. And to the elements under the concave of the moon's orb, it is the caufe of a period, converfant with mutations; an affimilation to the heavens; that which is without generation, in generated natures; that which abides in things which are moved; and whatever is bounded in partible effences. For all things are perpetual, through the circle of generation; and equability is every where preferved on account of the reciprocation of corruption. Since, if generation did not return, in a circular revolution, in a fhort fpace of time, the order, and all the ornament of the elements would vanifh. But again, the circle procures to animals and plants, that fimilitude which is found in generations: for thefe are produced from feeds, and feeds from thefe. Hence, generation here, and a circumvolution, alternately takes place, from the imperfect to the perfect, and the contrary; fo that corruption fubfifts together with
generation.

generation. But, befides this, to unnatural productions it impofes order, and reduces their indeterminate variety to the limitation of bound; and, through this, nature herfelf is gracefully ornamented in the laft veftiges of her powers. Hence, things contrary to nature have a revolution according to determinate numbers, and not only fertility, but alfo fterility, fubfifts according to the alternate convolutions of circles (as the difcourfe of the Mufes evinces), and all evils though they are difmiffed from the prefence of the gods, into the place of mortals, yet thefe roll round, fays Socrates, and to thefe there is prefent a circular revolution, and a circular order; fo that nothing immoderate and evil is deferted by the gods; but that providence, which is perfective of the univerfe, reduces alfo the infinite variety of evils, to bound, and an order convenient to their nature. The circle, therefore, is the caufe of ornament to all things, even to the laft participations, and leaves nothing deftitute of itfelf, fince it fupplies beauty, fimilitude, formation, and perfection to the univerfe. Hence too, in numbers it contains the middle centres of the whole progreffion of numbers, which revolves from unity to the decad (or ten). For five and fix exhibit a circular power, becaufe, in the progreffions from themfelves, they return again into themfelves, as is evident in the multiplication of thefe numbers. Multiplication, therefore, is an image of progreffion, fince it is extended into multitude; but an ending in the fame fpecies, is an image of regreffion into themfelves. But a circular power affords each of thefe, exciting, indeed, as from an abiding centre, thofe caufes which are productive of multitude; but converting multitude after the productions to their caufes. Two numbers, therefore, having the properties of a circle, poffefs the middle place between all numbers: of which one, indeed, precedes every convertible genus of males and an odd nature; but the other, recalls every thing feminine and even, and all prolific feries, to their proper principles, according to a circular power. And thus much concerning the perfection of the circle. Let us now contemplate the mathematical definition of the circle, which is every way perfect. In the firft place, therefore, he defines it a figure, becaufe, indeed, it is finite, and every where comprehended by one limit, and is not of an infinite nature, but affociated to bound. Likewife

plane,

plane, becaufe, fince figures are either beheld in fuperficies, or in folid bodies, a circle is the firft of plane figures, excelling folids in fimplicity, but poffeffing the proportion of unity to planes. But comprehended by one line, becaufe it is fimilar to one, by which it is defined, and becaufe it does not extrinfically receive a variety of furrounding terms. And again, that this line makes all the lines drawn to it from a certain point within equal, becaufe of the figures which are bounded by one line, fome have all the lines proceeding from the middle equal; but others not at all. For the ellipfis is comprehended by one line, yet all the lines iffuing from the centre, and bounded by its curvature, are not equal, but only two. Alfo the plane, which is included by the line called a ciffoid, has one containing line, yet it does not contain a centre, from which all the lines are equal. But, becaufe the centre in a circle is entirely one point (for there are not many centres of one circle), on this account, the geometrician adds, that lines falling from one point to the bound of the circle, are equal. For there are infinite points within it, but of all thefe, one only has the power of a centre. And becaufe this one point, from which all the lines drawn to the circumference of the circle are equal, is either within the circle, or without (for every circle has a pole, from which all the lines drawn to its circumference are equal), on this account he adds, *of the points within the figure,* becaufe, here he receives the centre alone, and not the pole. For he wifhes to behold all its properties in one plane, but the pole is more elevated than the fubjeᴄt plane. Hence, he neceffarily adds, in the end of the definition, that this point, which is placed within the circle, and to which all right lines drawn from it to the circumference, are equal, is the centre of the circle. For there are only two points of this kind, the pole and the centre. But the former is without, and the other within the plane. Thus, for inftance, if you conceive a perpendicular ftanding on the centre of a circle, its fuperior extremity is the pole: for all lines drawn from it to the circumference of the circle, are demonftrated to be equal. And, in like manner, in a cone, the vertex of the whole cone, is the pole of the circle at the bafe. And thus far we have determined what a circle is, and its centre, and what the nature is of its circumference, and the whole circular figure. Again, therefore, from thefe, let us return

to

to the fpeculation of their exemplars, contemplating in them the cen-
tre, according to one impartible and ftable excellence. But the diftances
from the centre, according to the progreffions which are made from
one, to multitude infinite in capacity. And the circumference of the
circle, according to the regreffion of the progreffions to the centre, by
means of which the multitude of powers are rolled round their union,
and all of them haften to its comprehenfion, and defire to energize
about its indivifible embrace. And, as in the circle itfelf, all things
fubfift together, the centre, intervals, and external circumference; fo
in thefe which are its image, one thing has not an effence pre-exiftent,
and another confequent in time; but all things are, indeed, together,
permanency, progreffion, and regreffion. But thefe differ from thofe,
becaufe the former fubfift indivifibly, and without any dimenfion; but
the latter with dimenfion, and in a divifible manner; the centre exift-
ing in one place, the lines emanating from the centre, in another;
and the external circumference terminating the circle, having a ftill
different fituation. But there all things abide in one: for if you
regard that which performs the office of a centre, you will find it the
receptacle of all things. If the progreffion diftant from the centre,
in this, likewife, you will find all things contained. And, in a fimi-
lar manner, if you regard its regreffion. When, therefore, you are
able to perceive all things fubfifting together, and have taken away
the defect proceeding from dimenfion, and have removed from your
inward vifion, the pofition about which partition fubfifts, you will
find the true circle, advancing to itfelf, bounding, and energizing in
itfelf, exifting both one and many, and abiding, proceeding, and
returning; likewife firmly eftablifhing that part of its effence which is
moft impartible, and efpecially fingular; but advancing from this
according to rectitude, and the infinity which it contains; and rolling
itfelf from itfelf to one, and exciting itfelf by fimilitude and identity
to the impartible centre of its nature, and to the occult power of the
one which it contains. But this one, which the circle contains, and
environs in its bofom, it emulates according to the multitude of its
own nature. For that which is convolved, imitates that which abides,
and the periphery is as a centre which is diftant with interval, and

Y 2   nods

nods to itself, haftening to receive, and to become one with the centre, and to terminate its regrefs where it received the principle of its pro-greffion. For the centre is every where in the place of that which is lovely, and the object of defire, prefiding over all things which fubfift about its nature, and exifting as the beginning and author of all pro-greffions. And this the mathematical centre alfo expreffes, by termi-nating all the lines falling from itfelf to the circumference, and by affording to them equality, as an image of proper union. But the oracles likewife define the centre, after this manner: *The centre is that from which and* ＊ *to which all the lines to the circumference are equal.* Indicating the beginning of the diftance of the lines, by the particle *from which*; but the middle of the circumference by the par-ticle *to which*: for this, in every part, is joined with the centre. But if it be neceffary to declare the firft caufe, through which a cir-cular figure appears and receives its perfection, I affirm, that it is the fupreme order of intelligibles. For the centre, indeed, is affimilated to the caufe of bound; but the lines emanating from this, and which are infinite, with refpect to themfelves, both in multitude and mag-nitude, reprefent infinity; and the line which terminates their exten-fion, and conjoins the circular figure with the centre, is fimilar to that occult ornament, confifting from the intelligible orders; which Orpheus alfo fays, is circularly borne, in the following words, *But it is carried with an unwearied energy, according to an infinite circle.* For, fince it is moved intelligibly, about that which is intelligible, having it for the centre of its motion, it is, with great propriety, faid to energize in a circular manner. Hence, from thefe alfo, the triadic god † proceeds, who contains in himfelf the caufe of the progreffion of right-lined figures. For on this account, wife men, and the moft myftic of theologifts, have fabricated his name. [‡ Hence too, it is manifeft, that a circle is the firft of all figures:] but a triangle is the

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* πρὸς ὃ, or, *to which*, is wanting in the original, and in all the publifhed collections of the Zoroaftrian oracles.

† That is Jupiter, who is called triadic, becaufe he proceeds from Saturn and Rhea; and becaufe his government is participated by Neptune and Pluto, for each of thefe is called Jupiter by Orpheus.

‡ This fentence, within the brackets, is omitted in the printed Greek.

firſt of ſuch as are right-lined. Figures, therefore, appear firſt in the regular ornaments of the gods; but they have a latent ſubſiſtence, according to pre-exiſtent cauſes, in intelligible eſſences.

DEFINITION XVII.

A DIAMETER of a Circle is a certain ſtraight Line, drawn
    through the Centre, which is terminated both ways
    by the Circumference of the Circle, and divides the
    Circle into two equal Parts.

EUCLID here perſpicuouſly ſhews, that he does not define every diameter, but that which belongs to a circle only. Becauſe there is a diameter of quadrangles and all parallelograms, and like-wiſe of a ſphere among ſolid figures. But in the firſt of theſe, it is denominated a diagonal: but in a ſphere, the axis; and in circles the diameter only. Indeed, we are accuſtomed to ſpeak of the axis of an ellipſis, cylinder, and cone; but of a circle, with propriety, the diameter. This, therefore, in its genus, is a right-line; but as there are many right-lines in a circle, as likewiſe infinite points, one of which is a centre, ſo this only is called a diameter, which paſſes through the centre, and neither falls within the circumference, nor tranſcends its boundary; but is both ways terminated by its com-prehenſive bound. And theſe obſervations exhibit its origin. But that which is added in the end, that it alſo divides the circle into two equal parts, indicates its proper energy in the circle, excluſive of all other lines drawn through the centre, which are not terminated both ways by the circumference. But they report, that Thales firſt de-monſtrated, that the circle was biſected by the diameter. And the cauſe of this biſection, is the indeclineable tranſit of the right line, through the centre. For, ſince it is drawn through the middle, and always preſerves the ſame inflexible motion, according to all its parts, it cuts off equal portions on both ſides to the circumference of the circle. But if you deſire to exhibit the ſame mathematically, con-ceive the diameter drawn, and one part of the circle placed on the
                                                                .other.

other *. Then, if it is not equal, it either falls within, or without; but the confequence either of thefe ways muft be, that a lefs right-line will be equal to a greater. Since all lines from the centre to the circumference are equal. The line, therefore, which tends to the exterior circumference, will be equal to that which tends to the interior. But this is impoffible. Thefe parts of the circle, then, agree, and are on this account equal. But here a doubt arifes, if two femicircles are produced by one diameter, and infinite diameters may be drawn through the centre, a double of infinities will take place, according to number. For this is objected † by fome againft the fection of magnitudes to infinity. But this we may folve by affirming, that magnitude may, indeed, be divided infinitely, but not into infinites. For this latter mode produces infinites in energy, but the former in capacity only. And the one affords effence to infinite, but the other is the fource of its origin alone. Two femi-circles, therefore, fubfift together with one diameter, yet there will never be infinite diameters, although they may be infinitely affumed. Hence, there

* Thus let a part A E B cut off by the diameter A B (fig. 1.) of the circle A E B D be
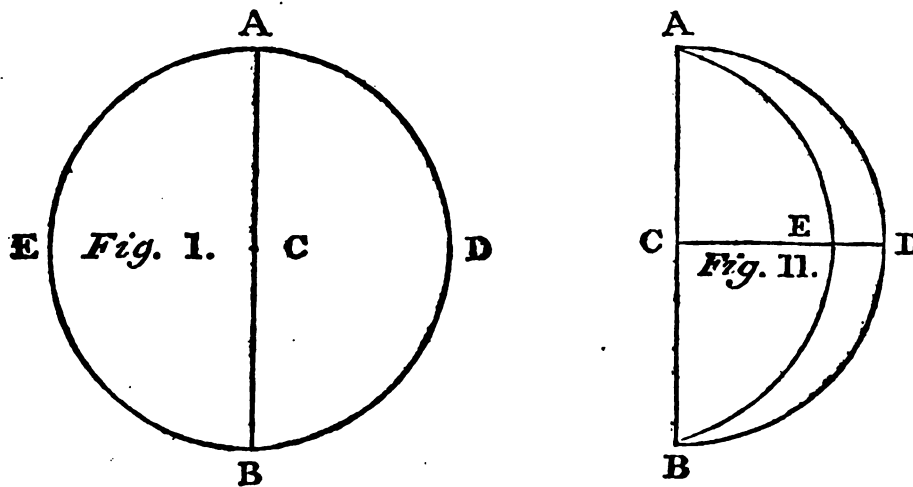
placed on the other part A D B, as in fig. II. Then, if it is not equal to the other part, either A E B will fall within A D B, or A D B within A E B: but in either cafe, C E will be equal to C D, which is abfurd.

† This objection is urged by Philoponus, in his book againft Proclus on the eternity of the world; but not, in my opinion, with any fuccefs. See alfo Simplicius, in his third digreffion againft Philoponus, in his commentary on the 8th book of Ariftotle's Phyfics.

can

can never be doubles of infinites; but the doubles which are conti-
nually produced, are the doubles of finites; for the diameters which
are always affumed, are finite in number. And what reafon can be
affigned why every magnitude fhould not have finite divifions, fince
number is prior to magnitudes, defines all their fections, pre-occupies
infinity, and always determines the parts which rife into energy from
dormant capacity?

DEFINITION XVIII.

A SEMICIRCLE is the Figure contained by the Diameter,
and that Part of the Circumference which is cut off by
the Diameter.

DEFINITION XIX. *

But the CENTRE of the Semi-circle, is the fame with that
of the Circle.

FROM the definition of a circle Euclid finds out the nature of
the centre, differing from all the other points which the circle
contains. But from the centre he defines the diameter, and feparates
it from the other right lines, which are defcribed within the circle.
And from the diameter, he teaches the nature of the femi-circle; and
informs us, that it is contained by two terms, always differing from
each other, viz. a right-line and a circumference: and that this right-
line is not any one indifferently, but the diameter of the circle. For
both a lefs and a greater fegment of a circle, are contained by a right-
line and circumference; yet thefe are not femi-circles, becaufe the
divifion of the circle is not made through the centre. All thefe figures,
therefore, are biformed, as a circle was monadic, and are compofed

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* This definition is no where extant but in the commentaries of Proclus. Inftead of it, in
almoft all the printed editions of Euclid, the following is fubftituted. *A fegment of a circle is
the figure contained by a diameter, and the part of the circumference cut off by the diameter.* This
Mr. Simfon has marked with commas, as a fymbol of its being interpolated: but he has taken
no notice of the different reading in the commentaries of Proclus. And what is ftill more
remarkable, this variation is not noticed by any editor of Euclid's Elements, either ancient or
modern.

from diffimilars. For every figure which is comprehended by two terms, is either contained by two circumferences, as the lunular: or by a right-line and circumference, as the above mentioned figures; or by two mixt lines, as if two ellipfes interfect each other (fince they enclofe a figure, which is intercepted between them), or by a mixt line and circumference, as when a circle cuts an ellipfis; or by a mixt and right-line, as the half of an ellipfis. But a femi-circle is compofed from diffimilar lines, yet fuch as are, at the fame time, fimple, and touching each other by appofition. Hence, before he defines triadic figures, he, with great propriety, paffes from the circle to a biformed figure. For two right-lines can, indeed, never comprehend fpace. But this may be effected by a right-line and circumference. Likewife by two circumferences, either making angles, as in the lunular figure ; or forming a figure without angles, as that which is comprehended by concentric circles. For the middle fpace intercepted between both, is comprehended by two circumferences ; one interior, but the other exterior, and no angle is produced. For they do not mutually interfect, as in the lunular figure, and that which is on both fides convex. But that the centre of the femi-circle is the fame with that of the circle, is manifeft. For the diameter, containing in itfelf the centre, completes the femi-circle, and from this all lines drawn to the femi-circumference are equal. For this is a part of the circumference of the circle. But equal right lines proceed from the centre to all parts of the circumference. The centre, therefore, of the circle and femi-circle is one and the fame. And it muft be obferved, that among all figures, this alone contains the centre in its own perimeter, I fay, among all plane figures. Hence you may collect, that the centre has three places. For it is either within a figure, as in the circle ; or in its perimeter, as in the femi-circle; or without the figure, as in certain conic lines *. What then is indicated by the femi-circles, having the fame centre with the circle, or of what things does it bear an image, unlefs that all figures which do not entirely depart from fuch as are firft, but participate them after a manner, may be concentric with them, and participate of the fame caufes ? For the femicircle communicates with the circle doubly, as well according to the diame-

* As in every hyperbola.

ter

ter, as according to the circumference. On this account, they poſſeſs
a centre alſo in common. And perhaps, after the moſt ſimple prin-
ciples, the ſemi-circle is aſſimilated to the ſecond co-ordinations,
which participate thoſe principles; and by their relation to them, al-
though imperfectly, and by halves, they are, neverthelefs, reduced
to that which _is_, and to their firſt original cauſe.

DEFINITION XX.

RECTILINEAR FIGURES are thoſe which are comprehended
by Straight Lines.

DEFINITION XXI.

TRILATERAL FIGURES, or TRIANGLES, by three Straight
Lines.

DEFINITION XXII.

QUADRILATERAL, by four Straight Lines.

DEFINITION XXIII.

MULTILATERAL FIGURES, or POLYGONS, by more than four
Straight Lines.

AFTER the monadic figure having the relation of a principle to
all figures, and the biformed ſemi-circle, the progreſſion of
right-lined figures in infinitum, according to numbers, is delivered.
For on this account alſo, mention was made of the ſemi-circle, as
communicating according to terms or boundaries; partly, indeed, with
the circle, but partly with right-lines: juſt as the duad is the medium,
between unity and number. For unity, by compoſition, produces
more than by multiplication; but number, on the contrary, is more
increaſed by multiplication than compoſition: and the duad, whether
multiplied into, or compounded with itſelf, produces an equal quan-
tity. As, therefore, the duad is the middle of unity and number, ſo
likewiſe, a ſemi-circle communicates, according to its baſe, with right-

lines; but according to its circumference, with the circle. But right-lined figures proceed orderly to infinity, attended by number and its bounding power, which begins from the triad. On this account, Euclid also begins from hence *. For he fays, trilateral and quadrilateral, and the following figures, called by the common name of multilateral : fince trilateral figures are alfo multilaterals; but they have likewife a proper, befides a common denomination. But, as we are but little able to purfue the reft, on account of the infinite progreffion of numbers, we muft be content with a common denomination. But he only makes mention of trilaterals and quadrilaterals, becaufe the triad and tetrad are the firft in the order of numbers; the former being a pure odd among the odd; but the latter, an entire even among even numbers. Euclid, therefore, affumes both in the origin of right-lined figures, for the purpofe of exhibiting their fubfiftence, according to all even and odd numbers. Befides, fince he is about to teach concerning thefe in the firft book, as efpecially elementary (I mean triangles and parallelograms) he does not undefervedly, as far as to thefe, eftablifh a proper enumeration: but he embraces all other right-lined figures by a common name, calling them multilaterals: but of thefe enough. Again, affuming a more elevated exordium, we muft fay, that of plane figures, fome are contained by fimple lines, others by fuch as are mixt, but others again by both. And of thofe which are comprehended by fimple lines, fome are contained by fimilars in fpecies, as right-lines; but others by diffimilars in fpecies, as femi-circles, and fegments, and apfides, which are lefs than femi-circles. Likewife of thofe which are contained by fimilars in fpecies, fome are comprehended by a circular line; but others by a right-line. And of thofe comprehended by a circular line, fome are contained by one, others by two, but others by more than two. By one, indeed, the circle itfelf. But by two, fome without angles, as the crowns † terminated by concentric circles; but others angular (γεγωνιωμένα) as the lunula.

* The Platonic reader muft doubtlefs be pleafed to find that Euclid was deeply fkilled in the philofophy of Plato, as Proclus every where evinces. Indeed, the great accuracy, and elegant diftribution of thefe Elements, fufficiently prove the truth of this affertion. And it is no inconfiderable teftimony in favour of the Platonic philofophy, that its affiftance enabled Euclid to produce fuch an admirable work.

† Concerning thefe crowns, or annular fpaces, confult the great work of that very fubtle and elegant mathematician Tacquet, entitled *Cylindrica et Annularia*.

And

And of thofe comprehended by more than two, there is an infinite proceffion. For there are certain figures contained by three and four and fucceeding circumferences. Thus, if three circles touch each other, they will intercept a certain trilateral fpace; but if four, one terminated by four circumferences, and in like manner, by a fucceffive progreffion. But of thofe contained by right lines, fome are comprehended by three, others by four, and others by a multitude of lines. For neither is fpace comprehended by two right-lines, nor much more by one right-line. Hence, every fpace comprehended by one boundary, or by two, is either mixt or circular. And it is mixt in a twofold manner, either becaufe the mixt lines comprehend it, as the fpace intercepted by the ciffoidal line; or becaufe it is contained by lines diffimilar in fpecies, as the apfis: fince mingling is two-fold, either by appofition or confufion. Every right-lined figure, therefore, is either trilateral, or quadrilateral, or gradually multilateral; but every trilateral, or quadrilateral, or multilateral figure, is not right-lined; fince fo great a number of fides is alfo produced from circumferences. And thus much concerning the divifion of plane figures. But we have already afferted *, that rectitude of progreffion is both a fymbol of motion and infinity, and that it is peculiar to the generative co-ordinations of the gods, and to the producers of difference, and to the authors of mutation and motion. Right-lined figures, therefore, are peculiar to thefe gods, who are the principles of the prolific energy of the whole progreffion of forms. On which account, generation alfo, was principally adorned by thefe figures, and is allotted its effence from thefe, fo far as it fubfifts in continual motion and mutation without end.

* In the preceding mark commentary.

DEFINITION XXIV.

Of three-fided FIGURES: an EQUILATERAL TRIANGLE is that which has three equal Sides.

DEFINITION XXV.

An ISOSCELES TRIANGLE, is that which has only two Sides equal.

DEFINITION XXVI.

A SCALENE TRIANGLE, is that which has three unequal Sides.

DEFINITION XXVII.

A RIGHT-ANGLED TRIANGLE is that which has a Right Angle.

DEFINITION XXVIII.

An OBTUSE-ANGLED TRIANGLE is that which has an Obtufe Angle.

DEFINITION XXIX.

An ACUTE-ANGLED TRIANGLE is that which has three Acute Angles.

THE divifion of triangles fometimes commences from angles, but fometimes from fides. And that, indeed, which originates from fides, precedes as known; but that from angles follows as a proper diftribution. For thefe three angles alone belong to right-lined figures, viz. the right, the obtufe, and the acute: but the equality and inequality of fides fubfift alfo in non-rectilinear figures. Euclid fays, therefore, that of triangles, fome are equilateral, others

ifofceles,

ifofceles, and others fcalene : for they have either all their fides equal, or all unequal, or only two equal. And again, that of triangles fome are right-angled, others obtufe-angled, and others acute-angled. And he defines a right-angled triangle, that which has one right angle, as likewife an obtufe-angled triangle, that which has one obtufe angle: for it is impoffible that a triangle can have more than one right, or obtufe angle *. But he defines an acute-angled triangle, that which has all its angles acute. For here it is not fufficient that it fhould have only one acute; fince, in this cafe, all triangles would be acute-angled, as every triangle has neceffarily two acute angles +. But, to poffefs three acute angles, is the property of an acute-angled triangle alone. But Euclid appears to me to have made a feparate divifion into angles and fides, from confidering this alone, that every triangle is not alfo trilateral. For there are quadrilateral triangles, which are called by mathematicians themfelves (ἀκιδοειδῆ) that is, fimilar to the point of a fpear ‡: but by Zenodorus (κοιλογώνια) that is, having an hollow angle. For on one of the fides of a trilateral figure, conftitute two right-lines inwardly; by this means a certain fpace will be enclofed, which is comprehended by external and internal right-lines, and which has three angles; one, indeed, contained by the external lines; but two comprehended by thefe and the internal lines, at the extremities in which thefe lines are conjoined. A figure of this kind, therefore, is a quadrilateral triangle. And hence, it does not immediately follow, that becaufe a figure has three angles (whether they

* This in confequence of every triangle poffeffing angles alone equal to two right.
+ This too, follows from the fame caufe as above.
‡ Thus the following figure A B D C has four fides, and but three angles.
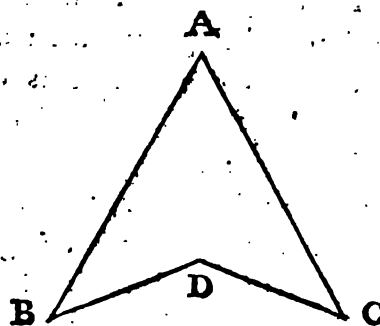
are all acute, or one right, or one obtufe), we fhall find it trilateral ;
for it may be, perhaps, quadrilateral.   In like manner, you may alfo
find quadrangles having more than four fides.   And therefore, we
muft not rafhly determine the number of fides from the multitude of
angles.   But of this enough.   But the Pythagoreans affirm that the
triangle is fimply the principle of generation, and of the formation of
generable natures.   On which account, Timæus fays, that natural
reafons, as well as thofe of the conftruction of the elements, are tri-
angular.   For they are diftant by a triple interval, are on all fides col-
lective of partible, and varioufly mutable natures, are replete with ma-
terial infinity, and bear before themfelves the conjunctions of material
bodies, loofened and free: as, indeed, triangles alfo are comprehended
by three right-lines, but they poffefs angles which collect the multi-
tude of lines, and afford to them an adventitious angle and conjunction.
With great propriety, therefore, Philolaus has confecrated the angle
of a triangle to four gods, Saturn, Pluto, Mars, and Bacchus, com-
prehending in thefe the whole quadripartite ornament of the elements
defcending from the heavens, or from the four fegments of the zodiac.
For Saturn conftitutes an effence wholly humid and frigid; but Mars
a nature totally fiery; and Pluto contains the whole terreftrial life;
but Bacchus governs a humid and hot generation; of which wine alfo
is a fymbol, for this is humid and hot.   Hence, all thefe gods differ
according to their operations in inferior concerns : but they are mu-
tually united according to their proper natures.   And on this account,
Philolaus collects their union according to one angle.   But if the
differences of triangles contribute to generation, we fhall very properly
confefs that a triangle is the principle and author of the conftitution
of fublunary natures.   For a right angle, indeed, affords them effence,
and determines the meafure of being ; and the reafon of a right-angled
triangle produces the effence of the elements of generable natures;
but an obtufe angle affigns to them univerfal diftance; and the rea-
fon of an obtufe angled triangle increafes material forms in magni-
tude, and in mutation of every kind.   But an acute angle effects their
divifible nature; and the reafon of an acute-angled triangle prepares
them to receive infinite divifion.   But fimply, a triangular reafon
conftitutes the effence of material bodies diftant with interval, and on

all

all fides divifible. And thus much fhould we fpeculate concerning the nature of triangles. But from thefe divifions you may underſtand, that all the fpecies of triangles are neither more nor lefs than feven. For the equilateral triangle is one, fince it is acute-angled only; but each of the reſt is triple. For the ifofceles is either right-angled, or obtufe-angled, or acute-angled; and, in like manner, the fcalene triangle poſſeſſes this triple difference. If then, thefe have a triple diſtinction, but the equilateral has but one mode of exiſtence, all the fpecies of triangles will be feven. But again, you will underſtand the proportion of triangles to the things which *are*, according to the divifion of fides; for the equilateral, entirely excelling in equality and fimplicity, is allied to divine fouls; fince it is the meafure and equality of things unequal, in the fame manner as divinity of all inferior concerns. But the ifofceles triangle is allied to the better genera, which govern a material nature, the greater part of which genera is held by the limitation of meafure; but their extremes extend to inequality and material immoderation; for the two fides of an ifofceles triangle are equal, but the bafe is unequal. But a fcalene triangle fymbolizes with partible lives, which are on all fides lame and defective, which prepare themfelves for generation, and are replete with matter and material imperfection.

DEFINITION XXX.

Of Quadrilateral Figures, a QUADRANGLE or SQUARE is that which has all its Sides equal, and all its Angles Right Angles.

DEFINITION XXXI.

An OBLONG is that which has all its Angles right Angles, but has not all its Sides equal.

DEFINITION XXXII.

A RHOMBUS, is that which has all its Sides equal, but its Angles are not right Angles.

DEFI.

DEFINITION XXXIII.

A RHOMBOID is that which has its oppofite Sides equal to one another, but all its Sides are not equal, nor its Angles Right Angles.

DEFINITION XXXIV.

All other Quadrilateral Figures befides thefe, are called TRAPEZIUMS.

IT is requifite that the firft divifion of quadrilateral figures fhould take place in two numbers; and that fome of them fhould be called parallelograms, but others non-parallelograms. But of parallelograms fome are rectangular and equilateral, as quadrangles; but others neither of thefe, as rhomboids: others again, are rectangular, but not equilateral, as oblongs: but others, on the contrary, are equilateral, but not rectangular, as the rhombufes. For it is requifite either to poffefs both, viz. equality of fides and rectitude of angles, or neither; or one of thefe, and this in a twofold refpect. Hence a parallelogram has a quadruple fubfiftence. But of non-parallelograms, fome have only two parallel fides, and not the reft; but others have none of their fides parallel. And thofe are called Trapeziums, but thefe Trapezoids. But of Trapeziums, fome, indeed, have the fides equal, by which the parallel fides of this kind are conjoined; but others unequal. and the former of thefe are called ifofceles trapeziums; but the latter fcalene trapeziums. A quadrilateral figure, therefore, is conftituted by us according to a feven-fold diftribution. For one is a quadrangle; but the other an oblong; the third a rhombus; the fourth a rhomboides; the fifth an ifofceles trapezium; the fixth a fcalene trapezium; the feventh a trapezoid. But Poffidonius makes a perfect divifion of right-lined quadrilateral figures into fo many members; for he eftablifhes feven fpecies of thefe; as likewife of triangles. But Euclid could not divide into parallelograms and non parallelograms, becaufe he neither mentions parallels, nor teaches us concerning the parallelogram itfelf. But trapeziums, and all trapezoids, he calls by a common

name,

name, defcribing trapeziums themfelves, according to the difference of thofe four figures *, in which the property of parallelograms is verified. And this is to have the oppofite fides and angles equal. For a quadrangle and an oblong, and a rhombus, have their oppofite fides and angles equal. But in a rhomboides he only adds this, *that its oppofite fides are equal,* left he fhould define it by negations alone, fince he neither calls it equilateral, nor rectangular. For where we want proper appellations, it is neceffary to ufe fuch as are common. But we fhould hear Euclid fhewing that this is common to all parallelograms. But a rhombus appears to be a quadrangle having its fides moved, nnd a rhomboides a moved oblong. Hence, according to fides, thefe do not differ from thofe; but they vary only according to the obtufenefs and acutenefs of angles; fince the quadrangle and the oblong are rectangular. For if you conceive a quadrangle or an oblong, having its fides drawn in fuch a manner, that while two of its oppofite angles are dilated, the other two are contracted; then the dilated angles will appear obtufe, and the contracted, acute. And the appellation of rhombus † feems to have been impofed from motion: For if you conceive a quadrangle moving after the manner of a rhombus, it will appear to you changed in order, according to its angles : juft as if a circle is moved after the manner of a fling, it will immediately exhibit the appearance of an ellipfis. But here you may perhaps enquire concerning the quadrangle, why it has this denomination ? and why the appellation of quadrangle may not be applied to other quadrilateral figures, as the name of triangle is common to all thofe which are neither equiangular nor equilateral, and in like manner of quinquangles or pentagons; for the geometrician, in thefe, adds only the particle *an equilateral triangle,* or *a quinquangle, which is equilateral and equiangular,* as if thefe could not be otherwife than fuch as they are? But when he mentions a quadrangle, he immediately indicates that it muft be equilateral and rectangular. But the reafon of this is as follows: a quadrangle alone has the beft fpace, both

* The Greek in this place is very erroneous, which I have reftored from the verfion of Barccius.

† For the Greek word ϸμβος is derived from the verb ϸμβω, which fignifies to have a circumvolute motion.

according to its fides and angles. For each of the latter is right, intercepting a meafure of angles, which neither receives intention nor remiffion. As it excels, therefore, in both refpects, it defervedly obtains a common appellation. But a triangle, though it may have equal fides, yet will in this cafe have all its angles acute, and a quinquangle all its angles obtufe. Since, therefore, of all quadrilateral figures, a quadrangle alone is replete with equality of fides, and rectitude of angles, it was not undefervedly allotted this appellation: for, to excellent forms, we often dedicate the name of the whole. But it appeared alfo to the Pythagoreans, that this property of quadrilateral figures, principally conveyed an image of a divine effence. For they particularly fignified by this, a pure and immaculate order. Since rectitude imitates inflexibility, but equality a firm and permanent power: for motion emanates from inequality, but quiet from equality itfelf. The gods, therefore, who are the authors to all things of ftable difpofition, of pure and uncontaminated order, and of indeclinable power, are defervedly manifefted as from an image, by a quadrangular figure. But, befides thefe, Philolaus alfo, according to another apprehenfion, calls a quadrangular angle, the angle of Rhea, Ceres and Vefta. For, fince a quadrangle conftitutes the earth, and is its proximate element, as we learn from Timæus, but the earth herfelf receives from all thefe divinities, genital feeds, and prolific powers, he does not unjuftly confecrate the angle of a quadrangle to thefe goddeffes, the beftowers of life. For fome call both the earth and Ceres, Vefta *, and they fay that Rhea totally participates her nature, and that all generative caufes are contained in her effence. Philolaus, therefore, fays that a quadrangular angle comprehends, by a certain terreftrial power, one union of the divine genera. But fome affimilate a quadrangle to univerfal virtue, fo far as every quadrangle from its perfection has four right angles. Juft as we fay that each of the virtues is perfect, content with itfelf, the meafure and bound of life, and the middle of every thing which, in morals, correfponds to the obtufe and acute. But it is by no means proper to conceal, that Philolaus attributes a triangular angle to four, but a quadrangular angle to three gods, ex-

* See the Orphic Hymns of Onomacritus to thefe deities; my tranflation of which I muft recommend to the Englifh reader, becaufe there is no other.

hibiting

hibiting their alternate tranſition, and the community of all things in
all, of odd natures in the even, and of even in the odd. Hence, the
tetradic ternary, and the triadic quaternary, participating of prolific and
efficacious goods, contain the whole ornament of generable natures,
and preſerve them in their proper ſtate. From which the duodenary,
or the number twelve, is excited to a ſingular unity, viz. the govern-
ment of Jupiter. For Philolaus ſays, that the angle of a dodecagon
(or twelve-ſided figure) belongs to Jove, ſo far as Jupiter contains and
preſerves, by his ſingular union, the whole number of the duodenary.
For alſo, according to Plato, Jupiter preſides over the duodenary *,
and governs and moderates the univerſe with abſolute ſway. And
thus much we have thought proper to diſcourſe concerning quadrila-
teral figures, as well declaring the ſenſe of our author, as likewiſe
affording an occaſion of more profound inſpections to ſuch as deſire
the knowledge of intelligible and occult eſſences.

DEFINITION XXXV.

PARALLEL RIGHT LINES are ſuch as being in the ſame
Plane, and produced both ways infinitely, will in no
part mutually coincide.

WHAT the elements of parallels are, and by what accidents in
theſe they may be known, we ſhall afterwards learn: but
what parallel right lines are, he defines in theſe words: " It is requiſite,
therefore (ſays he), that they ſhould be in one plane, and while they
are produced both ways have no co-incidence, but be extended in in-
finitum." For non-parallel lines alſo, if they are produced to a certain
diſtance, will not coincide. But to be produced infinitely, without
coincidence, expreſſes the property of parallels. Nor yet this abſo-
lutely, but to be extended both ways infinitely, and not coincide.

* Theſe twelve divinities, of which Jupiter is the head, are, Jupiter, Neptune, Vulcan,
Veſta, Minerva, Mars, Ceres, Juno, Diana, Mercury, Venus, and Apollo. The firſt triad of
theſe is demiurgic, the ſecond comprehends guardian deities, the third is vivific, or zoogonic,
and the fourth contains elevating gods. But, for a particular theological account of theſe divi-
nities, ſtudy Proclus on Plato's Theology, and you will find their nature unfolded, in page 403,
of that admirable work.

A a 2

For

For it is poffible that non-parallel lines may alfo be produced one way infinitely, but not the other; fince, verging in this part, they are far diftant from mutual coincidence in the other. But the reafon of this is, becaufe two right-lines cannot comprehend fpace; for if they verge to each other both ways, this cannot happen. Befides this, he very properly confiders the right-lines as fubfifting in the fame plane. For if the one fhould be in a fubject plane, but the other in one elevated, they will not mutually coincide according to every pofition, yet they are not on this account parallel. The plane, therefore, fhould be one, and they fhould be produced both ways infinitely, and not coincide in either part. For with thefe conditions, the right-lines will be parallel. And agreeable to this, Euclid defines parallel right-lines. But Poffidonius fays, parallel lines are fuch as neither incline nor diverge in one plane; but have all the perpendiculars equal which are drawn from the points of the one to the other. But fuch lines as make their perpendiculars always greater and lefs, will fome time or other coincide, becaufe they mutually verge to each other. For a perpendicular is capable of bounding the altitudes of fpaces, and the diftances of lines. On which account, when the perpendiculars are equal, the diftances of the right lines are alfo equal; but when they are greater and lefs, the diftance alfo becomes greater and lefs, and they mutually verge in thofe parts, in which the leffer perpendiculars are found. But it is requifite to know, that non-coincidence does not entirely form parallel lines. For the circumferences of concentric circles do not coincide: but it is likewife requifite that they fhould be infinitely produced. But this property is not only inherent in right, but alfo in other lines: for it is poffible to conceive fpirals defcribed in order about right lines, which if produced infinitely together with the right lines, will never coincide *. Geminus, therefore, makes a very proper divifion in this place, affirming from the beginning, that of lines fome are bounded, and contain figure, as the circle and ellipfis, likewife the ciffoid, and many others; but others are indeterminate, which may be produced infinitely, as the right-line, and the fection of a right-angled, and

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* For it is eafy to conceive a cylindric fpiral defcribed about a right-line, fo as to preferve an equal diftance from it in every part; and in this cafe the fpiral and right-line will never coincide though infinitely produced.

<div align="right">obtufe</div>

3

obtufe angled cone; likewife the conchoid itfelf.   But again, of thofe·
which may be produced in infinitum, fome comprehend no figure, as·
the right-line and the conic fections; but others, returning into them-·
lves, and forming figure, may afterwards be infinitely produced.·
1 of thefe fome will not hereafter coincide, which refift coincidence,
far foever they may be produced; but others are coincident, which
ne time or other coincide.   But of non-coincident lines, fome
.utually in one plane; and others not.   And of non-coincidents
.ubfifting in one plane, fome are always mutually diftant by an equal
interval; but others always diminifh the interval, as an hyperbola in
its inclination to a right-line, and likewife the conchoid *.   For thefe,
though

* As the choncoid is a curve but little known, I have fubjoined the following account of its
generation and principal property.   In any given right line A P, call P the pole, A the vertex,.
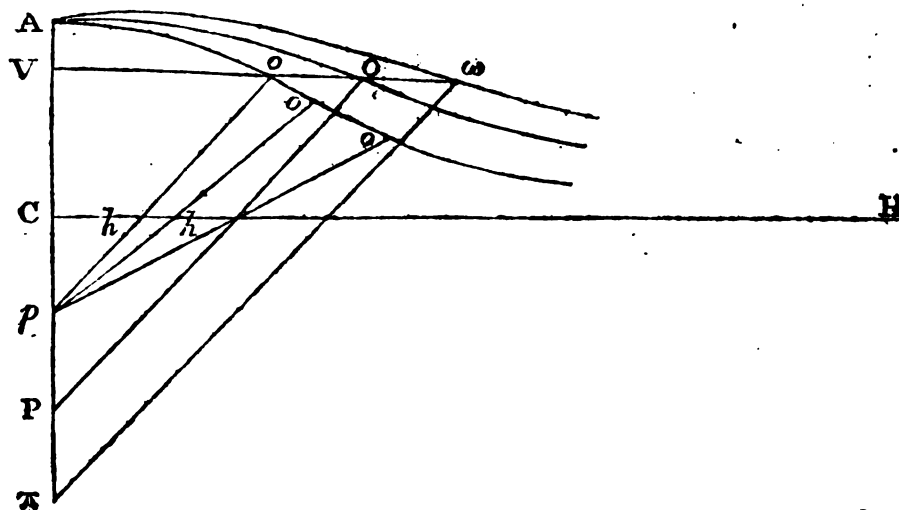

and any intermediate point C the centre of the choncoid: likewife, conceive an infinite right
line C H, which is called a rule, perpendicular to A P.   Then, if the right line A p continued
at p as much as is neceffary, is conceived to be fo turned about the abiding pole p, that the
point C may perpetually remain in the right line C H, the point A will defcribe the curve A o,
which the ancients called a conchoid.
    In this curve it is manifeft (on account of the right line P O, cutting the rule in H that
the point o will never arrive at rule C H; but becaufe b O is perpetually equal to C A,
and the angle of fection is continually more acute, the diftance of the point O from C H
will at length be lefs than any given diftance, and confequently the right line C H will be an
affymptote to the curve A O.

When

For it is poſſible that non-parallel lines may alſo '

infinitely, but not the other; ſince, vergi-

diſtant from mutual coincidence in '·'

is, becauſe two right-lines can-

to each other both wayf

properly conſiders r'

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And they

to each other ;

admirable, exhibiting

But the right-lines,

and which never dimi-

niſh

to C A, the choncoid A O deſcribed by the
and thoſe deſcribed from the poles p, and ϖ,
and theſe are either contracted or protracted.
the generative radius C A, which is called the

is greater or leſs than

of the conchoid, it may be eaſily inferred, that not only the exterior
with the right line C H, but this is likewiſe true of the
extending the right-line A ϖ, an infinite number of
between the exterior conchoid A ω, and the line C H, no one of
with the aſſymptote C H. And this paradoxical property of the
obſerved by any mathematician, is a legitimate conſequence of the
Not, indeed, that quantity admits of an actual diviſion in
and impoſſible; but it is endued with an unwearied capacity of
of being diffuſed into multitude, which can never be exhauſted. And
which it poſſeſſes ariſes from its participation of the indefinite duad; the
and innumerable multitude.
is not confined to the choncoid, but is found in the following
Conceive that the right line A C which is perpendicular to the indefinite line X Y, is
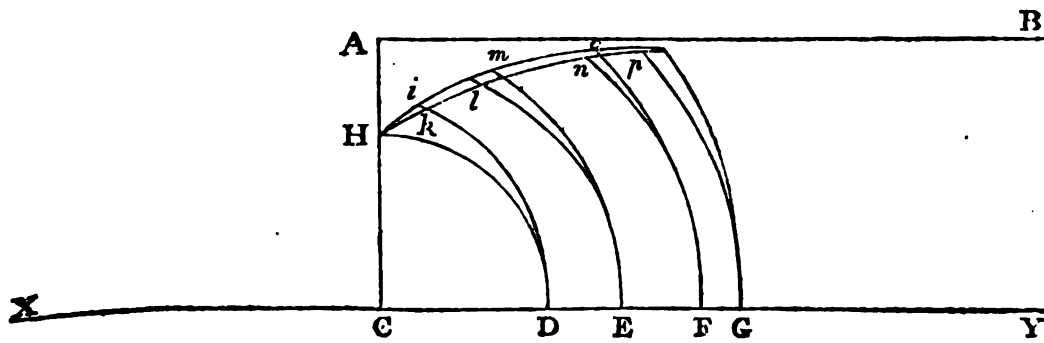
equal to the quadrantal arch H D, deſcribed from the centre C, with the radius C D: then
from the ſame centre C, with the ſeveral diſtances C E, C F, C G, deſcribe the arches E l,
F n, G p, each of which muſt be conceived equal to the firſt arch H D, and ſo on infinitely.
Now, if the points H, k, l, n, p, be joined, they will form a curve line, approaching conti-
nually nearer to the right-line A B (parallel to C Y) but never effecting a perfect coincidence.
This will be evident from conſidering that each of the ſines of the arches H D, l E, n F, &c.
being leſs than its reſpective arch, muſt alſo be leſs than the right-line A C, and conſequently
can never coincide with the right-line A B.

But

nifh the fpace placed between them in one plane, are parallel lines. And thus much we have extracted from the ftudies of the elegant Geminus, for the purpofe of explaining the prefent definition.*

But if other arches D *i*, E *m*, F *o*, &c. each of them equal to the right-line A C, and deſcribed from one centre, tangents to the former arches H D, *l* E, *n* F, &c. be fuppofed; it is evident that the points H, *i*, *m*, *o*, &c. being joined, will form a curve line, which fhall paſs beyond the foimer curve, and converge ſtill nearer to the line A B, without a poſſibility of ever becoming coincident: for fince the arches D *i*, E *m*, F *o*, &c. have leſs curvature than the former arches, but are equal to them in length, it is evident that they will be fubtended by longer fines, and yet can never touch the right-line A B. In like manner, if other tangent arches be drawn to the former, and fo on infinitely, with the fame conditions, an infinite number of curve-lines will be formed, each of them paſſing between H *p* and A B, and continually diverging from the latter, without a poſſibility of ever coinciding with the former. This curve, which I invented fome years fince, I fufpect to be a parabola; but I have not yet had opportunity to determine it with ccainty.

END OF THE FIRST VOLUME.